

Competitive equilibrium in exchange economies with indivisible goods

The first ideas of competitive equilibrium and its relation to optimality in economics date back to Adam Smith in 18th century. The first roots of its formalism can be traced in the works of L. Walras, F. Edgeworth and V. Pareto more than 100 years later. In the 20th century first rigorous proofs of the existence of equilibrium were given by K. Arrow and G. Debreu. However, the prevailing assumption was the infinite divisibility of all goods.

If the goods are indivisible, coming in discrete units, the needed mathematical tools are completely different. The general existence problem is NP-complete. In a special case where each agent owns and wants to hold only one unit of the good (this model is known as the 'housing market'), competitive equilibrium always exists and can be found by a very simple and intuitive Top Trading Cycles algorithm due to D. Gale. In this talk we show that already a very simple modifications of the basic housing market make the problem intractable. For the hard cases we study the parameterized complexity and approximability of the deficiency of the market.

Cake cutting. Computability of equitable divisions.

People, firms, organizations or nations often have to divide between them some scarce resource, it might be the birthday cake at a childrens' party, or the country's education budget between schools and universities, or the rights of nations in their use of the world's oceans. Which division is fair? How to arrive at a fair division? This talk is an introduction into the mathematical theory of fair division. We shall deal with an infinitely divisible resource called a cake, represented by unit interval of reals. We show how to model preferences of players over various pieces of cake, define several notions of fairness and demonstrate the relations between them. Finally, we shall present our own results concerning the existence and computability of equitable divisions. (For those who want to dive deeper into the topic, a page of problems is prepared.)