# Estimating C-CAPM and the Equity Premium over the Frequency Domain

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**Abstract**

In this paper we estimate the single-factor Consumption Capital Asset Pricing Model (C-CAPM) over the frequency domain. We modify the standard two-step methodology (Fama and French, 1992) to account for the spectral properties of consumption risk and we find that its lower frequencies explain up to 98% of the cross-sectional variation of expected returns and that the equity premium puzzle is eliminated. These results are robust to the definitions of the variables, the sample span and the set of portfolios utilized, and the maturity of interest rates.

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## 1. Introduction

A persistent puzzle in the macroeconomics and finance literature has been the failure of the Consumption Capital Asset Pricing model (C-CAPM), which measures risk by consumption beta, to explain empirically the differences in expected stock returns by the variation in the covariance of consumption and returns. However, several studies have emphasized that this puzzle is likely to emerge primarily at short horizons due for instance to habits, information delays, and transaction costs, and that short-run consumption should be replaced with a portfolio that exhibits higher correlations with long-run movements in consumption.[1] Singleton (1990) has first pointed out that the inclusion of consumption growth over longer time intervals improves the empirical fit of the single-good asset pricing model. Brainard et al. (1991) have shown that the performance of the C-CAPM improves as the horizon increases, a finding confirmed by Daniel and Marshall (1997) who have found that at lower frequencies aggregate returns and consumption growth are more correlated and the behavior of the equity premium becomes less puzzling. Parker (2001, 2003) and Parker and Julliard (2005) have allowed for long-term consumption dynamics by focusing on the ultimate risk to consumption, defined as the covariance between an asset's return during a quarter and consumption growth over the quarter of the return and several following quarters, and have found that it explains the cross-sectional variation in returns surprisingly well, but also show that the equity premium puzzle persists. Bansal and Yaron (2004), Bansal et al. (2005), and Hansen et al. (2008) show that when consumption risk is measured by the covariance between long-run cashflows from holding a security and long-run consumption growth in the economy, the differences in consumption risk provide useful information about the expected return differentials across assets.

In this paper we assess the explanatory power of consumption risk over the frequency domain by performing within the single-factor C-CAPM a dynamic analysis of consumption risk over the spectrum rather than over the time domain. As pointed out by Granger and Hatanaka back in 1964, according to the spectral representation theorem a time series can be seen as the sum of waves of different periodicity and, hence, there is no reason to believe that economic variables should present the same lead/lag cross-correlation at all frequencies. We incorporate this rationale into the context of the single-factor C-CAPM by using standard techniques to

[1] See, among others, Mankiw and Shapiro (1986), Grossman et al. (1987), Wheatley (1988), and Breeden et al. (1989). Mehra (2003) and Cochrane (2005) provide extensive surveys of the relevant literature.

estimate the coherency (the analog of the correlation coefficient in the time domain) and the gain (the analog of the regression coefficient) between consumption risk over the frequency domain and returns.

To attain our goal we modify the standard two-step Fama and French (1992) estimation procedure typically employed in the literature, which involves regressing the portfolio return on consumption growth and using the estimated coefficients in a cross-section regression, in the following way. We employ a spectral decomposition of the series at hand and we then use these estimates in the cross-section equation to obtain the equity premium for each frequency.[2] Assessing the portfolio risk of consumption over the whole frequency domain enables us to separate different layers of dynamic behavior within the standard C-CAPM by distinguishing between the short run (fluctuations of 2 to 6 quarters), the medium run or business cycle (lasting from 8 to 32 quarters), and the long run (oscillations of duration above 32 quarters). If consumption risk is a more persistent process than suggested by the conventional analysis, identifying the impact of lower frequencies of consumption risk can alter the implied long-run riskiness in ways that are empirically important and cannot be addressed by standard time-domain techniques, which aggregate over the entire frequency band and are not robust when frequency variations are large. Our approach can thus circumvent several caveats associated with unmodeled frictions, time aggregation or measurement error in consumption data, which are often found to account for the short-run predictability of the pricing errors.

Our findings indicate that at high frequencies of consumption risk the evidence coincides with those reported by the existing literature: consumption risk does not explain satisfactorily the variation in returns. However, when lower frequencies of consumption risk are examined and thus the horizon of consumption growth increases (eventually reaching infinity) consumption risk can explain up to 98% of the cross-sectional variation of expected returns and the equity premium puzzle is eliminated. These findings are robust to the definitions of the variables, the sample span and the set of portfolios utilized. Moreover, given the importance of long-run consumption risk for the dynamics of the C-CAPM, we address the impact of long-term risk-free rates within this spectral approach and we find that the model preserves its significance for low frequencies of consumption risk.

We are thus able to provide additional insights into the relationship between returns and

[2]See sections 2 and 3 for more details on this procedure.

long-term consumption dynamics by confirming that consumption risk can provide useful information for the variation of excess returns when examined over the frequency domain. In this respect, we further highlight the importance of long-run consumption risk by explaining a larger share of cross-sectional variation of expected returns. It is worth noting that the spectral estimation of consumption-based models has also been considered by Berkowitz (2001) and Cogley (2001). Berkowitz (2001) has proposed a one-step Generalized Spectral estimation technique for estimating parameters of a wide class of dynamic rational expectations models in the frequency domain. By applying his method to the C-CAPM he finds that when the focus is oriented towards lower frequencies, risk aversion attains more plausible values at the cost of a risk-free rate puzzle generated by low estimates of the discount factor. Cogley (2001) decomposes approximation errors over the frequency domain from a variety of stochastic discount factor models and finds that their fit improves at low frequencies, but only for high degrees of calibrated risk aversion. In this paper we show how low frequencies of consumption risk can be incorporated in the standard Fama and French (1992) two-step estimation methodology in an easily implementable way, which addresses satisfactorily the equity premium puzzle and the cross-sectional variation of returns.

The rest of the paper is organized as follows. Section 2 presents the modified version of long-term consumption risk within the C-CAPM in the context of spectral analysis and outlines the empirical methodology. Section 3 presents the data and section 4 discusses the empirical results. Section 5 presents some robustness tests and section 6 investigates the impact of long-term risk-free rates. Section 7 concludes the paper.

## 2. Expected returns and the risk to consumption over the frequency domain

The standard C-CAPM assumes that the representative household maximizes the expected present discounted value of utility flows from consumption by allocating wealth to consumption and different investment opportunities. At the optimal allocation a marginal investment at time $t$ in any asset should yield the same expected marginal increase in utility at $t+1$, which for the constant relative risk aversion utility function implies that:

$$E_t[C_{t+1}^{-\gamma} R_{j,t+1}] = E_t[C_{t+1}^{-\gamma}] R_{t,t+1}^{f} \tag{1}$$

where $C_{t+1}$ is consumption at $t+1$, $R_{j,t+1}$ is the gross real return on portfolio $j$ of stocks unknown at $t$ and known at $t+1$, $R^f_{t,t+1}$ is the gross real return on a risk-free asset between $t$ and $t+1$, and $\gamma$ is the representative household's constant coefficient of relative risk aversion. Equation (1) can be written as a model of average cross sectional returns by manipulating it to a beta representation or factor model, in which the expectation of the equity premium, $E[R^e_{j,t+1}] = E[R_{j,t+1} - R^f_{t,t+1}]$, is given in terms of covariances by:

$$E[R^e_{j,t+1}] = \alpha_0 + \beta_{j,0}\lambda_0 \tag{2}$$

where $\alpha_0 = 0$, $\beta_{j,0} = \frac{Cov[\Delta \ln C_{t+1}, R^e_{j,t+1}]}{Var[\Delta \ln C_{t+1}]}$, $\lambda_0 = \frac{\gamma Var[\Delta \ln C_{t+1}]}{E[1-\gamma \Delta \ln C_{t+1}]}$. Equation (2) provides an external test of the structure embodied in the model with consumption growth, $\Delta \ln C_{t+1}$, being the stochastic discount factor that prices returns. The estimated $\alpha_0$ should be equal to zero and the expected excess return on a portfolio is equal to the scaled consumption risk of the portfolio, $\beta_{j,0}\lambda_0$. Equations (1) to (2) evaluate the risk of a portfolio based solely on its covariance with contemporaneous consumption growth. They maintain the assumption that the intertemporal allocation of consumption is optimal from the perspective of the textbook model of consumption smoothing, so that any change in marginal utility is reflected instantly and completely in consumption.

Parker (2001, 2003) and Parker and Julliard (2005) have allowed for the slow response of consumption to market returns and have evaluated the risk/return trade-off among portfolios of stocks by focusing on the ultimate consumption risk measured by the covariance of the return at $t+1$ and the change in consumption from $t$ to $t+1+s$, where $s$ is the horizon over which the consumption response is studied:

$$Cov[\ln\left(\frac{C_{t+1+s}}{C_t}\right), R^e_{j,t+1}] \tag{3}$$

In terms of beta representation we have:

$$E[R^e_{j,t+1}] = \alpha_s + \beta_{j,s}\lambda_s \tag{4}$$

where $\alpha_s = 0$, $\beta_{j,s} = \frac{Cov[\ln\left(\frac{C_{t+1+s}}{C_t}\right), R^e_{j,t+1}]}{Var[\ln\left(\frac{C_{t+1+s}}{C_t}\right)]}$, $\lambda_s = \frac{\gamma_s Var[\ln\left(\frac{C_{t+1+s}}{C_t}\right)]}{E[1-\gamma_s \ln\left(\frac{C_{t+1+s}}{C_t}\right)]}$. When $S=0$, equation (4) yields the standard beta representation (2). Equation (4) shows a modification of the standard

C-CAPM over the time domain. In their empirical results, Parker and Julliard (2005) find a model improvement as the horizon increases accompanied by lower estimates of the risk-free rate, but do not report results beyond 15 quarters as the trade-off between a larger horizon and optimal inference leads to a choice of 11 quarters as the preferred specification.

Clearly by varying the horizon, $S$, consumption risk takes a range of values from the short-run to the long-run along with the corresponding asset pricing implications of these risks. Defining $\ln(C_{t+1+s}/C_t) \equiv \Delta^s \ln C_t$, it is straightforward to show that we can obtain $\Delta^s \ln C_t$ from $\Delta \ln C_t$ through the transformation $H(L) = (1 + L + L^2 + ... + L^s)$, where $L$ is the lag operator. The spectrum of $\Delta^s \ln C_t$, $f_{\Delta^s \ln C_t}$, is then linked with the one of $\Delta \ln C_t$ by $f_{\Delta^s \ln C_t} = H(e^{-i\omega})H(e^{i\omega})f_{\Delta \ln C_t}$, where the frequency $\omega$ is a real variable in the range $0 \preceq \omega \preceq \pi$; for example, for $s = 2$, $f_{\Delta^s \ln C_t} = (2 + 2\cos\omega)f_{\Delta \ln C_t}$. For $\omega = 0$, the variance of $\Delta^2 \ln C_t$ is 4 times the variance of $\Delta \ln C_t$, while the respective variance for $\omega = \pi$ is eliminated. This transformation strengthens lower frequencies (long-run) and attenuates the impacts of the higher ones (short-run).

Now, we can measure consumption growth at any frequency, $\omega$, and in turn calculate a measure of consumption risk over the frequency domain given the covariance between consumption growth at a given frequency, $\Delta \ln C_t(\omega)$, and the excess return, $R^e_{j,t}$. In this respect, after dropping the time subscript for notational simplicity, the beta-form representation given in (2) and (4) can be modified to its frequency domain counterpart that yields the response of excess returns to consumption risk over the whole band of frequencies, $\omega$, as follows:

$$E[R^e_j] = \alpha_\omega + \beta_{j,\omega}\lambda_\omega \tag{5}$$

where $\alpha_\omega = 0$ and $\lambda_\omega$ is the price of risk at each frequency $\omega$. The beta coefficient, $\beta_{j,\omega}$, is the *gain* between returns and consumption growth denoted by $G_{R^e_j,\Delta \ln C}(\omega)$ and defined as the ratio of the cross-spectrum of the series at hand over the spectrum of consumption growth at a given frequency:

$$G_{R^e_j,\Delta \ln C}(\omega) \equiv \frac{\left| f_{R^e_j,\Delta \ln C}(\omega) \right|}{f_{\Delta \ln C,\Delta \ln C}(\omega)} \tag{6}$$

The *gain* provides us with a scalar measure of the amplitude of the relationship between the

$\omega-$frequency component of $R_j^e$ on the corresponding component of $\Delta \ln C$ since it is a fact of cross-spectral densities that the covariance between consumption growth in a given frequency and returns at any frequency is the same as the covariance between consumption growth at a given frequency and returns on the same frequency.[3]

Estimation of (2) is typically performed in the literature within a two-step approach (Fama and French, 1992). The first step involves a time series regression of the return of the $j$ portfolio, $R_{j,t+1}^e$, onto a constant and consumption growth, $\Delta \ln C_{t+1}$, in order to obtain an estimate of the slope coefficient $\beta_{j,0}$. As a second step, the estimated coefficients are employed in the cross-section regression (2) in order to get the estimate of the price of risk, $\lambda_0$.[4] By employing excess returns, we can test whether our model contains an equity premium by simply testing the significance of the constant. The adjusted $R^2$ of this equation measures the fraction of the cross-sectional variation explained by the data.

Our methodology differs from the one described above only in the first step, i.e. in the way betas are obtained. Specifically, employing the spectral decomposition of the series (described in detail below), we calculate the gain between each portfolio's excess return and consumption growth for each frequency as the ratio between the co-spectra of the series and the spectrum of consumption growth as given by formula (6).[5] The estimated gains/betas, $\beta_{j,\omega}$, for each portfolio $j$, are then employed as regressors in the following cross-section equation:

$$E[R_j^e] = \widehat{\alpha}_\omega + \beta_{j,\omega}\widehat{\lambda}_\omega$$

which yields a testable form of the C-CAPM in the frequency domain. Our handling of this equation is the same as for any cross-sectional regression. We obtain the estimates $\widehat{\alpha}_\omega, \widehat{\lambda}_\omega$, by means of ordinary least squares estimation correcting for possible heteroscedasticity by employing Newey-West standard errors. The adjusted $R^2$ of the equation gives as a measure of success

[3]Notice that once the price of risk, $\lambda_\omega$, is estimated from the cross-section regression (5), we could calculate an implied coefficient of ('pseudo') relative risk aversion at each frequency as $\gamma_\omega = \frac{\lambda_\omega}{E[\Delta \ln C]\lambda_\omega + f_{\Delta \ln C, \Delta \ln C}(\omega)]}$. We would refer to $\gamma_\omega$ as 'pseudo' relative risk aversion because the integral of $\gamma_\omega$ over the whole band of frequencies is not equal to its time domain counterpart due to the non-additiveness property of the above formula. In fact, the definition of preferences over frequency components of a stochastic process is not as straightforward as it might seem; see Otrok (2001) on spectral welfare cost functions. Furthermore, the calculation of $\gamma_\omega$ involves the mean of consumption growth over the time domain in the denominator while the remaining elements of the formula are related to a specific frequency.

[4]Alternatively, the Fama and MacBeth (1973) methodology can be employed.

[5]The demeaned series are used before any spectral measure is estimated, as spectral analysis pertains to stationary zero-mean processes.

of the frequency domain C-CAPM to explain the cross-sectional variation of returns and, in turn, the statistical significance of the intecept, $\widehat{\alpha}_{\omega}$, is directly related to the existence of an equity premium.

## 3. Data and spectral properties

For our portfolios and returns series we use quarterly returns on the 25 Fama and French portfolios, which are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (B/M). B/M used during a fiscal year is based on the book equity for the previous fiscal year divided by ME for December of the previous year. The B/M breakpoints are the NYSE quintiles. The portfolios include all NYSE, AMEX, and NASDAQ stocks for which there is market equity data for December and June of the previous fiscal year, and (positive) book equity data for the previous fiscal year. The series are available on a monthly basis and excess returns are constructed by subtracting the three-month Treasury Bill rate, which proxies the risk-free rate. To match consumption data we use a quarterly frequency and set our timing convention so that $R_{j,t+1}$ represents the return on portfolio $j$ during the quarter $t+1$. We measure consumption as personal consumption expenditures on nondurable goods from the National Income and Product Accounts. We make the 'end-of-period' timing assumption that consumption during quarter $t$ takes place at the end of the quarter. The data are made real using a chain weighted price deflator, spliced across periods, produced by the Bureau of Economic Analysis. These series determine the sample, which covers the second quarter of 1947 to the last quarter of 2001, and the frequency (quarterly) utilized.[6]

Before moving on with the estimation methodology, we report some evidence on the co-movement between returns and consumption growth in the frequency domain. The spectra and co-spectra of a vector of time-series for a sample of $T$ observations can be estimated for a set of frequencies $\omega_n = 2\pi n/T$, $n = 1, 2, ..., T/2$. The relevant quantities are estimated through the periodogram, which is based on a representation of the observed time-series as a superposition of sinusoidal waves of various frequencies; a frequency of $\pi$ corresponds to a time period of two quarters, while a zero frequency corresponds to infinity. However, the estimated periodogram

[6] We obtained the Fama and French portfolio data from Kenneth French's web page (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The rest of the data were obtained from Jonathan Parker's web page (http://www.princeton.edu/~jparker/research/crisk.html); see Parker and Julliard (2005) for a more detailed description of the dataset.

is an unbiased but inconsistent estimator of the spectrum because the number of parameters estimated increases at the same rate as the sample size. Consistent estimates of the spectral matrix can be obtained by either smoothing the periodogram, or by employing a lag window approach that both weighs and limits the autocovariances and cross-covariances used.[7] We use here the Bartlett window that assigns linearly decreasing weights to the autocovariances and cross-covariances in the neighborhood of the frequencies considered and zero weight thereafter.[8]

Figures 1A and 1B depict the spectra of the demeaned series under scrutiny (along with 95% confidence intervals) and can be interpreted as the variance decompositions over various frequency bands (stated as a fraction of $\pi$).[9] As can be readily observed, the variability of excess returns does not exhibit substantial changes over the frequency domain.[10] On the other hand, the variability of non-durables consumption is muted for 2 to 32 quarters; however, for horizons exceeding 32 quarters a steep increase is prevalent. As $t$ approaches infinity, the variance of consumption is seven times greater than its 32-quarter value and 52 times greater than its short-run value. The concentration of variance in low frequencies is an indication of short-term correlation in consumption growth, such as an AR(1) process with a positive coefficient, rather than an indication of non-stationarity of the process, which can be ruled out for the series at hand.[11]

For expositional purposes, we also employ as a measure of comovement between returns and consumption risk over the frequency domain, the well-known squared *coherency*, $c^2_{R^e_j,\Delta\ln C}(\omega)$, defined here as:

$$c^2_{R^e_j,\Delta\ln C}(\omega) \equiv \frac{\left| f_{R^e_j,\Delta\ln C}(\omega) \right|^2}{f_{\Delta\ln C,\Delta\ln C}(\omega) f_{R^e_j,R^e_j}(\omega)} = \frac{C^2_{R^e_j,\Delta\ln C} + Q^2_{R^e_j,\Delta\ln C}}{f_{\Delta\ln C,\Delta\ln C}(\omega) f_{R^e_j,R^e_j}(\omega)} \qquad (7)$$

where $0 \leq c_{R^e_j,\Delta\ln C}(\omega) \leq 1$, $f_{R^e_j,\Delta\ln C}(\omega) = C_{R^e_j,\Delta\ln C}(\omega) - iQ_{R^e_j,\Delta\ln C}(\omega)$ is the cross-spectrum

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[7]For example, the spectrum of $x_t$ is estimated by $f_{xx}(\omega) = \frac{1}{2\pi} \sum_{k=-(T-1)}^{T-1} w(k)\widehat{\rho_k} e^{-ik\omega}$, where the kernel, $w(k)$, is a series of lag windows.

[8]The Bartlett window has the following form: $\lambda(s) = \{ \begin{array}{c} 1 - |s|/k, |s| \preceq k \\ 0, |s| \succ k \end{array}$ while the bandwidth, $k$, is set using the rule $k = 2\sqrt{T}$, as suggested by Chatfield (1989), where $T$ is the sample size.

[9]Confidence intervals were derived based on a normal approximation of the spectra of the series; see Priestley (1981) for a detailed description.

[10]The pattern of the spectra of excess returns rules out the possibility of non-stationarity of the data. For instance, if excess returns contained a unit root the spectral density of the series would tend to infinity as the frequency tends to zero.

[11]See Campbell (2003, section 3.2) and the references cited therein for some evidence on the properties of US consumption growth.

between any two variables, which is complex-valued and therefore can be decomposed into its real and imaginary components, $C_{R_j^e,\Delta \ln C}(\omega)$ termed the *co-spectrum*, and $Q_{R_j^e,\Delta \ln C}(\omega)$ termed the *quadrature spectrum,* respectively.[12] Intuitively, coherency provides a measure of the correlation between the market excess return and non-durables consumption growth at each frequency and can be interpreted as the frequency domain analog of the correlation coefficient.

Figure 1C presents the coherency (along with 95% confidence intervals) between the two series. This analysis has been undertaken for every portfolio but to save space we report only the results for the aggregate market return. Overall our estimates suggest that the correlation (measured by coherency) between returns and consumption growth exhibits an upward trend as we move from high to low frequencies. Specifically, as regards the short-run correlation for frequencies between $\pi$ and $7\pi/8$ corresponding to around 2 quarters, coherency fluctuates around 20%. Then it plunges to around 5% and steadily increases to reach a local peak of 60% at frequencies corresponding to 3-4 quarters. Two more cycles are observable with peaks at 6 and 16 quarters. The maximum is reached at zero frequency, i.e. for an infinite horizon. In this case, the coherency between the series at hand is estimated at 79%. On the whole, the short-run correlation between returns and consumption growth is low, the business-cycle correlation amounts on average to roughly 50%, while the long-run correlation exceeds 70%.

## 4. Empirical findings

This section asks whether consumption risk explains the cross-sectional variation in expected returns for various frequencies. In particular, the questions we seek to answer are the following. First, does consumption risk at various frequencies explain a large share of variation of average returns? Second, does the estimate of $\alpha_\omega$ corroborate the existence of an equity premium?

To allow for comparisons with the rest of the literature, in this section we take the standard route and we estimate the model by employing non-durables consumption and gross excess returns from the 25 Fama-French portfolios. As a first step, we estimate model (5) by imposing the coefficient restriction $\alpha_\omega = 0$. Column (1) of Table 1 reports the estimation results for a range of frequencies corresponding from 2 quarters to infinity. The first row reports the evidence for the highest frequency considered (which corresponds to two quarters in the time domain). Our results suggest that at this frequency consumption risk does not explain the variation in returns

[12] See Hamilton (1994) for a general overview of spectral analysis.

and are in line with those typically reported in the literature on the C-CAPM. As we move to lower frequencies (and consequently increase the time horizon) consumption risk still fails to explain a larger share of the cross-sectional variation. When even lower frequencies are taken into account the performance of the model improves substantially. For the 32-quarter horizon, consumption risk is positive and significant and explains 66% of the cross-sectional variation of the returns. The performance of the C-CAPM is further improved at zero frequency (infinite horizon). The model succeeds in explaining 98.1% of the cross-sectional variation of returns.

Next, we assess model (5) by estimating $\alpha_\omega$ rather than imposing $\alpha_\omega = 0$. In this respect, we separately evaluate the ability of the model to explain the equity premium and the cross section of expected stock returns, and we are able to measure the extent to which the model addresses the equity premium puzzle. Columns (2) to (4) of Table 1 report the estimation results. The evidence suggests that at a high frequency consumption risk does not explain variation in returns and is associated with a significant equity premium of the magnitude of 2.3% per quarter. This poor performance of contemporaneous consumption risk is also depicted in the upper left panel of Figure 2, which plots the consumption betas (gains) and the average realized returns along with the second-stage regression line associated with this frequency. The overall picture indicates an almost flat relationship between consumption risk and returns at this frequency. Figure 3 plots in turn the predicted and average returns of the portfolios. The horizontal distance between a portfolio and the 45-degree line is the extent to which the expected return based on fitted consumption risk (on the vertical axis) differs from the observed average return (on the horizontal axis). As expected, at the 2-quarter horizon there is almost no relation between predicted and realized returns. When we move to lower frequencies consumption risk explains a larger share of the cross-sectional variation, reaching 12% for the 8-quarter horizon. However, the implied premium remains large and significant. This picture is also depicted in the regression line in the upper right part of Figure 3.

As lower frequencies are further considered the performance of the model improves substantially. For the 32-quarter horizon, consumption risk is positive and significant, and explains 66% of the cross-sectional variation of the returns. These findings are depicted in the lower left panel of Figures 2 and 3. The regression line is positive, quite steep and suggests a strong relationship between betas and returns. As expected, the deviation between fitted and realized returns is sufficiently reduced. Associated with this horizon is an insignificant equity premium of -0.3%. The performance of the C-CAPM is further improved at zero frequency (infinite horizon). The model succeeds in explaining 98.6% of the cross-sectional variation of returns coupled with a negligible, though significant, pricing error of -0.2%.[13] These features are also illustrated in the lower right part of Figures 2 and 3, in which the average realized and fitted returns are almost perfectly aligned on the regression line and the 45-degree line, respectively.

To sum up, we find that when higher frequencies of consumption risk are considered the results replicate the typical findings of the literature, i.e. the C-CAPM fails to explain the differences in expected stock returns by the variation in the covariance of consumption and returns. In contrast, as lower frequencies of consumption risk are taken into account, consumption risk explains almost entirely the cross-sectional variation of expected returns and the equity premium puzzle is eliminated.

## 5. Robustness tests

In this section we present some sensitivity tests on the relationship between consumption risk and the expected returns over the frequency domain. We first consider the impact of alternative specifications by using a smaller sample size as well as alternative definitions of returns and consumption, and subsequently we examine the impact of alternative portfolios on our results.

### *5.1. Alternative specifications*

Some studies (including, among others, Fama and French, 1992, 1993, and Lettau and Ludvigson, 2001) have used a shorter time period than the one analyzed in our baseline results. To allow for comparisons, Panel A of Table 2 shows the results of estimating our model on a sample of returns that starts in the third quarter of 1963. In this sub-period, the pattern of coefficients and the fit tell a similar story, except that low-frequency consumption risk does even better at explaining expected returns. Around 67% and almost 100% of the variation in expected returns is explained by consumption risk over the 32-quarter and infinite horizons. Similar to the baseline specification, the fitted model understates the average return on all portfolios by 0.5% and 0.2%. The fit of the model for the infinite horizon is depicted on the upper

[13] Further robustness tests (see sections 5 and 6) show that the significance of the pricing errors is eliminated at the zero frequency.

left part of Figure 4. Second, we measure consumption risk using total consumption instead of non-durables consumption. Ait-Sahalia et al. (2004) have argued that the consumption risk of equity is understated by NIPA nondurable goods because it contains many necessities and few luxury goods. As pointed out by the authors, consumers have more discretion over their consumption of luxury goods than essential goods, and consumption of the former is found to covary more strongly with stock returns.[14] Panel B of Table 2 shows that using total consumption risk in place of nondurable consumption risk leads to a slightly different picture. Long-run total consumption risk fits the cross-section of expected returns somewhat better than non-durables consumption. The upper right part of Figure 4 plots the performance of the specification with total consumption. Finally, we use consumption risk over the frequency domain to price long-horizon returns. Long-horizon returns are calculated as cumulative returns over the next 11 quarters.[15] Panel C of Table 2 shows some improvements of our model for shorter horizons compared to the baseline specification. Specifically, for an 8-quarter horizon, the model succeeds in explaining almost half the cross-sectional variation of returns. As we move to lower frequencies, and specifically to the 32-quarter horizon the explanatory power of the model is lower than the baseline specification (42.2% as opposed to 65.5%). This specification yields similar findings to the baseline specification for the infinite horizon and its performance is depicted at the bottom part of Figure 4.

### *5.2. Other portfolios*

The C-CAPM as any asset pricing model should be able to explain expected returns on any set of portfolios. So far, the portfolios considered are the double-sorted 25 Fama-French B/M and ME value-weighted portfolios, which basically aim at capturing the value and size premia. We consider here alternative portfolios sorted on both firm characteristics and overall economic factors or systematic risk factors in order to check whether consumption risk over the frequency domain succeeds in explaining risk premia generated by these portfolios.

As a first step, we consider a slightly different set of returns, namely the 25 equal-weighted Fama-French portfolios that are also examined by Parker and Julliard (2005). In line with

[14]See also Parker (2001). The usual concern when total consumption is used is that it contains the flow of expenditures on durable goods instead of the -theoretically desired- stock of durable goods. However, expenditures and stocks are cointegrated and, hence, the long-term movement in expenditures following an innovation to equity returns also measures the long-term movement in consumption flows.

[15]For comparison purposes, the choice of the horizon is the one that corresponds to the selected model of Parker and Juliard (2005).

these authors, low-frequency consumption risk does an even better job of explaining the cross-sectional pattern of expected returns for these portfolios (see Panel A of Table 3). A slightly increased proportion of the variation in expected returns is explained; the fit of the model for the infinite horizon is depicted on the upper left part of Figure 5. Second, we consider a set of single sorted portfolios, namely the 10 size (ME), 10 book to market (B/M) and 10 dividend yield (D/P) portfolios of Fama and French. These portfolios sort firms on the basis of their characteristics that lead to cross-sectional dispersion in measured risk premia and are behind the factor models of Fama and French (1993).[16] This set of portfolios aims at disentangling the value and size premia. To the extent that the C-CAPM holds, we expect to find growth firms to have less exposure to consumption risk than value firms and smaller firms to be exposed to higher consumption risk when compared to larger firms; see also Jagganathan and Wang (2005) and Cochrane (2005). Our results (reported in Panel B of Table 3) are in line with those of our baseline specification. At a high frequency, the C-CAPM explains 13% of the cross-sectional variation in expected returns, but the fit of the model improves with the frequency decline. At the 32-quarter horizon, half of the variation is explained, while at an infinite horizon, the respective figure is 93.9%. The upper right part of Figure 5 plots the actual and the predicted returns for this set of portfolios. Third, we use the 20 risk-sorted portfolios employed by Campbell and Vuolteenaho (2004).[17] The authors follow Daniel and Titman's (1997) point that sorting only on firm characteristics could generate a spurious link between premia and risk measures, and sort common stocks into 20 portfolios according to their past loadings with state variables that are useful in predicting the aggregate market return.[18] The purpose of their strategy is to generate portfolios with a large spread in these loadings and

[16]The 10 size value-weighted portfolios are formed on the basis of market capitalization and include all NYSE, AMEX, and NASDAQ stocks in the CRSP database which are ranked at the end of June of each year using NYSE capitalization breakpoints. The 10 B/M portfolios are formed at the end of each June using NYSE breakpoints. The BE used in June of year $t$ is the book equity for the last fiscal year ending in $t$-*1* and ME is price times shares outstanding at the end of December of $t$-*1*. The 10 D/P portfolios include all NYSE, AMEX, and NASDAQ stocks for which ME for June of year $t$, and at least 7 monthly returns (to compute the dividend yield) from July of $t$-*1* to June of $t$ are available. Portfolios are formed on D/P at the end of each June using NYSE breakpoints. The dividend yield used to form portfolios in June of year $t$ is the total dividends paid from July of $t$-*1* to June of $t$ per dollar of equity in June of $t$. The returns on these portfolios are taken from Kenneth French's web site, where more details on their construction can be found.

[17]These portfolios are available at http://post.economics.harvard.edu/faculty/vuolteenaho/papers.html.

[18]These state variables include the excess log return on the market, the term yield spread (computed as the difference between ten-year and short-term bonds) and the small stock-value spread (computed as the difference between the log(B/M) of the small high B/M portfolio and the small low B/M portfolio). More details can be found in the Appendix of Campbell and Vuolteenaho (2004), which is available at http://kuznets.fas.harvard.edu/~campbell/papers.html.

thus overcome Daniel and Titman's (1997) problem. Panel C of Table 3 reports our results for this set of portfolios. Interestingly, the C-CAPM fails in at least one of its aspects for all the frequencies under consideration with the exception of the infinite horizon. For this horizon, 82% of the cross-sectional variation of the returns is explained. Figure 5 (bottom part), which plots realized returns versus predicted returns, shows that the spread in returns across portfolios is lower than the one generated by the portfolios considered so far explaining the somewhat worse performance of this model. Fourth, we consider 34 industry-sorted portfolios, which have posed a particularly challenging feature from the perspective of systematic risk measurement (see Fama and French, 1997). Value-weighted industry portfolios are formed by sorting all NYSE, AMEX, and NASDAQ stocks by their CRSP four-digit SIC Code at the end of June of each year.[19] Similar to the previous set of portfolios, our findings suggest that systematic industry-specific risk is priced only for the infinite horizon (see Panel D of Table 3).

## 6. Long-run risk-free rates and estimates of C-CAPM over the frequency domain

The previous sections have established that the low frequencies of consumption risk, which are associated with the long-run pattern of C-CAPM, improve the empirical fit of the model. An extension of this approach envisages the impact of risk-free rates of longer maturity, which are likely to embed useful information when the horizon of consumption risk widens. Intuitively, if long-term interest rates are negatively related to consumption growth, they provide a hedge against bad states and individuals will sell short-term bonds and buy long-term bonds to receive payoffs when their consumption level is expected to be lower, thus resulting in a falling or negative term structure. On the flip side, if long-run rates earn a low return when consumption growth is negative, holding long-term bonds exacerbates consumption risk resulting in a rising term premium.[20]

To assess the impact of long-run risk-free rates and consumption risk over the frequency domain, we follow Parker and Juliard (2003) and develop a variant of the model presented in section 2 that incorporates risk-free rates of longer maturity in the C-CAPM. In particular, the

[19] The industry definitions are available at Kenneth French's web site. We include in our analysis the portfolios for which we have returns for the whole sample period.

[20] Estrella and Mishkin (1996) have found that inverted yield curves can be leading indicators of recessions and hence of reduced consumption growth rates. The empirical implications of long-run risk-free rates (and the associated term structure) for the C-CAPM have been investigated by several studies including, among others, Harvey (1988, 1989, 1991, 1993), Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Kamara (1997), Roma and Torous (1997), and Hamilton and Kim (2002). The general empirical consensus from these studies is that the slope of the term spread is positively associated with future economic activity.

solution of the investor optimization problem implies that:

$$E_{t+1}[\frac{1}{1+\rho}(1+R^f_{t+1,t+1+s})\frac{u'(C_{t+1+s})}{u'(C_{t+1})}]=1 \tag{8}$$

where $\rho$ is the rate of time preference and $R^f_{t+1,t+1+s}$ is the risk-free rate with $s$-periods ahead maturity. In turn, we can re-write the Euler equation (1) as:

$$E_t[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}R_{j,t+1}]=E_t[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]R^f_{t,t+1} \tag{9}$$

Assuming that $R^f_{t+1,t+1+s}$ is orthogonal to $R^j_{t,t+1}$, we can get the following beta representation for the excess return of portfolio $j$**:**

$$E[R^e_{j,t+1}]=\alpha^s+\beta^s_j\lambda^s \tag{10}$$

where

$$\alpha^s=\frac{Cov[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)},R^f_{t,t+1}]}{E[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]},\ \beta^s_j=\frac{Cov[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)},R^e_{j,t+1}]}{Var[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]},$$

$$\text{and } \lambda^s=-\frac{Var[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]}{E[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]}$$

Equation (10) renders an alternative specification to (2) and shows how risk-free rates of longer maturity affect the single factor C-CAPM with the interaction of the long-term risk-free rate scaling consumption growth over the corresponding period. In turn, defining $R^f_{t+1,t+1+s}\equiv R^f_{s,t}$ for notational simplicity and adopting the standard constant relative risk aversion parametrization, which implies that $\frac{u'(C_{t+1+s})}{u'(C_t)}\simeq 1-\gamma^s\Delta^s\ln C_t$, we get that:

$$\alpha^s=\frac{Cov[R^f_{s,t}(1-\gamma^s\Delta^s\ln C_t),R^f_{t,t+1}]}{E[R^f_{s,t}(1-\gamma^s\Delta^s\ln C_t)]},\ \beta^s_j=\frac{Cov[R^f_{s,t}\Delta^s\ln C_t,R^e_{j,t+1}]}{Var[R^f_{s,t}\Delta^s\ln C_t]},$$

$$\text{and } \lambda^s=\frac{\gamma^s Var[R^f_{s,t}\Delta^s\ln C_t]}{E[R^f_{s,t}(1-\gamma^s\Delta^s\ln C_t)]}$$

Following the spectral approach adopted in section 2, equation (10) can be estimated over the frequency domain as:

$$E[R^e_{j,t+1}] = \alpha^s_\omega + \beta^s_{j,\omega}\lambda^s_\omega \tag{11}$$

where its components, after dropping the time subscript, are given by:

$$\alpha^s_\omega = \frac{f_{R^f_s(1-\gamma^s_\omega\Delta^s \ln C),R^f}(\omega)}{E[R^f_s(1-\gamma^s_\omega\Delta^s \ln C)]},\ \beta^s_{j,\omega} = G_{R^e_j,R^f_s\Delta^s \ln C}(\omega),\ \lambda^s_\omega = \frac{\gamma^s_\omega f_{R^f_s\Delta^s \ln C,R^f_s\Delta^s \ln C}(\omega)}{E[R^f_s(1-\gamma^s_\omega\Delta^s \ln C)]} \tag{12}$$

To estimate equation (11) we use data on long-term risk-free interest rates. Since data for each maturity, $s$, are not readily available to match our consumption and return series, we employ risk-free interest rates with maturities of 1, 3, 5 and 10 years starting in 1953:Q2.[21] Risk-free interest rates are made real by employing as a measure of inflation the quarter-to-quarter change in the chain weighted price deflator, spliced across periods, produced by the Bureau of Economic Analysis. In this respect, we proxy expected interest rates and expected inflation with their realized counterparts over the holding period of the corresponding risk-free asset.

Before presenting the main empirical results, we briefly discuss the spectral properties of the data. The first row of Figure 6 presents the log-spectra of 1, 3, 5, and 10-year consumption growth for all frequencies (stated as a fraction of $\pi$). As expected, the volatility of consumption growth at any horizon increases sharply for lower frequencies and, given the propeties of consumption growth over the time domain, the low-frequency variability of consumption growth is amplified when the time horizon increases. Again, the relative concentration of fluctuations in low frequencies is an indication of short-term correlation in consumption growth. The second row of Figure 6 plots the estimated coherencies between the long-term returns and the corresponding measures of consumption growth and shows that the relationship remains fairly stable over the whole frequency domain for all four horizons considered. The third row of Figure 6 plots the respective estimated gains over the frequency domain and, as can be readily seen, as the horizon of returns and the corresponding consumption growth rates increase, the gains for higher frequencies are substantially lower.

Table 4 presents the estimates for the four maturities considered. The evidence from the 1-year interest rates (Panel A) replicates the usual failure of the C-CAPM; the high frequency of

[21] The series codes are GS1, GS3,GS5 and GS10 and are available from the Board of Governors of the Federal Reserve System (http://www.research.stlouisfred.org/fred2/).

consumption risk explains only a small fraction of the variation in returns and is associated with a significant equity premium of the magnitude of 3.4% per quarter. For the 16-quarter horizon the coefficient of risk equity premium falls to 2%, but the model is overall unable to explain the cross-section of returns. As we move to lower frequencies, the picture changes starkly. For the 32-quarter horizon, the performance of the model improves dramatically and the equity premium is negligible. The picture is further improved at the zero frequency, where the model explains 96.4% of the variability in returns with a zero equity premium. A similar picture emerges from other risk-free rates of long-term maturities (Panels B to D of Table 5). In all cases, the model fails at high frequencies, but its performance is consistent with the C-CAPM at lower frequencies. These patterns are corroborated by Figure 8, in which the average realized and fitted returns from the various risk-free rates are found to be closely aligned.

## 7. Conclusions

In this paper we re-evaluated the C-CAPM by adopting a spectral approach to measure the covariance of an asset's return with consumption growth over the frequency domain and its impact on expected stock returns. We established that when lower frequencies of consumption risk are considered the validity of the C-CAPM is restored. For low frequencies the C-CAPM can explain almost entirely the cross-sectional variation of expected returns accompanied by a decrease in the equity premium.

The paper is part of the upcoming literature that aims at capturing the behavior of aggregate and cross-sectional stock returns via the long-term dynamics of consumption. The approach adopted here remains, however, agnostic about the driving force of these dynamics. For instance, our findings are consistent with the general class of models that relax the assumption of costless adjustment in consumption plans by including the time spent to calculate and implement a new consumption-savings decision, or constraints in information and search costs that lead investors in making infrequent consumption and portfolio allocation decisions at discrete points in time. The impact of consumption risk measured over the frequency domain can also be consistent with models that entail monitoring costs and heterogeneous agents, in which only a fraction of households adjusts its consumption over discrete intervals.[22]

Recently, there have been some attempts to bring together longer-term consumption dy-

[22]See, for instance, Grossman and Laroque (1990), Lynch (1996), Marshall and Parekh (1999), Gabaix and Laibson (2001) and Jagganathan and Wang (2005).

namics with theoretical explanations. Panageas and Yu (2006) claim that over the short run, consumption growth is dominated by small frequent shocks, while unpredicted and large technological innovations, which are embodied in the capital stock, prevail in the long run. The authors then show that this framework implies that consumption growth over the long run can reveal information about the degree to which the economy has absorbed a major technological shock. Malloy et al. (2009) show that in a model with recursive preferences the covariance of returns with long-run consumption growth of households who bear stock market risk captures the cross-sectional variation of average stock returns better than the covariance of returns with long-run aggregate or non-stockholder consumption growth. In this context, our frequency-domain evidence on the C-CAPM could be reconciled with a preference structure, in which utility depends on the variance of consumption at different frequencies. Thus, determining the implications for preferences over consumption at different times through a spectral approach remains an open issue and offers a promising route for further research.

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**Table 1. Expected excess returns and consumption risk frequencies**

| *Frequency* | *(Quarters)* | *Adjusted R-sq ($a_\omega$=0)* | *equity premium* | *standard error* | *Adjusted R-sq* |
|---|---|---|---|---|---|
| | | (1) | (2) | (3) | (4) |
| 1 | (2.000) | -2.728 | 2.322 | 0.292 | -0.026 |
| 15/16 | (2.133) | -4.460 | 3.210 | 0.300 | 0.257 |
| 7/8 | (2.286) | -1.931 | 2.917 | 0.325 | 0.006 |
| 13/16 | (2.462) | -3.428 | 2.895 | 0.392 | 0.042 |
| 3/4 | (2.667) | -5.218 | 2.593 | 0.235 | -0.030 |
| 5/8 | (3.200) | -1.325 | 2.528 | 0.554 | -0.043 |
| 1/2 | (4.000) | -0.298 | 1.864 | 0.467 | -0.003 |
| 3/8 | (5.333) | -2.076 | 3.715 | 0.605 | 0.242 |
| 1/4 | (8.000) | -3.160 | 3.125 | 0.436 | 0.121 |
| 3/16 | (10.667) | -1.401 | 3.113 | 0.625 | 0.022 |
| 1/8 | (16.000) | -0.443 | 2.159 | 0.790 | -0.032 |
| 1/16 | (32.000) | 0.660 | -0.344 | 0.330 | 0.655 |
| 0 | (inf) | 0.981 | -0.201 | 0.080 | 0.986 |

*Notes:*
*1) Frequency is expressed as a fraction of $\pi$.*
*2) Newey-West heteroskedasticity and autocorrelation corrected standard errors.*
*3) The equity premium and its standard errors are scaled by $10^2$.*

**Table 2.**

**Expected excess returns and consumption risk frequencies: Robustness tests**

| *Frequency* | *(Quarters)* | *Adjusted R-sq* | *equity premium* | *standard error* |
|---|---|---|---|---|
| A. Original Fama-French start date (1963:03) | | | | |
| 1 | (2) | 0.296 | 1.696 | 0.172 |
| 1/2 | (4) | -0.008 | 2.808 | 0.887 |
| 1/4 | (8) | 0.314 | 3.282 | 0.328 |
| 1/8 | (16) | -0.016 | 1.745 | 0.718 |
| 1/16 | (32) | 0.671 | -0.531 | 0.201 |
| 0 | (inf) | 0.995 | -0.211 | 0.046 |
| B. Total consumption | | | | |
| 1 | (2) | 0.016 | 1.795 | 0.602 |
| 1/2 | (4) | 0.000 | 1.976 | 0.465 |
| 1/4 | (8) | 0.098 | 1.206 | 0.552 |
| 1/8 | (16) | -0.033 | 2.200 | 0.718 |
| 1/16 | (32) | 0.712 | 0.321 | 0.252 |
| 0 | (inf) | 0.989 | -0.049 | 0.075 |
| C. Long-horizon returns | | | | |
| 1 | (2) | -0.006 | 50.567 | 7.801 |
| 1/2 | (4) | 0.022 | 40.704 | 3.327 |
| 1/4 | (8) | 0.421 | 52.874 | 3.045 |
| 1/8 | (16) | 0.272 | 20.540 | 7.091 |
| 1/16 | (32) | 0.422 | 1.295 | 9.299 |
| 0 | (inf) | 0.979 | 1.657 | 1.760 |

*Notes: See Table 1.*

**Table 3.**

**Expected excess returns and consumption risk frequencies: Alternative portfolios**

| *Frequency* | *(Quarters)* | *Adjusted R-sq* | *equity premium* | *standard error* |
|---|---|---|---|---|
| A. Equally weighted portfolios | | | | |
| 1 | (2) | 0.092 | 2.146 | 0.259 |
| 1/2 | (4) | -0.025 | 3.185 | 1.060 |
| 1/4 | (8) | 0.034 | 3.177 | 0.403 |
| 1/8 | (16) | 0.091 | 1.434 | 0.664 |
| 1/16 | (32) | 0.733 | 0.063 | 0.260 |
| 0 | (inf) | 0.992 | -0.081 | 0.045 |
| B. 10 size, 10 B/M and 10 D/P portfolios | | | | |
| 1 | (2) | 0.132 | 1.911 | 0.116 |
| 1/2 | (4) | 0.110 | 1.665 | 0.180 |
| 1/4 | (8) | 0.052 | 1.888 | 0.158 |
| 1/8 | (16) | 0.198 | 1.367 | 0.246 |
| 1/16 | (32) | 0.507 | 0.835 | 0.174 |
| 0 | (inf) | 0.939 | -0.032 | 0.105 |
| C. 20 risk-sorted portfolios | | | | |
| 1 | (2) | -0.056 | 1.939 | 0.100 |
| 1/2 | (4) | 0.024 | 2.195 | 0.196 |
| 1/4 | (8) | -0.051 | 1.978 | 0.138 |
| 1/8 | (16) | -0.046 | 1.999 | 0.169 |
| 1/16 | (32) | -0.054 | 1.912 | 0.157 |
| 0 | (inf) | 0.818 | -0.299 | 0.272 |
| D. 34 industry portfolios | | | | |
| 1 | (2) | -0.019 | 1.936 | 0.156 |
| 1/2 | (4) | 0.003 | 2.161 | 0.194 |
| 1/4 | (8) | 0.062 | 1.602 | 0.210 |
| 1/8 | (16) | -0.030 | 1.957 | 0.378 |
| 1/16 | (32) | 0.066 | 1.731 | 0.172 |
| 0 | (inf) | 0.853 | 0.247 | 0.141 |

*Notes: See Table 1.*

**Table 4.**

**Expected excess returns and consumption risk frequencies: Long-term interest rates**

| *Frequency* | *(Quarters)* | *Adjusted R-sq* | *equity premium* | *standard error* |
|---|---|---|---|---|
| A. 1-year interest rate | | | | |
| 1 | (2) | 0.143 | 3.378 | 0.561 |
| 1/2 | (4) | 0.156 | 1.282 | 0.557 |
| 1/4 | (8) | 0.103 | 3.241 | 0.539 |
| 1/8 | (16) | -0.017 | 2.023 | 0.614 |
| 1/16 | (32) | 0.730 | -0.176 | 0.340 |
| 0 | (inf) | 0.964 | 0.026 | 0.123 |
| B. 3-year interest rate | | | | |
| 1 | (2) | 0.127 | 2.904 | 0.325 |
| 1/2 | (4) | 0.059 | 1.614 | 0.639 |
| 1/4 | (8) | 0.499 | 3.549 | 0.307 |
| 1/8 | (16) | 0.215 | 0.832 | 0.510 |
| 1/16 | (32) | 0.659 | 1.015 | 0.154 |
| 0 | (inf) | 0.968 | 0.195 | 0.109 |
| C. 5-year interest rate | | | | |
| 1 | (2) | 0.008 | 2.318 | 0.214 |
| 1/2 | (4) | 0.008 | 2.153 | 0.350 |
| 1/4 | (8) | 0.215 | 3.557 | 0.445 |
| 1/8 | (16) | 0.266 | 0.052 | 0.774 |
| 1/16 | (32) | 0.760 | 1.028 | 0.161 |
| 0 | (inf) | 0.939 | 0.276 | 0.170 |
| D. 10-year interest rate | | | | |
| 1 | (2) | 0.021 | 2.294 | 0.191 |
| 1/2 | (4) | 0.011 | 2.121 | 0.334 |
| 1/4 | (8) | 0.295 | 3.354 | 0.293 |
| 1/8 | (16) | 0.072 | 1.179 | 0.590 |
| 1/16 | (32) | 0.442 | -0.282 | 0.288 |
| 0 | (inf) | 0.929 | 0.052 | 0.182 |

*Notes: See Table 1.*

**Figure 1A. Spectrum of non-durables consumption growth**

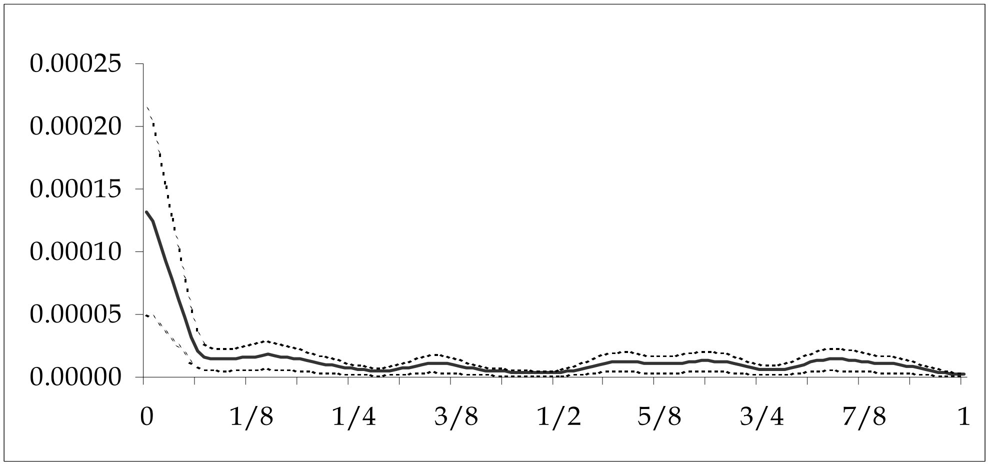



**Figure 1B. Spectrum of excess returns**

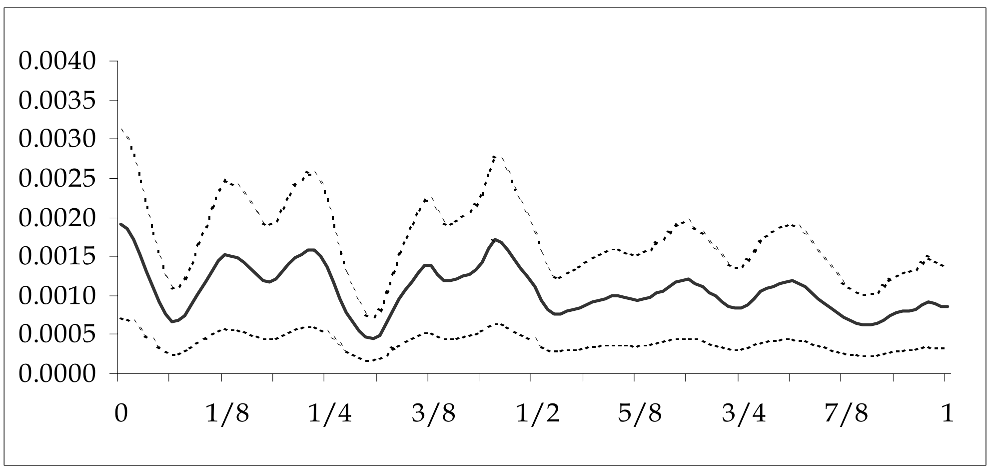
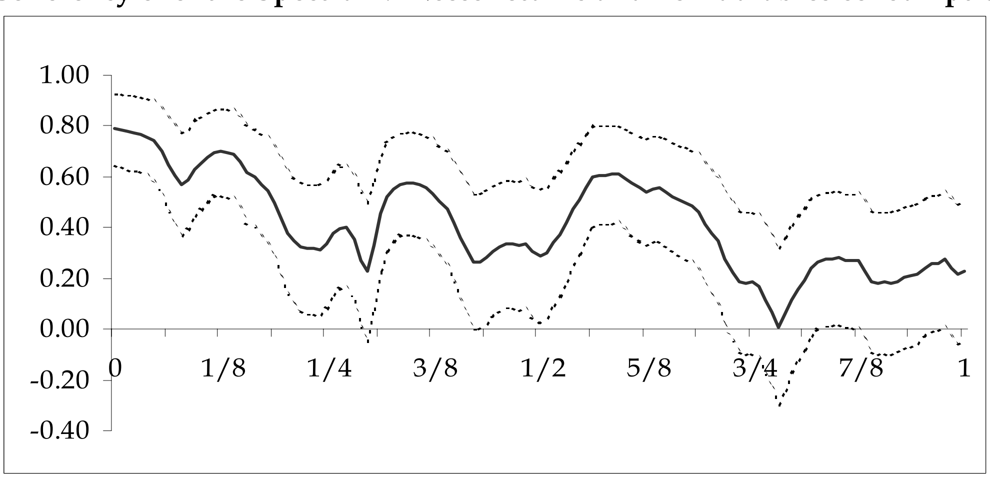



**Figure 1C. Coherency over the Spectrum: Excess returns and non-durables consumption growth**




*Notes: 95% confidence intervals in dashed lines.*

**Figure 2. Average returns and betas**

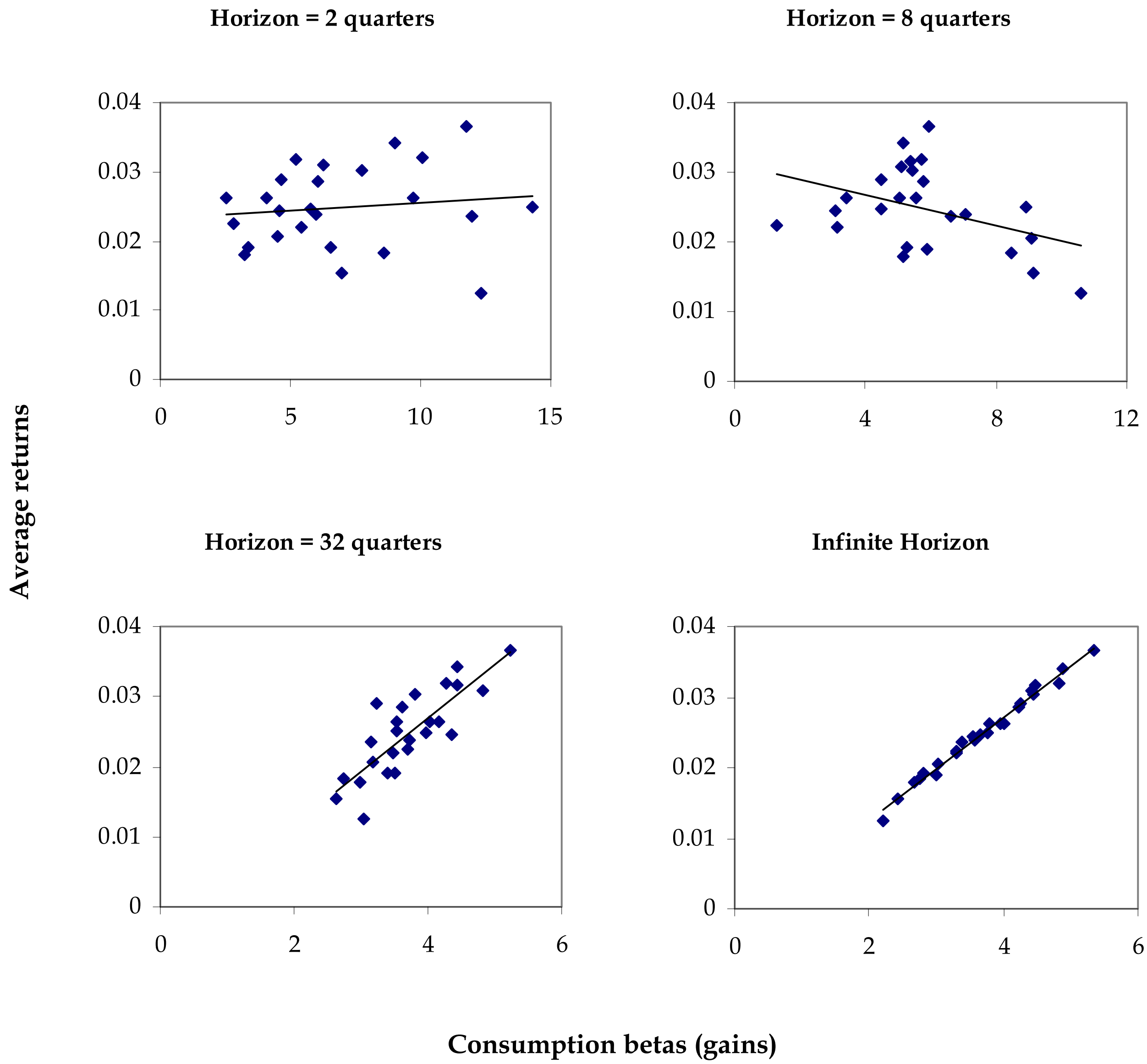


**Figure 3. Fitted and average returns**

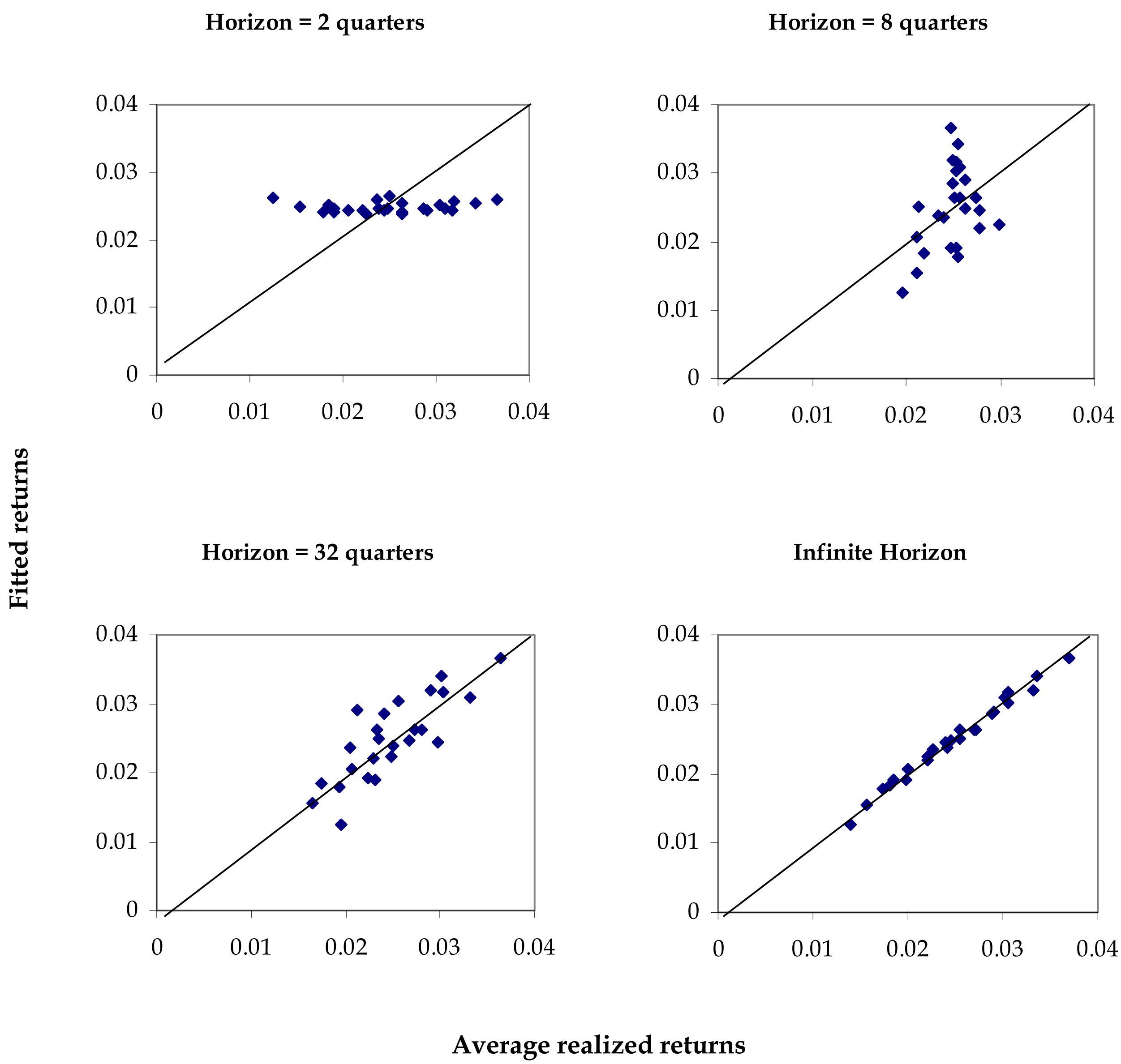


**Figure 4. Fitted and average returns (alternative specifications, infinite horizon)**

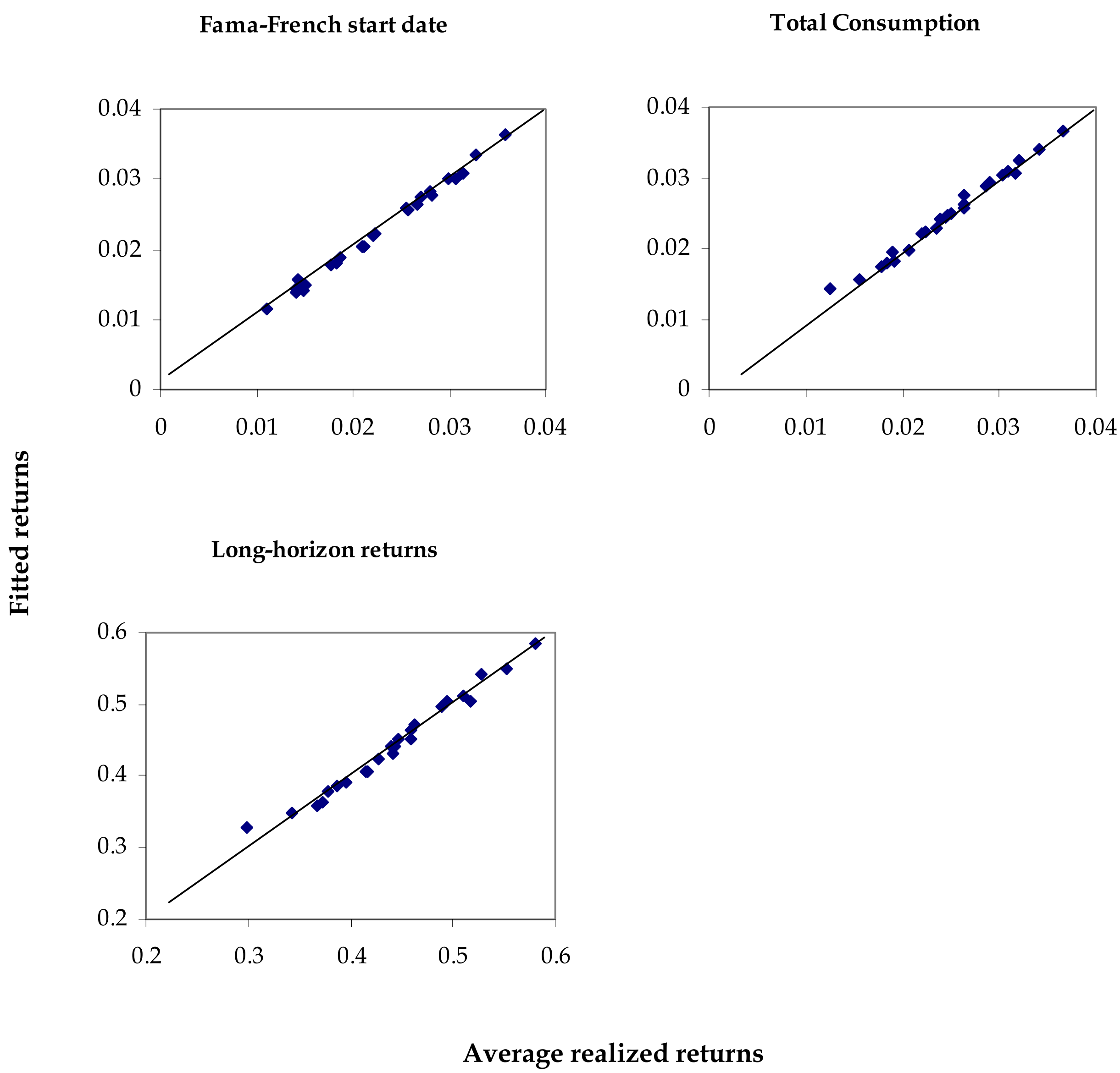


**Figure 5. Fitted and average returns (alternative portfolios, infinite horizon)**

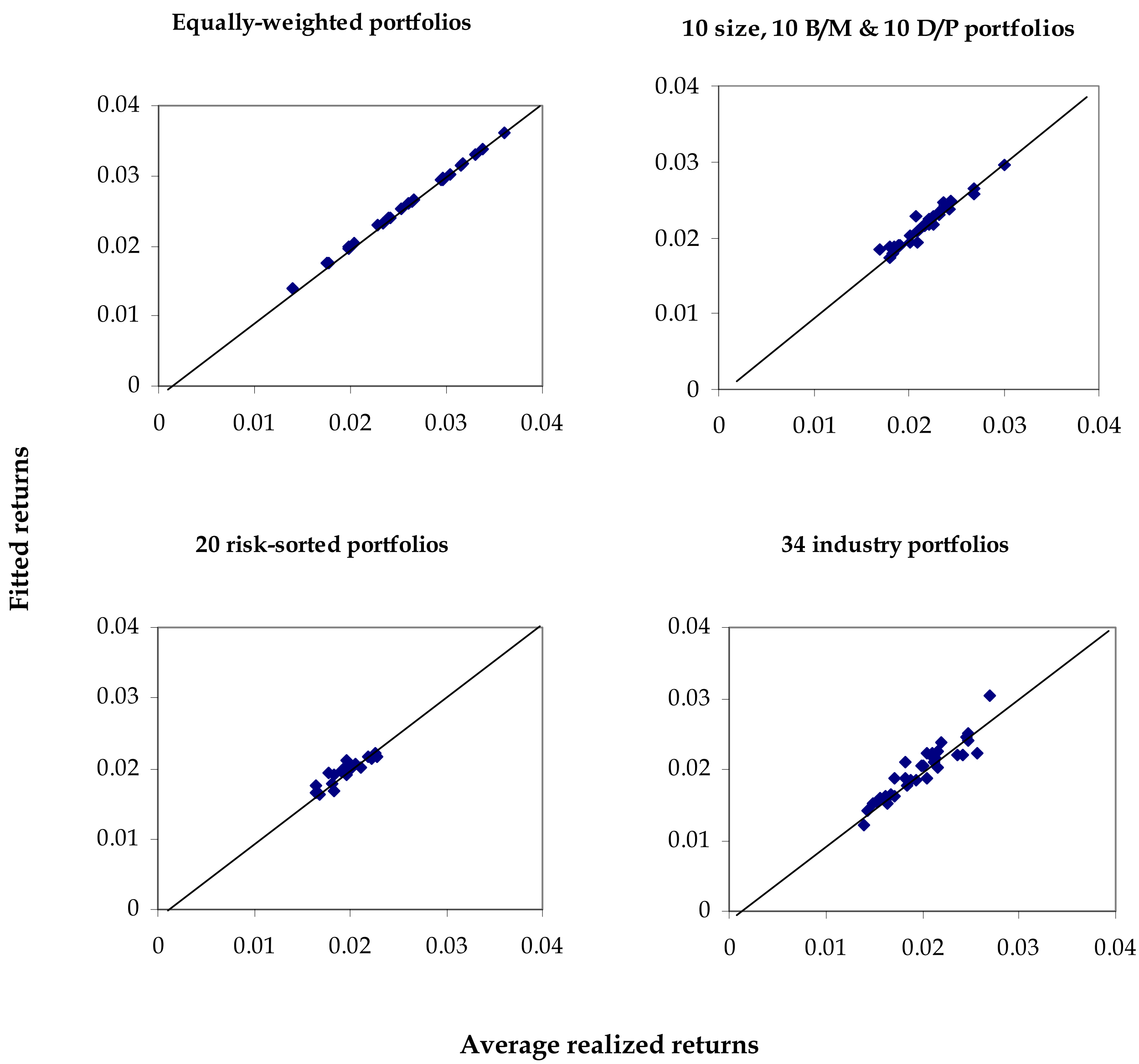


**Figure 6. Spectral properties of C-CAPM and the term structure of interest rates**

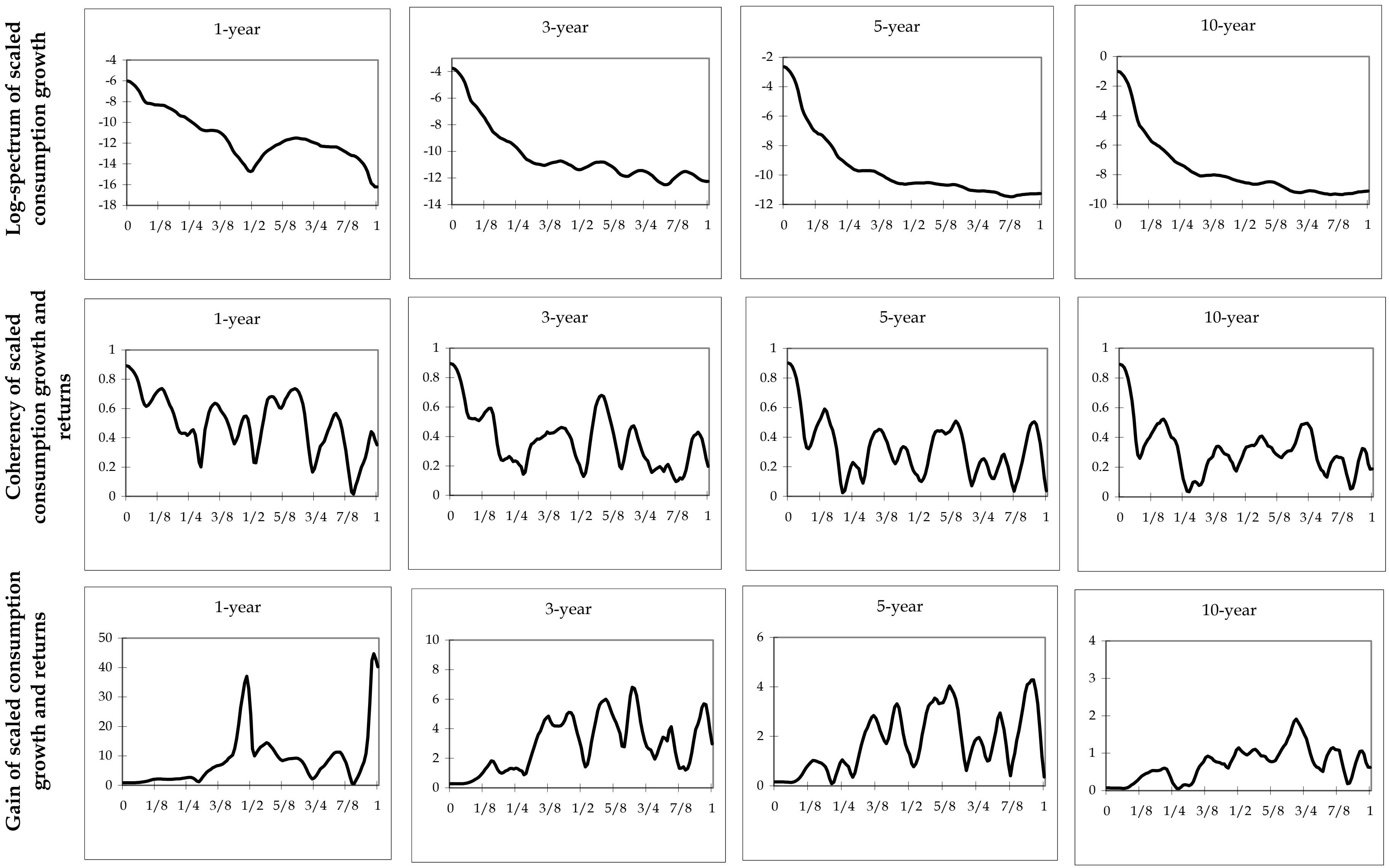


**Figure 7. Fitted and average returns (term structure, infinite horizon)**