

# On the Macroeconomic Implications of Maintenance in Public Capital\*

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## **On the Macroeconomic Implications of Maintenance in Public Capital**

**Abstract:** An infrastructure-led growth model is constructed where the durability of public capital is endogenous and varies according to its usage and the level of maintenance expenditure. Policy changes in total expenditures and the maintenance share are shown to be important for the steady state and the dynamic behavior of the economy. The optimal (growth-maximizing) taxation burden which goes to both ‘new’ investment and maintenance expenditure is, in contrast to standard results from other growth models, larger than the elasticity of infrastructure in the production function. The optimal shares of maintenance expenditure and investment in ‘new’ capital, that ensure maximum utilization of public resources, are calculated.

**Keywords:** endogenous growth, public capital, maintenance cost, optimal policy.

**JEL classification numbers:** E22, E62, H21, H54.

## 1. Introduction

Government productive activities play unambiguously a prominent role in the new generation of endogenous growth models. Both theoretically (after the work of Barro, 1990, Barro and Sala-i-Martin, 1992, and others) and empirically (see Aschauer, 1989, and the voluminous empirical literature that followed) government productive activities are widely recognized as key determinants of long-run growth and productivity. Their positive spillovers are transmitted in the economy via private firms' production function either as a flow (government productive services) or as a stock (public or infrastructural capital).

The evolution of public capital typically takes place via a capital accumulation equation, where added capital stock in the economy is the difference between new investment and depreciated capital. This framework explicitly or implicitly assumes that total expenditure related to the public capital accumulation process of the economy is oriented to the formation of 'new' public capital. However, by considering capital accumulation as an exogenously given technical relationship, this approach neglects a crucial factor embedded in the definition of the capital stock and the implementation of investment decisions. The capital accumulation process mirrors as a whole society's choice on the future welfare. To this extent, policymakers, when facing the consumption-investment dilemma as part of policy design on the efficient allocation of resources do not view public capital as a final good with an inherently fixed terminal period. Rather, they face an option concerning the policy mix between building '*new*' infrastructure and extending the durability of existing public capital through increased *maintenance* expenditure.

Take, for instance, the case of a major infrastructure project, like an airport. Deciding on the profitability of a 'new' airport does not necessary imply that any existing plants are rendered obsolete. Most likely, the options available to policymakers include

both the building of a ‘new’ airport, as well as the continuation of the operation after some improvement of the existing plant. The cost paid for the latter option comes as maintenance cost and, strictly speaking, should be classified under the budgetary term ‘public investment’ as it fulfills two basic criteria: (i) it is financed by taxation or government borrowing and (ii) it does not result in public consumption expenditure, but instead increases the public capital stock available in the economy.

Of course, the issue of capital maintenance is not new in the literature. Early writings, dating back to Hayek (1935), recognized the significance of durability for the fixed assets of a private firm. Hicks (1942) emphasized that “...*the ‘maintenance’ work done in the present is not a contribution to current final output, but to the final output of future years*”, while Haavelmo (1960, p. 130) noted that “...*some maintenance and repair work yields a very high return in terms of preventing wear and tear of capital equipment*”. In a series of papers during the seventies, several authors investigated the firm’s problem between optimal maintenance level and the maintenance dependent depreciation rate.<sup>1</sup> Subsequent empirical studies found that although decaying constant rates of depreciation often provide a reasonable approximation at a given point in time, there is mounting evidence that capital deterioration is endogenous and, in particular, associated with maintenance expenditure.<sup>2</sup>

However, little work has been done so far in exploring at the macroeconomic or general equilibrium level the effects of maintenance policies on *public* capital formation and growth. Existing work is usually performed in the context of cost-benefit analysis and is primarily concerned with road damage and optimal user charges, which rely on required repairs and their timing (see Newberry, 1988). The only attempt to analyze the trade-off between maintenance and new investment has been recently

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<sup>1</sup>See, among others, Schmalensee (1974), Feldstein and Rothschild (1974), Su (1975) and Parks (1979).

<sup>2</sup>For a brief survey on related empirical findings, see Nelson and Caputo (1997) and the references cited therein.

made by Rioja (2002). The author sets up a growth model where domestic tax revenues finance maintenance expenditure, whereas ‘new’ investment is financed solely by foreign donors. Rioja (2002) shows that the optimal maintenance level (as a share of GDP) depends upon various parameters and presents calibration results from Latin American countries that confirm the importance of maintenance for the pattern of growth in these countries.

Despite the consensus on the crucial weight of maintenance in total public investment expenditure, there has been no systematic attempt to investigate the macroeconomic impact of maintenance in the context of (i) the trade-off between maintenance and ‘new’ investment when both types of expenditure are domestically financed, and (ii) the implications of optimal maintenance policies for public capital formation on growth. Along this line, we develop in this paper an infrastructure-led two sector endogenous growth model where private and public capital stocks enter directly in the production function, as in Turnovsky (1997). Following conventional wisdom, we assume that it is the stock of available capital (such as roads, power systems, water supply etc.) that affects output, rather than its flow. As in Rioja (2002), we assume that the depreciation rate of public capital and, consequently, its accumulation rate are determined by public expenditure on maintenance. A rise in maintenance expenditure as a ratio of output (where output can be considered as a measure of the burden on public infrastructure) raises the durability of public capital. At the same time, ‘new’ investment, which is also financed by tax revenues, adds directly to public capital.

Thus, our model extends Rioja’s (2002) specification by endogenizing the decision for both ‘new’ investment and maintenance expenditures. So, we are able to examine the behavior of the economy when the government uses these two policy instruments related to expenditures on capital formation to achieve a higher growth rate. First, the government can alter the composition of public expenditure between ‘new’ investment

and maintenance for a given level of total public expenditure. It is shown that the equilibrium private to public capital ratio will fall, as the economy suffered from a waste of private capital due to the misallocation between ‘new’ investment and maintenance. We show that the economy had initially a shortage of public capital, which is eliminated by the change in allocation and the subsequent accumulation of infrastructure. This increases the shadow price of private capital and the economy reaches a higher steady-state growth rate, because at the given government size the economy utilizes more efficiently the existing private capital stock. Second, given the allocation between ‘new’ investment and maintenance the government can change the level of total public expenditure. Here, the dynamic trajectory of the economy depends on the current level of the tax rate. In particular, we confirm the inverse U-shaped tax rate effect of Barro (1990), which is now intensified by the impact of maintenance expenditure on public capital accumulation.

Finally, we focus on the optimal setting of public expenditure policies which maximizes the long-run growth rate of the economy. We find that under the assumption of maintenance expenditure, which affects the durability of public capital, the optimal taxation is now larger than the elasticity of infrastructure in the production function. This contrasts standard results derived by, among other, Barro (1990), Glomm and Ravikumar (1994), and Devarajan et al. (1998). Here, maintenance provides the economy with an additional benefit stemming from reduced capital decay, which was not encountered in previous growth models. Under the new setting, we are able to derive the optimal levels for maintenance and ‘new’ investment expenditure. An immediate consequence of our findings is that expenditure on ‘new’ investment as a fraction of output should be smaller than the elasticity of public capital in the production function, because an additional unit of ‘new’ investment generates the need for extra maintenance, in order to sustain the extra usage of public capital.

The rest of the paper is organized as follows. Section 2 sets up a representative firm

model and derives the equilibrium of the model. Section 3 analyses the transitional dynamics of tax policies related to the allocation of total public expenditure between investment in ‘new’ public capital and maintenance. Section 4 derives the optimum levels of taxation and maintenance expenditures. Section 5 outlines some empirical implications generated by the theoretical results and, finally, section 6 concludes the paper.

## 2. A growth model with maintenance in public capital

### 2.1. Model description

We consider a closed economy populated by identical agents who consume and produce a single commodity,  $Y$ . There is no population growth. The labor force is equal to the population, with labor supplied inelastically. Individuals have identical additively time separable utility functions, which imply that, in the steady state, the growth rate of consumption will be determined by the constant rate of time preference and the interest rate.

On the production side of the economy, a representative firm  $i$  produces its output,  $Y_i$ , using a Cobb-Douglas technology:

$$Y_i = K_i^\alpha (hL_i)^{1-\alpha} \quad (1)$$

where  $0 < \alpha < 1$ ,  $K_i$  denotes the stock of private capital, and  $L_i$  the labor used by firm  $i$ . The productivity of labor,  $h$ , is a function of the existing aggregate stock of private capital,  $K$ , and public capital,  $K_g$ , per worker so that:

$$h = \frac{K^\beta K_g^{1-\beta}}{L} \quad (2)$$

where  $L$  is the total labour force and  $0 < \beta < 1$ . Equation (2) follows Alogoskoufis (1995) and is in the spirit of Romer’s (1986) approach where each individual firm

benefits from an increase in economy-wide labor productivity triggered by a rise in the public or private capital stock available in the economy. The individual firm takes  $h$  as given. Equation (2) is a general form which can nest two interesting special cases. When  $\beta = 1$  we get the learning-by-doing model of knowledge accumulation, while when  $\beta = 0$  we get a version of Barro's (1990) well known model of endogenous growth through productive government expenditure.<sup>3</sup>

Private capital depreciates at the constant rate  $\delta_k$ .<sup>4</sup> Therefore, letting  $I$  denote gross private investment, the net private capital stock accumulates at the rate:

$$\dot{K} = I - \delta_k K \quad (3)$$

New output may be transformed to either type of capital, but in the case of private capital this process involves adjustment costs. The cost of investment faced by local firms is:

$$\Psi(I, K) = \left(1 + \frac{\phi}{2} \frac{I}{K}\right) I \quad (4)$$

where  $\phi > 0$  is an adjustment cost parameter. As typically put forward by the relevant literature, the adjustment cost of private capital is proportional to the rate of investment per unit of installed capital.<sup>5</sup>

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<sup>3</sup>Notice here that we eliminate the scale effect on the growth rate by defining the labor productivity in per capita terms.

<sup>4</sup>The relevant literature has shown that maintenance expenditure is also very important for private capital durability. However, the focus of our paper is on the macroeconomic policy implications of maintenance for public capital formation, and so we simplify our model by abstracting from maintenance expenditure for private capital. This simplification does not affect the theoretical results derived later on.

<sup>5</sup>The inclusion of adjustment costs is useful because it allows the presence of transitional dynamics when comparative statics are utilized to analyze the dynamic behavior of the model. Notice that we will not include adjustment costs for public capital formation, as this would only complicate the solution of the model without adding further insights.

Typically, models without maintenance expenditure assume a constant depreciation rate for the public capital stock. To incorporate maintenance expenditure we define, following Bitros (1976), as maintenance for public capital the deliberate utilization of all public resources which preserve the operative state of public capital goods. So, in our model the stock of public capital depreciates at the rate  $\delta_g$ , which depends negatively on maintenance expenditures  $M$  and positively upon usage measured by the aggregate economic activity  $Y$ . In particular, the depreciation rate  $\delta_g$  entering the public capital accumulation equation is assumed here to be a function of the ratio of maintenance expenditure on public capital to aggregate output. If we let  $G$  denote gross public investment for ‘new’ public capital, then the net public capital stock accumulates as follows:<sup>6</sup>

$$\dot{K}_g = G - \delta_g \left( \frac{M}{Y} \right) K_g, \quad \text{with } \delta'_g(.) < 0. \quad (5)$$

The government finances its total expenditure (‘new’ public investment plus maintenance) through tax revenues collected via a tax rate  $\tau$  imposed on total output produced by firms. Therefore, the government budget constraint is:

$$G + M = \tau Y \quad (6)$$

It is to be noted here that although  $G$  and  $M$  may differ in the level of technology embodied, they result in a homogeneous public capital good and can be treated in a similar manner without further implications for the aggregate state of the economy. We define the share of total government expenditure that goes towards maintenance and ‘new’ investment as  $\mu$  and  $(1 - \mu)$ , respectively:

$$M = \mu \tau Y \quad \text{and} \quad G = (1 - \mu) \tau Y \quad (7)$$

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<sup>6</sup>Equation (5) makes the distinction between ‘new’ investment and *net* investment clear. ‘New’ investment, as defined here, corresponds to *gross* investment as it refers to the creation of previously non-existing capital goods, irrespectively of whether they replace utilized capital or not.

We assume, for the moment, that both the tax rate  $\tau$  and its shares  $\mu$  and  $(1 - \mu)$  are fixed and constant over time. So the government can set both variables arbitrarily. Since, however, these policy instruments are going to affect the long-run growth rate of the economy, the optimal tax rate and the component shares that maximize the growth rate will also be derived later on.

## 2.2. Model solution

The representative firm  $i$  in our economy solves the following infinite horizon profit maximization problem:

$$\begin{aligned} \max \int_0^\infty e^{-rt} \left[ (1 - \tau) Y_i - w_i L_i - \left( 1 + \frac{\phi I_i}{2 K_i} \right) I_i \right] dt \\ \text{s.t.} \quad \dot{K}_i = I_i - \delta_k K_i \end{aligned}$$

where  $r$  is the real interest rate,  $w$  is the real wage rate, and the price of the commodity is set equal to one. The familiar optimality conditions with respect to  $L_i$ ,  $I_i$  and  $K_i$  are respectively:

$$w = (1 - \tau) (1 - \alpha) \left( \frac{K_i}{L_i} \right)^\alpha h^{1-\alpha} \quad (8)$$

$$1 + \phi \frac{I_i}{K_i} = q \quad (9)$$

$$r = \frac{1}{q} \left[ \dot{q} + (1 - \tau) \alpha \left( \frac{K_i}{L_i} \right)^{\alpha-1} h^{1-\alpha} + \frac{\phi}{2} \left( \frac{I_i}{K_i} \right)^2 \right] - \delta_k \quad (10)$$

where  $q$  is the shadow value of the private capital stock. Equation (8) equates the wage rate to the value of the marginal product of labor.<sup>7</sup> Equation (9) equates the

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<sup>7</sup>Given that, at equilibrium, all firms in the economy will pay the same wage and will hire the same amount of labor and private capital, equation (8) can be written as

$$\frac{wL}{Y} = (1 - \tau) (1 - \alpha)$$

implying that the labor share of income remains constant in the long run. As a result, the steady state wage rate and per capita income grow at the same rate.

marginal cost of investment to the shadow value of capital. Finally, equation (10) is the arbitrage condition that equates the interest rate to the rate of return of private capital, net of physical depreciation. The rate of return to private capital consists of three components: the change in its shadow value, the value of its marginal product, and its effect on the cost of investment.

In addition, the following transversality condition must hold:

$$\lim_{t \rightarrow \infty} (qe^{-rt} K_i) = 0 \quad (11)$$

Substituting (9) into (10), replacing for (2) and aggregating across firms, the optimality conditions with respect to  $I$  and  $K$  can be written as:

$$\frac{I}{K} = \frac{q-1}{\phi} \quad (12)$$

$$\dot{q} = (r + \delta_k)q - (1 - \tau)\alpha \left( \frac{K_g}{K} \right)^{(1-\alpha)(1-\beta)} - \frac{(q-1)^2}{2\phi} \quad (13)$$

From equations (5) to (3), and (12), the growth rates of private and public capital are given by:

$$\frac{\dot{K}}{K} = \frac{q-1}{\phi} - \delta_k \quad (14)$$

$$\frac{\dot{K}_g}{K_g} = (1 - \mu)\tau \left( \frac{K}{K_g} \right)^\omega - \delta_g (\mu\tau) \quad (15)$$

since the aggregate production function can be written as:

$$Y = K^\omega K_g^{1-\omega} \quad (16)$$

where  $\omega = \alpha + \beta(1 - \alpha) < 1$ . Now, let us define  $z$  as the ratio of private to public capital stock  $z = \frac{K}{K_g}$ . Then, by (14) and (15) we can write  $\frac{\dot{z}}{z}$  and (13) as:

$$\frac{\dot{z}}{z} = \delta_g (\mu\tau) + \frac{q-1}{\phi} - (1-\mu)\tau z^\omega - \delta_k \quad (17)$$

$$\dot{q} = (r + \delta_k)q - (1-\tau)\alpha z^{\omega-1} - \frac{(q-1)^2}{2\phi} \quad (18)$$

Equations (17) and (18) form a system of two differential equations given the independent policy parameters  $\tau$  and  $\mu$ . In the next subsection we shall analyze the steady-state behavior of the economy.

### 2.3. Steady state

The stationary solution of this system ( $\dot{z} = \dot{q} = 0$ ) must have at least one real solution, in order for output and the capital stocks,  $K$  and  $K_g$ , to follow a balanced growth path  $\left(\bar{g}_Y = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{K}_g}{K_g}\right)$ . From equation (14) and from the fact that, at the steady state, output, private and public capital grow at the same rate, we have that the steady-state value of private capital is:

$$\bar{q} = 1 + \phi(\bar{g}_Y + \delta_k) \quad (19)$$

The steady-state shadow price of private capital is a positive function of the growth rate, as higher growth rates are associated with increased output and profits (see also Alogoskoufis, 1995). The depreciation rate of private capital also affects positively its shadow price because a higher depreciation rate requires a higher shadow price of private capital due to larger associated adjustment costs.

Finally, the transversality condition (11) can be expressed in terms of the state variable  $z$ , as:

$$\lim_{t \rightarrow \infty} (q e^{-(r-g_Y)t} z) = 0 \quad (20)$$

which implies that in the steady state the rates of growth of output, private and public capital must not exceed the real interest rate (see also Turnovsky, 1997).

The dynamics of our economy are described by equations (17) and (18). These two equations can be used to draw the phase diagram in the  $q, z$  space. The equilibrium solutions for  $q$  and  $z$  are jointly determined by the following equations:

$$\dot{z}=0 \implies q = 1 + \phi [(1 - \mu) \tau z^\omega - \delta_g (\mu \tau) + \delta_k] \quad (21)$$

$$\dot{q}=0 \implies (1 - \tau) \alpha z^{\omega-1} = (r + \delta_k) q - \frac{(q - 1)^2}{2\phi} \quad (22)$$

The slope of the  $\dot{z}=0$  locus is positive since

$$\left. \frac{dq}{dz} \right|_{\dot{z}=0} = \phi (1 - \mu) \tau \omega z^{\omega-1} > 0. \quad (23)$$

with  $\left. \frac{d^2q}{dz^2} \right|_{\dot{z}=0} < 0$ . The slope of the  $\dot{q}=0$  locus is given by

$$\left. \frac{dq}{dz} \right|_{\dot{q}=0} = - \frac{(1 - \tau) \alpha (1 - \omega) z^{\omega-2}}{(r + \delta_k) - \frac{q-1}{\phi}} \quad (24)$$

The  $\dot{q}=0$  locus slopes upward when  $q > 1 + \phi(r + \delta_k)$ . For values of  $q$  such that  $q < 1 + \phi(r + \delta_k)$ , the slope of the  $\dot{q}=0$  locus is negative. Notice, however, that the equilibrium occurs at the negative part of the  $\dot{q}=0$  locus, since  $r + \delta_k > \frac{q-1}{\phi}$  by the transversality condition (20). Hence, we are interested only in the negative part of the  $\dot{q}=0$  locus. In the next section we shall analyze the dynamic behavior of the economy around the steady state.

### 3. Transitional dynamics and policy changes

#### 3.1. Linearization around the steady state and equilibrium dynamics

Since the dynamic system that describes our economy is nonlinear, we proceed to analyze the dynamics of our economy by considering the linearized system around

steady state. It can be shown that the linearized dynamics around the steady-state values  $(\bar{z}, \bar{q})$  are represented in matrix notation by:

$$\begin{bmatrix} \dot{z} \\ \dot{q} \end{bmatrix} = A \begin{bmatrix} z - \bar{z} \\ q - \bar{q} \end{bmatrix} \quad (25)$$

where

$$A = \begin{bmatrix} -(1 - \mu)\omega\tau\bar{z}^\omega & \frac{\bar{z}}{\phi} \\ \alpha(1 - \omega)(1 - \tau)\bar{z}^{\omega-2} & r + \delta_k - \frac{\bar{q}-1}{\phi} \end{bmatrix} < 0. \quad (26)$$

The determinant of the Jacobian matrix,  $A$ , is negative, which implies that the two roots of the matrix are of opposite sign and, therefore, the equilibrium is a saddle point with a downward sloping stable branch. The dynamics of the system are shown in Figure 1. Since the initial value of  $q$  is not predetermined, we can choose a unique value that is consistent with the stable manifold, given the inherited public to private capital ratio,  $z$ .

In the next subsections we shall investigate the dynamic response of the system to changes in two key policy variables: (i) the share of maintenance expenditure in public capital  $\mu$ , and (ii) the tax rate  $\tau$ . To isolate the impact of these policy changes in each exercise and obtain a more clear view of the adjustment of the economy, we shall assume that the other policy instrument is already set at the steady-state level.

### 3.2. A change in the ratio of maintenance to ‘new’ investment expenditures

The new element in the model is the presence of two types of capital expenditure, namely for ‘new’ investment and maintenance. Can the economy benefit from a reallocation between ‘new’ investment and maintenance at the given taxation rate? Interestingly, a first intuition comes from the relationship between the growth rate of the economy and the share of maintenance expenditure, which is not monotonic. The derivative  $\frac{\partial g_Y}{\partial \mu}$  is positive for relatively small values of  $\mu$  and negative for relatively high values of  $\mu$ . Therefore, the government can improve the growth rate of the economy

by reducing (increasing) the share of maintenance expenditure in total expenditure if it is set at a high (low) level.

What are the transitional dynamics of the economy towards the new steady state when the ratio of maintenance to ‘new’ investment is changed by the government to achieve a higher growth rate? Under given total expenditure, if maintenance  $M$  is at a low level, then a rise in  $M$  reduces expenditure for ‘new’ investment  $G$ . The economy is now able to accumulate public capital at a faster rate by utilizing the same amount of total public expenditure resources. Conversely, if  $M$  is relatively high, then ‘new’ investment is low. Again, the economy can benefit from the reallocation by exploiting more efficiently the existing tax revenues. Therefore, in both cases the improved reallocation triggers a rise in public capital accumulation. The private to public capital ratio falls, as the economy in both cases had a surplus of private capital. By accumulating more infrastructure, the economy can now take advantage of the existing private capital stock. Consequently, the shadow price of private capital and the growth rate rise as the economy moves towards the new composition between  $G$  and  $M$ . As depicted in Figure 2, a reallocation moves the  $\dot{z}=0$  locus to the left and the new steady state of the economy occurs at a higher shadow price of capital and growth rate, while the equilibrium private to public capital ratio is now lower.

An interesting point which comes out from this analysis is that the *relative* government size (the inverse of the private to public capital ratio) will increase after the reallocation between maintenance and ‘new’ investment although the *absolute* government size (the tax rate) has not changed. Public capital is now more efficiently utilized, and there is more scope for public capital formation at the same tax rate.

### 3.3. A change in the tax rate with constant allocation of expenditures

An alternative scenario that would be of interest under the presence of maintenance is to assume that the share of maintenance expenditure is already set. Next, we can

examine the dynamics after a change in the tax rate. In particular, we discriminate between two cases: first, when the economy has excess tax revenues, and, second, when the economy is short of tax revenues.

In the first case, the economy suffers from the distortionary impact of excess taxation, which reduces the marginal product of private capital and impedes private capital accumulation. Therefore, a tax reduction boosts initially private investment and triggers growth. As seen in Figure 3, the fall in the tax rate moves the  $\dot{z}=0$  locus and the  $\dot{q}=0$  locus to the right. The equilibrium shadow price of capital  $q$  and the growth rate  $g_Y$  are now higher, while the new steady-state private to public capital ratio  $z$  of the economy is also higher. The dynamic adjustment of the economy can be described as follows. At the given ratio, taxation finances both maintenance  $M$  and ‘new’ investment  $G$  at the given proportion. When the tax rate declines, private investment becomes more profitable and private capital accumulation increases. On the other hand, there are less available tax resources at the given proportion between  $M$  and  $G$ , and public capital accumulation decreases. The decline is triggered by two factors: first, expenditure for ‘new’ investment  $G$  is now lower and, second, the depreciation rate  $\delta_g\left(\frac{M}{Y}\right)$  is higher due to the fall in  $M$  (the durability of public capital is now shorter since  $\delta'_g\left(\frac{M^*}{Y}\right) < 0$ ). As the economy approaches the new steady state, the initial jump in the shadow price of private capital is now moderated by the fall in public capital. In turn, private capital accumulation starts to fall until, at the new steady state, it equals the public capital accumulation rate. The latter has been increasing as the economy grows faster, and both  $G$  and  $M$  rise. The economy ends up with a higher ratio of private to public capital.

The inverse picture (not depicted here) emerges when the economy has a low tax rate. The rise in taxation reduces private capital accumulation as after-tax profits are lower, but at the same time raises public capital accumulation as it provides resources for financing both  $M$  and  $G$  at the optimal proportion. The rise of public

infrastructure renders private capital more profitable and its accumulation rate starts to rise gradually until it reaches the corresponding one of public capital. As a result, the new equilibrium is characterized by higher growth and a lower private to public capital ratio.

#### 4. Optimal policies

The above discussion shows that both total expenditures and the share of maintenance are crucial for the long-run growth rate of the economy. We now address the question of optimal policy by considering the optimal values of the two policy variables, i.e. the optimal tax rate and the optimal allocation of public expenditure between ‘new’ investment and maintenance. To obtain the set of optimal policies we assume that the optimal taxation and the optimal maintenance share are those that maximize the long-run growth rate of the economy. Equivalently, the government can maximize the shadow price of private capital which, by (19), is a monotonic function of the growth rate at the steady state. The determination of the optimal policies is achieved in two steps: the government determines the optimal steady-state allocation between ‘new’ investment and maintenance for any level of taxation, and then derives the optimal steady-state taxation level at the given optimal allocation.

The following Proposition demonstrates the condition satisfied by maintenance expenditure at the optimum.

**Proposition 1.** *The optimal level of maintenance expenditure satisfies the following optimality condition*

$$\left(\frac{K}{K_g}\right)^\omega = -\delta'_g \left(\frac{M^*}{Y}\right). \quad (27)$$

**Proof.** *From equation (19) we see that the optimal growth rate of the economy is:*

$$\bar{g}_Y = \frac{\bar{q}(\mu, \tau) - 1}{\phi} - \delta_k \quad (28)$$

The optimal share of maintenance expenditure is the one that maximizes the growth rate of the economy. That is:

$$\frac{\partial \bar{g}_Y}{\partial \mu} = 0 \quad \implies \quad \frac{\partial \bar{q}(\mu, \tau)}{\partial \mu} = 0 \quad (29)$$

From equation (22) we can write  $z = z(q, \tau)$ . By substituting this function into equation (21) we get that at the steady state:

$$\bar{q} = 1 + \phi [(1 - \mu) \tau [z(\bar{q}, \tau)]^\omega - \delta_g (\mu \tau) + \delta_k] \quad (30)$$

From the above equation we get that

$$\frac{\partial \bar{q}(\mu, \tau)}{\partial \mu} = 0 \quad \implies \quad z^\omega = -\delta'_g (\mu^* \tau) \Leftrightarrow \left(\frac{K}{K_g}\right)^\omega = -\delta'_g \left(\frac{M^*}{Y}\right) \quad (31)$$

The intuition of the above proposition is that the optimal combination of maintenance expenditure,  $M^*$ , and public investment,  $G^*$ , is such that

$$\frac{\partial \dot{K}_g}{\partial G} = \frac{\partial \dot{K}_g}{\partial M} \quad (32)$$

In other words, the last dollar spent on maintenance should have the same contribution to the increase in the public capital stock level as the last dollar spent on public investment.

A question that remains open though is the optimal taxation level. Recall that, in our framework, the government affects the growth of the economy through two channels. Taxation affects negatively the marginal product of private capital, while government expenditure increases the productivity of labor. At low values of  $\tau$  the positive effect of government expenditure dominates, and, hence, the growth rate of the economy rises with the tax rate. At higher tax rates, however, the negative impact of taxation eventually dominates, and the growth rate declines as  $\tau$  rises. The optimal

tax rate that maximizes the growth rate is the one that equates the marginal cost of government expenditure to its marginal benefit. The following Proposition determines the optimal tax rate for the case where maintenance expenditure is already set at its optimal level.

**Proposition 2.** *The optimal tax rate that maximizes the growth rate of the economy is*

$$\tau^* = \frac{1 - \omega}{1 - \mu^* \omega} \quad (33)$$

**Proof.** *Since the optimal tax rate is the one that maximizes the growth rate of the economy, we have:*

$$\frac{\partial \bar{g}_Y}{\partial \tau} = 0 \quad \implies \quad \frac{\partial \bar{q}(\mu^*, \tau)}{\partial \tau} = 0 \quad (34)$$

*From equation (30) we get that*

$$\begin{aligned} \frac{\partial \bar{q}(\mu^*, \tau)}{\partial \tau} = 0 &\implies (1 - \mu^*) \omega \tau z^{-1} \frac{\partial z}{\partial \tau} \Big|_{(22)} = -1 &\implies \frac{(1 - \mu^*) \omega \tau}{(1 - \omega)(1 - \tau)} = 1 \\ &\implies \tau^* = \frac{1 - \omega}{1 - \mu^* \omega}. \end{aligned} \quad (35)$$

This result alters the typical finding obtained by standard endogenous growth models with a constant depreciation rate, which states that the optimal tax rate is equal to the elasticity of public capital in the aggregate production function (see Barro, 1990, Glomm and Ravikumar, 1994, Devarajan et al., 1998). Here, Proposition 2 shows that in the presence of maintenance in public capital the optimal tax rate is greater than the elasticity of public capital in the aggregate production function due to the positive effect of maintenance expenditure on the accumulation of public capital, which raises the benefits of taxation compared to the standard model.

A straightforward implication of Proposition 2 is the following Lemma.

**Lemma 1.** *Under optimal tax policy and optimal maintenance expenditure, the optimal ratio of public investment to GDP is smaller than the elasticity of the stock of public capital in the aggregate production function. That is,*

$$\frac{G^*}{Y} < (1 - \omega). \quad (36)$$

**Proof.** *Under optimal maintenance expenditure, equation (27), the optimal tax rate of Proposition 2 can be written as*

$$\tau^* = (1 - \omega) + \omega \frac{M^*}{Y}. \quad (37)$$

*The above equation and equation (6) then imply inequality (36).*

The result reveals the importance of maintenance in public capital for the long-run growth rate of the economy. In the standard model of no-maintenance in public capital, the marginal benefit of a unit of investment is given by the elasticity of public capital in the production function. The interpretation of this condition emerges when one considers that efficiency in public finance requires that at the optimum the marginal cost of public investment should equal its marginal benefit. A unit increase in  $G$  ( $\Delta G = 1$ ) raises the cost by one unit. At the same time, the marginal benefit from this unit equals the marginal product of public investment, which operates through the accumulation of public capital and its impact via the production function. However, the impact of public capital accumulation differs in the presence of maintenance in the public capital accumulation process. The accumulation of public capital via ‘new’ investment increases output, but at the same time generates an extra cost, due to the increased usage of public capital which leads to a shorter durability (faster depreciation) of public capital. Therefore, any rise in ‘new’ investment must be accompanied by increased maintenance to compensate for the extra usage cost. In turn, the ratio of ‘new’ investment to GDP has to be smaller than the corresponding elasticity in the production function.

## 5. Empirical implications

In this section we briefly highlight some important empirical implications that can be derived from the results presented in sections 3 and 4. According to the comparative statics exercise performed in section 3, a positive effect of  $\mu$  on growth implies that a reallocation of government expenditure towards maintenance will increase the growth rate of the economy. Or, using the Devarajan et al. (1996) terminology, maintenance is productive while infrastructure investment is unproductive at the current allocation of total government expenditure. To this extent, estimation of growth equations which include the *share* of maintenance as an explanatory variable could reveal whether an economy directs adequate resources towards maintenance. In a similar vein, a positive (negative) impact of total expenditures devoted to ‘new’ investment and maintenance on growth indicates that a rise (fall) of total expenditures as a fraction of GDP would *ceteris paribus* enhance (reduce) growth.

Ideally, one would like to test the implications of the model in the context of a wide cross-country panel data set consisting of data on maintenance and expenditures in ‘new’ capital. Unfortunately, published data on maintenance is very scarce involving only a handful of private activities due to inherent problems in the measurement of the maintenance expenditures, which traditionally appear under various categories in national accounts systems, such as repairs or renovations. In a recent paper, McGrattan and Schmitz (1999) describe the difficulties in constructing aggregate measures of maintenance and repair. The authors point out that, for instance, in the United States maintenance activities are, by and large, carried out outside the market and, thus, recorded transactions are usually incomplete. When such transactions exist, there is no systematic data collection, except for some manufacturing data, which renders the estimation of empirical relationships hard to implement in practice.<sup>8</sup>

Another potential application involves simulations with calibrated parameters in

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<sup>8</sup>At the aggregate level there is only one source of long-run data on maintenance expenditures,

the context of a full-fledged general equilibrium model. In this vein, the theoretical results derived in section 3.2 are confirmed by Rioja (2002) for Latin America. The author shows that substantial welfare gains are obtained when investment is reallocated between new projects and maintenance expenditures in favor of the latter. As pointed out by the author, it seems that at the present stage these countries would substantially benefit from a doubling of maintenance expenditures financed by a cut in expenditures for new projects.<sup>9</sup>

Finally, a straightforward empirical implication of Lemma 1 in section 4 is that standard tests on the impact of public capital on growth obtained via the production function approach may have underestimated its positive impact by ignoring the channel which operates through maintenance expenditures on public capital durability. In particular, if the assumptions of the model are correct, the optimal size of the public capital stock (and consequently the optimal government size) may well exceed its traditional elasticity in the production function, as originally put forward by Barro (1990). This may leave room for further exploration regarding the optimal size of government expenditures, provided that they are efficiently managed.

## 6. Concluding remarks

The aim of the paper was to highlight the macroeconomic aspects of maintenance expenditure in public capital. The main tool of our approach was an infrastructure-namely the Canadian survey on ‘Capital and Repair Expenditures’ conducted over the period 1956-1993, which contains data on maintenance expenditures for both private firms and government organizations. For a description, see McGrattan and Schmitz (1999).

<sup>9</sup>The concept of maintenance is particularly appealing in developing countries, where the transportation network is a prerequisite for sustained development. Quoting from the World Bank (1994) Annual Development Report: “...the rates of return from World Bank assisted road *maintenance* projects are nearly twice those of road *construction* projects. Timely maintenance expenditures of \$12 billion would have saved reconstruction cost of \$45 billion in Africa in the past decade” [our emphasis].

led endogenous growth model. In contrast to existing models which assume that the durability of public capital is exogenously given, in our model the depreciation rate of public capital varied according to the level of maintenance expenditure. It was shown that the government can benefit from a more efficient allocation of public resources between ‘new’ investment and maintenance to attain a higher growth rate. Our principal conclusion was that, in contrast to standard results derived by endogenous growth models with public infrastructure, the optimal taxation burden that maximizes the long-run growth rate of the economy is now larger than the elasticity of infrastructure in the production function. This is the result of the beneficial role of maintenance expenditure on public capital formation, which provides the economy with an additional benefit stemming from reduced capital decay. In this setup, we were able to calculate the optimal amount of maintenance expenditure and the allocation between ‘new’ investment and maintenance that a government should pursue to ensure maximum utilization of public resources.

We think that the paper makes a persuasive case for a strong theoretical link between maintenance expenditure, ‘new’ investment, and growth. However, absent any empirical evidence on public expenditure in maintenance, the broad cross-country empirical testing of the relationships outlined in the paper is, at the present stage of data availability, not possible. Albeit scarce existing evidence from other sectors of economic activity reveals that maintenance in developed economies covers a substantial fraction of total expenditure in capital formation, there has been no particular interest (with the exception of Canada in the past) in collecting data on maintenance expenditure in public capital. This lack of data is largely due to the nature of maintenance expenditure: as there is no market and recorded transactions for maintenance, data collection requires the planning of surveys in public organizations to obtain an accurate estimate of maintenance expenditure and its effect on the depreciation rate of public capital. Hopefully, in the future data collection in the public sector either

via a unified survey or by the inclusion of related questions in existing surveys will yield valuable information for the conduct of optimal public investment policy.

## References

Alogoskoufis G., 1995, 'The two faces of Janus: Institutions, policy regimes and macroeconomic performance in Greece', *Economic Policy*, 20, 149-192.

Aschauer D., 1989, 'Is public expenditure productive?', *Journal of Monetary Economics*, 23, 177-200.

Barro R., 1990, 'Government spending in a simple model of endogenous growth', *Journal of Political Economy*, 98, 103-125.

Barro R.J. and X. Sala-i-Martin, 1992, 'Public finance in models of economic growth', *Review of Economic Studies*, 59, 645-661.

Bitros G., 1976, 'A statistical theory of expenditures in capital maintenance and repair', *Journal of Political Economy*, 84, 917-936.

Devarajan S., V. Swaroop and H. Zou, 1996, "The composition of public expenditure and economic growth", *Journal of Monetary Economics*, 37, 313-344.

Devarajan S., D. Xie and H. Zou, 1998, 'Should public capital be subsidized or provided?', *Journal of Monetary Economics*, 41, 319-331.

Feldstein M. and M. Rothschild, 1974, 'Towards an economic theory of replacement investment', *Econometrica*, 42, 393-423.

Glomm G. and B. Ravikumar, 1994, 'Public investment in infrastructure in a simple growth model', *Journal of Economic Dynamics and Control*, 18, 1173-1187.

Haavelmo T., 1960, *A Study in the Theory of Investment*, University of Chicago Press, Chicago.

Hayek F., 1935, 'The maintenance of capital', in I. Kirzner (ed.), *Classics in Austrian economics: A sampling in the history of a tradition. Volume 3: The age of Mises and Hayek* (1994), Pickering and Chatto, London.

Hicks J.R., 1942, 'Maintaining capital intact: A further suggestion', *Economica*, 9, 174-179.

McGrattan E. and J. Schmitz, 1999, 'Maintenance and repair: Too big to ignore',

*Federal Reserve Bank of Minneapolis Quarterly Review*, 23, 2-13.

Nelson R. and M. Caputo, 1997, 'Price changes, maintenance, and the rate of depreciation', *Review of Economics and Statistics*, 79, 422-30.

Newberry D., 1988, 'Road damage externalities and road user charges', *Econometrica*, 56, 295-316.

Parks R., 1979, 'Durability, maintenance and the price of used assets', *Economic Inquiry*, 17, 197-217.

Rioja F., 2002, 'Filling potholes: Macroeconomic effects of maintenance vs. new investment in public infrastructure', *Journal of Public Economics* (forthcoming).

Romer P., 1986, 'Increasing returns and long-run growth', *Journal of Political Economy*, 94, 1002-1037.

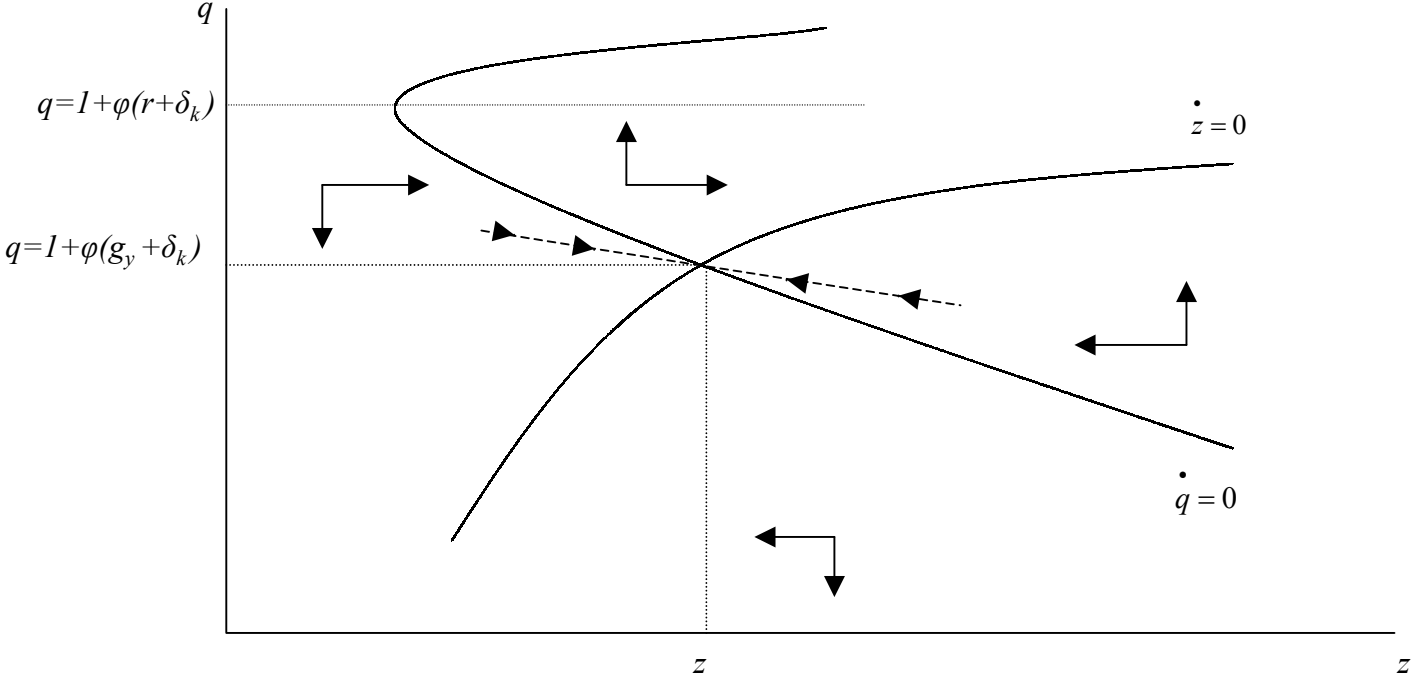
Schmalensee R., 1974, 'Market structure, durability, and maintenance effort', *Review of Economic Studies*, 41, 277-87.

Su T., 1975, 'Durability of consumption goods reconsidered', *American Economic Review*, 65, 148-57.

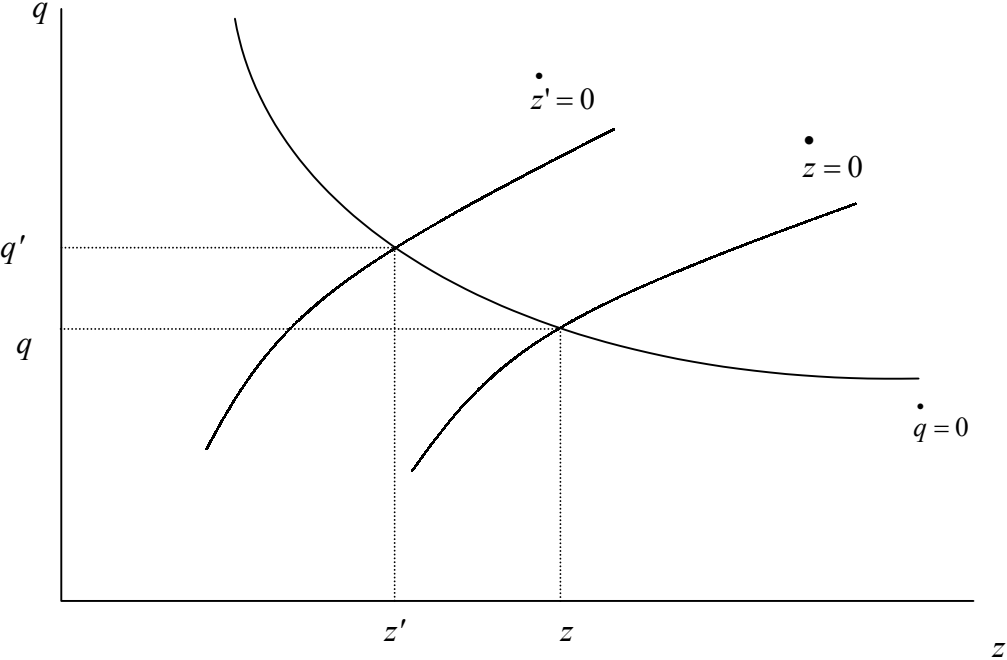
Turnovsky S., 1997, 'Public and private capital in an endogenously growing open economy', in B.S. Jensen and K. Wong (eds.), *Dynamics, Economic Growth and International Trade*, Ann Arbor: The University of Michigan Press.

World Bank, 1994, *World Development Report 1994: Infrastructure for Development*, Oxford University Press, New York.

**FIGURE 1. Dynamics of shadow price of private capital and private to public capital ratio**



**FIGURE 2. A reallocation between maintenance and ‘new’ infrastructure**



**FIGURE 3. A decline in excess taxation with given maintenance to 'new' infrastructure ratio**

