# A Control Chart for Monitoring Process Variability

S. Psarakis[1], J. Panaretos and M. Kiani
*Department of Statistics*
*Athens University of Economics and Business*
*76 Patision St, 10434 Athens Greece*

## ABSTRACT

The Shewhart and the Bonferroni-adjustment S control charts are usually applied to monitor the standard deviation of a quality characteristic. The control limits of these charts are constructed using approximately the normal distribution in case that the standard deviation parameter is known or unknown. In this paper, we establish a new S chart that is based approximately on the normal distribution. The control limits of the new chart are depended on both the sample group size, *k*, and sample subgroup size, *n*. Additionally, the unknown standard deviation for the proposed approach is estimated by a uniformly minimum variance unbiased estimator. This estimator has variance less than the estimators used for the Shewhart and Bonferroni approach. Meanwhile, for our proposed approach with unknown standard deviation, the out-of-control average run length is slightly less than the Shewhart approach and considerably less than Bonferroni-adjustment approach as demonstrated through Monte Carlo simulation experiments.

KEY WORDS: Shewhart, Bonferroni-adjustment, Average run length

## 1. Introduction

The Shewhart standard deviation control chart was introduced by Shewhart [14]. Ott [9], Ryan [13], Smith [15], Quesenberry [12], Montgomery [8] extended the Shewhart standard deviation control chart. The Shewhart procedure usually is based on at least 20 to 25 sample group sizes (*k*) and at least 4 to 6 sample subgroup sizes (*n*). The Shewhart chart with known and unknown standard deviation parameter is based approximately on a random variable that is assumed to follow the normal distribution. The values of the subgroup standard deviations ($S_i = \sum_{j=1}^{n} (X_{ij} - \overline{X}_{i.})^2$) are plotted on the Shewhart chart that includes the center line $E(S_i)$ and the following control limits

---

[1] e-mail for correspondence: psarakis@aueb.gr

$$E(S_i) \pm Z_{\alpha/2}\sqrt{Var(S_i)} \qquad (1)$$

where, the quality characteristic $X_{ij}$ for $i = 1,2,...,k$ and $j = 1, 2,...,n$ ($j^{th}$ observation in $i^{th}$ subgroup) is supposed to be identically independent normal distribution with mean $\mu$ and variance $\sigma^2$, and $\overline{X}_{i.} = \sum_{i=1}^{n} X_{ij} / n$. Further, $E(S_i)$ and $\sqrt{Var(S_i)}$ are the mean and standard deviation of $S_i$, respectively.

Ryan [13] introduced the Bonferroni-adjustment control limits as an alternative of the Shewhart approach. The control limits are given by:

$$E(S_i) \pm Z_{\alpha/2k}\sqrt{Var(S_i)} \qquad (2)$$

Here, the value $\alpha$ of the Shewhart control limits is replaced by the value $\alpha / k$ to construct the Bonferroni-adjustment control limits.

In this paper we introduce a new S control chart. In case that the standard deviation ($\sigma$) is known, the new chart is constructed similarly to the Shewhart and Bonferroni control chart. When the standard deviation is unknown the proposed chart is estimated using a statistic with variance less than that of the Shewhart and Bonferroni-adjustment chart. Furthermore, the constant value for the new chart with unknown standard deviation is depended on the sample subgroup and group sizes (*n*, *k*) whereas the constant value of the Shewhart and Bonferroni chart is depended only on the sample subgroup size (n). The in-control average run length ($ARL_0$) with fixed significance value $\alpha$ for Bonferroni approach is less than the $ARL_0$ for the Shewhart and proposed approach. On the other hand, the out-of-control average run length ($ARL_1$) for the new approach with unknown parameter $\sigma$ is less than both Shewhart and Bonferroni approach.

In this paper, the Shewhart, the Bonferroni, and the proposed chart are introduced in sections 2 and 3. In sections 4 and 5 the in-control and out-of-control average run length are presented.

## 2. The Shewhart and Bonferroni S Chart

If the quality characteristic $X_{ij} \overset{iid}{\sim} N(\mu, \sigma^2)$, for $i = 1,2,...,k$ and $j = 1, \ 2,...,n$, then the random variable $\sqrt{(n-1)}S_i / \sigma$ is identically independent chi distributed with n-1 degrees of freedom. The mean and standard deviation of this random variable are $\sigma c_4$ and $\sigma\sqrt{1-c_4^2}$, respectively. Here,

$$c_4 = \sqrt{\frac{2}{n-1}}\Gamma\left[\frac{n}{2}\right]\Big/\Gamma\left[\frac{(n-1)}{2}\right].$$

The Shewhart control chart with known and unknown parameter $\sigma$ are based on the random variables $\left(S_i - E(S_i)\right) / \sqrt{Var(S_i)}$ and $\left(S_i - E(\hat{S}_i)\right) / \sqrt{Var(\hat{S}_i)}$, respectively. These variables with large sample size such as $k \geq 20$ and $n \geq 4$ are approximately standard normal distributed.

The Shewhart control limits for known parameter $\sigma$ with confidence $1-\alpha$ are given below:

$$\sigma(c_4 \pm Z_{\alpha/2}\sqrt{1-c_4^2}) \tag{3}$$

The center line of the Shewhart control chart is $\sigma c_4$, and the constant value $c_4$ depends only on the sample subgroup size (*n*).

If the standard deviation of the quality characteristic is unknown then it is estimated by the unbiased statistic $\overline{S} / c_4$. For the Shewhart chart with unknown standard deviation

$$E(\hat{S}_i) = \overline{S} \ ; \ \sqrt{Var(\hat{S}_i)} = \overline{S}\sqrt{1-c_4^2} / c_4 \tag{4}$$

As a result, the center line and the control limits for the Shewhart chart with unknown parameter ($\sigma$) would be

$$U\hat{C}L = (\overline{S} / c_4)(c_4 + z_{\alpha/2}\sqrt{1-c_4^2}), \quad \hat{C}L = \overline{S}, \quad L\hat{C}L = (\overline{S} / c_4)(c_4 - z_{\alpha/2}\sqrt{1-c_4^2}) \tag{5}$$

The Bonferroni-adjustment control chart to improve the probability of one or more false alarm of the Shewhart chart is suggested by Ryan [13]. The Bonferroni-adjustment control limits with known standard deviation parameter are given by

$$\sigma(c_4 \pm Z_{\alpha/2k}\sqrt{1-c_4^2}) \tag{6}$$

Furthermore, the Bonferroni control limits with unknown parameter are

$$U\hat{C}L = (\overline{S} / c_4)(c_4 + z_{\alpha/2k}\sqrt{1-c_4^2}) \text{ and } L\hat{C}L = (\overline{S} / c_4)(c_4 - z_{\alpha/2k}\sqrt{1-c_4^2}) \tag{7}$$

The center lines for Bonferroni chart with known and unknown standard deviation are $\sigma c_4$ and $\overline{S}$, respectively.

## 3. The New S Chart

To construct the new standard deviation chart, the estimators of the standard deviation are given by Glasser [3], Khan [6], Markowitz [7], Gurland [4], Prescott [10], Prescott [11], Healy [5], Arnholt and Hebert [1], Donatos [2], and Watson [17], Vardeman [16]. However, the suggested authors did not introduce an UMVU estimator of the standard deviation of the normal distribution $N(\mu,\sigma^2)$ with *k* subgroups of size *n*.

The random variable $k(n-1)S^2/\sigma^2$ is chi-square distributed. Therefore, random variable $H=\sqrt{k(n-1)}S/\sigma$ would follow the chi distribution with $k(n-1)$ degrees of freedom. The statistic $S$ is an injective function of the complete sufficient statistic $S^2$ and the statistic $S/\psi$ is an unbiased estimator of $\sigma$. Therefore, according to *Lehman-Scheffe theorem* the statistic $S/\psi$ is an UMVU estimator of $\sigma$, where,

$$S=\sqrt{\sum_{i=1}^{k}\sum_{j=1}^{n}(X_{ij}-\overline{X}_{i.})^2/\left(k(n-1)\right)}\,,\qquad \psi=\left(\sqrt{\frac{2}{\sqrt{k(n-1)}}}\Gamma\left(\frac{k(n-1)+1}{2}\right)/\Gamma\left(\frac{k(n-1)}{2}\right)\right).$$

The constant value $\psi$ depends on both the sample subgroup size (*n*) and the sample group size (*k*).

The control limits on the average of quality characteristic depend on the variability of production process. The quality characteristic $X_{ij}$ for $i=1,2,\dots,k$ and $j=1,2,\dots,n$ follows identically independent normal distribution with mean $\mu$ and variance $\sigma^2$. The new standard deviation control chart with known standard deviation as the Shewhart S control chart is given by

$$UCL=\sigma(c_4+z_{\alpha/2}\sqrt{1-c_4^2})\quad ;\quad CL=\sigma c_4\quad ;\quad LCL=\sigma(c_4-z_{\alpha/2}\sqrt{1-c_4^2})$$

To establish the proposed control chart with unknown standard deviation, we estimate $E(S_i)=\sigma c_4$ and $\sqrt{Var(S_i)}=\sigma\sqrt{1-c_4^2}$ by the UMVU estimators $Sc_4/\psi$ and $S\sqrt{1-c_4^2}/\psi$, respectively. Therefore, the control limits for the proposed chart with unknown standard deviation would be

$$U\hat{C}L=(S/\psi)(c_4+z_{\alpha/2}\sqrt{1-c_4^2})\quad ;\quad \hat{C}L=Sc_4/\psi\quad ;\quad L\hat{C}L=(S/\psi)(c_4-z_{\alpha/2}\sqrt{1-c_4^2})$$

In sections 4 and 5, the in-control and out-of-control average run length for the control charts are considered.

## 4. In-Control Average Run Length

The in-control average run length (called $ARL_0$) is *the average number of subgroup standard deviations* that should be plotted before a subgroup standard deviation indicates an out-of-control condition. The $ARL_0$ can be calculated from $ARL_0=1/p$.

In this section, the average run length is considered for *initial group and groups 2, 3, …* with known and unknown parameter when the process is in control. To illustrate, let the individual events $G_i$ denote that the subgroup standard deviations $S_i$ exceeds the control limits of the in control process ($\sigma = \sigma_0$).

*For the initial group with unknown parameter (*$\sigma$*),* the events $G_i$ and $G_j$ for $i \neq j = 1,2,...,k$ are not independent. In case that the events $G_i$ were independent, then the sequence of trials, comparing $S_i$ with $U\hat{C}L$ would be a sequence of Bernoulli trials and the run length between occurrences of $G_i$ would be a Geometric random variable with probability $\alpha = P(G_i)$. In addition, the in-control average run length would be $1/P(G_i)$ or $1/\alpha$ where, $P(G_i) = P(S_i \leq L\hat{C}L \ or \ S_i \geq U\hat{C}L \,|\, \sigma = \sigma_0)$.

However, the statistics $S_i - U\hat{C}L$ and $S_j - U\hat{C}L$ for *initial group with unknown parameter* are not independent events. Therefore, the in-control $ARL$ for initial group with unknown parameter can not be calculated.

*For the initial group with known parameter and groups 2,3, …***,** the correlation between random variables $S_i - UCL$ and $S_j - UCL$ can be obtained to be 0. Thus, the events $G_i$ and $G_j$ for initial group with known parameter are uncorrelated. Consequently, the sequence of events $\{G_i\}$, for the initial group with known parameter and the groups 2,3,… with known and unknown parameter, would be Bernoulli trials and the run length between occurrences of $G_i$ would be a Geometric random variable with probability, $P(G_i)$. The probability $P(G_i)$ for both the Shewhart and the new approach is $\alpha$, and for the Bonferroni-adjustment approach is $\alpha / k$. As a result, the in-control average run length ($ARL_0$) for the Shewhart and the proposed chart ($1/\alpha$) is less than the $ARL_0$ for Bonferroni-adjustment chart ($k/\alpha$, for $k \geq 2$).

## 5. Out-of-Control Average Run Length

The ability of the standard deviation control charts to detect shifts in process quality is described by the out-of-control average run length ($ARL_1$). The probability of a false alarm and the probability of one or more false alarm of the process in control were improved by using the

Bonferroni-adjustment approach. In this section, the power of control limits for the usual approach (Shewhart and Bonferroni) and the new approach are compared by using $ARL_1$.

If the in-control value of the standard deviation shifts from $\sigma_0$ to $\sigma_1 = \lambda\sigma_0 > \sigma_0$, ($\lambda > 1$) the probability of not detecting standard deviation shift ($\beta$) is calculated by

$$\beta = P\left(LCL \le S_i \le UCL \,\middle|\, \sigma = \lambda\sigma_0\right)$$

This probability for both the Shewhart and proposed approach with known parameter ($\sigma$) is obtained

$\beta = P(\frac{-Z_{\alpha/2}\sqrt{1-c_4^2} + c_4(1-\lambda)}{\lambda\sqrt{1-c_4^2}} \le Z_i \le \frac{Z_{\alpha/2}\sqrt{1-c_4^2} + c_4(1-\lambda)}{\lambda\sqrt{1-c_4^2}})$. The probability $\beta$ for the Bonferroni-adjustment approach with known parameter ($\sigma$) is obtained

$\beta = P(\frac{-Z_{\alpha/2k}\sqrt{1-c_4^2} + c_4(1-\lambda)}{\lambda\sqrt{1-c_4^2}} \le Z_i \le \frac{Z_{\alpha/2k}\sqrt{1-c_4^2} + c_4(1-\lambda)}{\lambda\sqrt{1-c_4^2}})$. Usually the parameter $\sigma$ is unknown. Under this condition, we obtain the probability $\beta$ for the control limits with unknown parameter. The standard deviation for the Shewhart, Bonferroni, and new approach the same is estimated by the UMVU estimator $S/\psi$ (called $S_t$). Thus, the probability $\beta$ for the Shewhart, the Bonferroni and the new approach with unknown parameter are given below,

$$\beta = P\left(\frac{(\bar{S}/c_4)(c_4 - Z_{\alpha/2}\sqrt{1-c_4^2}) - \lambda S_t c_4}{\lambda S_t\sqrt{1-c_4^2}} \le Z_i \le \frac{(\bar{S}/c_4)(c_4 + Z_{\alpha/2}\sqrt{1-c_4^2}) - \lambda S_t c_4}{\lambda S_t\sqrt{1-c_4^2}}\right),$$

$$\beta = P\left(\frac{(\bar{S}/c_4)(c_4 - Z_{\alpha/2k}\sqrt{1-c_4^2}) - \lambda S_t c_4}{\lambda S_t\sqrt{1-c_4^2}} \le Z_i \le \frac{(\bar{S}/c_4)(c_4 + Z_{\alpha/2k}\sqrt{1-c_4^2}) - \lambda S_t c_4}{\lambda S_t\sqrt{1-c_4^2}}\right),$$

$$\beta = P\left(\frac{(S/\psi)(c_4 - Z_{\alpha/2}\sqrt{1-c_4^2}) - \lambda S_t c_4}{\lambda S_t\sqrt{1-c_4^2}} \le Z_i \le \frac{(S/\psi)(c_4 + Z_{\alpha/2}\sqrt{1-c_4^2}) - \lambda S_t c_4}{\lambda S_t\sqrt{1-c_4^2}}\right).$$

The probability $\beta$ with known standard deviation for various sample sizes *n* and coefficient $\lambda$ is exhibited by the operating-characteristic (OC) curves.

**Figure 1** Operating characteristic curves for the Shewhart and new chart (k=20 , a=0.01).

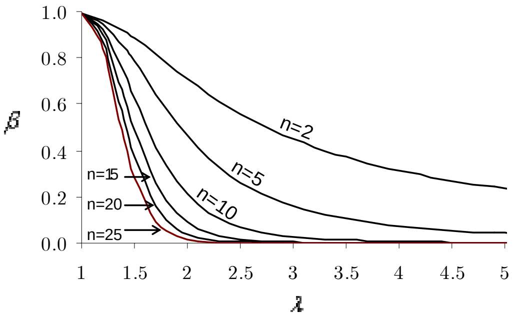
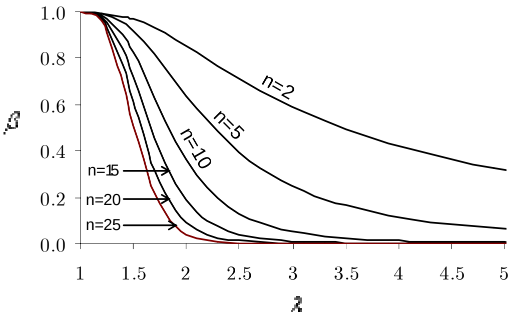


**Figure 2** Operating characteristic curves for the Bonferroni chart (k=20 , a=0.01).



Figures 1 and 2 indicate that the Shewhart and the new approach are considerably effective in detecting small and large shifts on the first sample following the shift.

The probability $\beta$ with unknown standard deviation for various sample sizes *n* and *k*, and coefficient $\lambda$ is exhibited by Table 1 and Table 2. These tables are constructed by using Monte Carlo simulation experiments.

**Table 1** Estimated probability of not detecting standard deviation shift ( $\beta$ ) for $\alpha = 0.1$

|  |  | n=2 |  |  |  | n=10 |  |  |  | n=25 |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| k | Approach/ $\lambda$ | **1.5** | **2** | **2.5** | **3** | **1.5** | **2** | **2.5** | **3** | **1.5** | **2** | **2.5** | **3** |
| 10 | Shewhart | 0.757 | 0.618 | 0.529 | 0.431 | 0.431 | 0.116 | 0.025 | 0.008 | 0.226 | 0.011 | 0.000 | 0.000 |
|  | Bonferroni | 0.881 | 0.772 | 0.648 | 0.560 | 0.686 | 0.246 | 0.078 | 0.016 | 0.467 | 0.049 | 0.001 | 0.000 |
|  | New | 0.745 | 0.598 | 0.523 | 0.425 | 0.419 | 0.116 | 0.023 | 0.009 | 0.222 | 0.010 | 0.000 | 0.000 |
| 20 | Shewhart | 0.772 | 0.616 | 0.529 | 0.442 | 0.446 | 0.122 | 0.025 | 0.007 | 0.226 | 0.012 | 0.001 | 0.000 |
|  | Bonferroni | 0.909 | 0.780 | 0.591 | 0.594 | 0.743 | 0.314 | 0.096 | 0.035 | 0.467 | 0.065 | 0.004 | 0.000 |
|  | New | 0.768 | 0.602 | 0.527 | 0.437 | 0.445 | 0.120 | 0.023 | 0.007 | 0.222 | 0.011 | 0.001 | 0.000 |
| 40 | Shewhart | 0.767 | 0.634 | 0.538 | 0.442 | 0.442 | 0.132 | 0.035 | 0.007 | 0.226 | 0.011 | 0.001 | 0.000 |
|  | Bonferroni | 0.910 | 0.809 | 0.709 | 0.615 | 0.777 | 0.342 | 0.124 | 0.045 | 0.467 | 0.085 | 0.007 | 0.000 |
|  | New | 0.761 | 0.630 | 0.521 | 0.441 | 0.441 | 0.132 | 0.034 | 0.006 | 0.222 | 0.012 | 0.001 | 0.000 |
| 120 | Shewhart | 0.772 | 0.634 | 0.530 | 0.453 | 0.448 | 0.131 | 0.034 | 0.012 | 0.225 | 0.012 | 0.000 | 0.000 |
|  | Bonferroni | 0.939 | 0.839 | 0.750 | 0.652 | 0.834 | 0.423 | 0.165 | 0.058 | 0.462 | 0.129 | 0.010 | 0.001 |
|  | New | 0.769 | 0.632 | 0.530 | 0.453 | 0.447 | 0.129 | 0.032 | 0.011 | 0.221 | 0.013 | 0.000 | 0.000 |

The results in Tables 1 and 2 show that with unknown standard deviation the new chart performs better than both the Shewhart and the Bonferroni-adjustment approach. Consequently, the out-of-control average run length would be $ARL_1 = 1/(1-\beta)$ Using the OC curves 1 and 2, the $ARL_1$ for the Shewhart and new approach is demonstrated less than the Bonferroni-adjustment approach. In addition, for unknown parameter $\sigma$, we demonstrate trough Monte Carlo simulation experiments that the $ARL_1$ for the new chart is slightly less than the Shewhart chart and considerably less than the Bonferroni-adjustment chart.

**Table 2** Estimated probability of not detecting standard deviation shift ( $\beta$ ) for $\alpha = 0.01$

| | | n=2 | | | | n=10 | | | | n=25 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| k | Approach/ $\lambda$ | **1.5** | **2** | **2.5** | **3** | **1.5** | **2** | **2.5** | **3** | **1.5** | **2** | **2.5** | **3** |
| 10 | Shewhart | 0.904 | 0.760 | 0.656 | 0.550 | 0.634 | 0.214 | 0.038 | 0.020 | 0.264 | 0.010 | 0.000 | 0.000 |
| | Bonferroni | 0.956 | 0.840 | 0.748 | 0.636 | 0.816 | 0.328 | 0.084 | 0.042 | 0.478 | 0.024 | 0.002 | 0.000 |
| | New | 0.896 | 0.752 | 0.652 | 0.548 | 0.628 | 0.214 | 0.037 | 0.020 | 0.258 | 0.011 | 0.000 | 0.000 |
| 20 | Shewhart | 0.889 | 0.763 | 0.657 | 0.551 | 0.635 | 0.215 | 0.039 | 0.021 | 0.262 | 0.015 | 0.000 | 0.000 |
| | Bonferroni | 0.959 | 0.841 | 0.745 | 0.638 | 0.818 | 0.329 | 0.084 | 0.042 | 0.477 | 0.029 | 0.002 | 0.000 |
| | New | 0.889 | 0.755 | 0.655 | 0.549 | 0.629 | 0.214 | 0.039 | 0.021 | 0.254 | 0.016 | 0.000 | 0.000 |
| 40 | Shewhart | 0.889 | 0.766 | 0.658 | 0.553 | 0.637 | 0.212 | 0.043 | 0.022 | 0.266 | 0.015 | 0.000 | 0.000 |
| | Bonferroni | 0.955 | 0.841 | 0.750 | 0.632 | 0.819 | 0.327 | 0.088 | 0.043 | 0.481 | 0.028 | 0.002 | 0.000 |
| | New | 0.887 | 0.756 | 0.656 | 0.549 | 0.634 | 0.214 | 0.042 | 0.023 | 0.265 | 0.013 | 0.000 | 0.000 |
| 120 | Shewhart | 0.895 | 0.761 | 0.654 | 0.552 | 0.639 | 0.214 | 0.045 | 0.022 | 0.267 | 0.012 | 0.000 | 0.000 |
| | Bonferroni | 0.957 | 0.843 | 0.747 | 0.639 | 0.821 | 0.327 | 0.089 | 0.044 | 0.479 | 0.025 | 0.004 | 0.000 |
| | New | 0.892 | 0.759 | 0.654 | 0.551 | 0.639 | 0.214 | 0.043 | 0.023 | 0.267 | 0.012 | 0.000 | 0.000 |

## 6. Conclusions

It has been shown that, for unknown standard deviation parameter, the new standard deviation control chart has three advantages over the Shewhart and the Bonferroni-adjustment standard deviation control chart: first the constant values to construct the standard deviation control chart for the new approach are based on both sample subgroup size and sample group size; second for fixed value $\alpha$ the $ARL_1$ for the new approach is less than the $ARL_1$ for the Shewhart and Bonferroni-adjustment approach; third the new approach is based on the statistic with variance less than one of the Shewhart and Bonferroni-adjustment approach.

## References

[1]. Arnholt, A. T., and Hebert, J. L. (1995). Estimating the Mean With Known Coefficient of Variation. *The American Statistician*, Vol. 49, No. 4, pp. 367-369.

[2].Donatos, G. S. (1989). A Monte Carlo Study of $k$-Class Estimators for Small Samples with Normal and Non-Normal Disturbances. *The Statistician*, Vol. 38, No. 1, pp. 11-20.

[3].Glasser, G. J. (1962). On Estimators for Variances and Covariances. *Biometrika*, Vol. 49, No. ½, pp. 259-262.

[4].Gurland, J., and Tripathi, R. C. (1971). A Simple Approximation for Unbiased of the Standard Deviation. *The American Statistician*, Vol. 25, No. 4, pp. 30-32.

[5].Healy, M. J. R. (1978). A Mean Difference Estimator of Standard Deviation in Symmetrically Censored Normal Samples. *Biometrika*, Vol. 65, No. 3, pp. 643-646.

[6].Khan, R. A. (1968). A Note on Estimating the Mean of a Normal Distribution with Known Coefficient of Variation. *Journal of the American Statistical Association*, Vol. 63, No. 323, pp. 1039-1041.

[7].Markowitz, E. (1968). Minimum Mean-Square-Error Estimation of the Standard Deviation of the Normal Distribution. *The American Statistician*, Vol. 22, No. 3, pp. 26-26.

[8].Montgomery, D. C. (2001). *Design and Analysis of Experiments*. 5th edition. Wiley, New York.

[9].Ott, E. R. (1975). *Process Quality Control. McGraw-Hill Book Company*, New York.

[10].Prescott, P. (1971a). Use of a Simple Range-Type Estimator of standard deviation in Test of Hypotheses. *Biometrika*, Vol. 58, No. 2, pp. 333-340.

[11].Prescott, P. (1971b). Distribution of the Normal Scores Estimator of the Standard Deviation of a Normal Population. *Biometrika*, Vol. 58, No. 3, pp. 631-636.

[12].Quesenberry, C. P. (1997). *SPC Methods for Quality Improvement*. John Wiley & Sons.

[13].Ryan, T. P. (1989). *Statistical Methods for Quality Improvement.* John Wiley & Sons.

[14].Shewhart, W. A. (1931). *Economic Control of Quality of Manufactured Product*, D. van Nostrand Co., New York.

[15].Smith. G. M. (1998). *Statistical Process Control and Quality Improvement*. Prentice-Hall, Inc.

[16].Vardeman, S. B. (1999). A Brief Tutorial on the Estimation of the Process Standard Deviation. *IIE Transactions,* Vol. 31, pp. 503-507.

[17].Watson, S. (1997). Evaluation of Semivariance Estimators Under Normal Conditions. *The Statistician,* Vol. 46, No. 4, pp. 495-503.