An applied methodology for the prediction of time series' local optima

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Abstract

Time series prediction is directly connected with the major problem of portfolio optimization; its solution is detected in numerous research and in many statistical methodologies which have implied towards the investigation of the best set of assets. In finance, the time series theory is mainly applied for the prediction of the stock market prices or in applications regarding the currency levels.

Recently, have proposed gradient unconstrained optimization algorithms that are being used in the process of the Lipschitz constant estimation towards the approximation of the objective functions optima. More detailed, the estimation of the Lipschitz constant is calculated on sequenced points that come from the repetitive process of optima finding, and their function values, as well. Furthermore, it is proved that the use of this step size, given from the Lipschitz constant estimation, leads to local optimum point.

This paper attempts to forecast a time series future optima by applying the Steepest Descent with the Adaptive Step size (SDAS) algorithm. Thus, $n-1$ past and known points from the time series are chosen in such way that all necessary conditions are applied; furthermore these points represent sequence points of a repetitive process that leads to local optima of the objective function.

The proposed methodology was tested on the daily closing prices of the Athens' Stock Market. The results obtained provide clues that the proposed methodology predicts the local maxima and minima in a rather successive rate. What, however, should be furtherly investigated is the degree that each characteristic of the sample and their occasional fluctuations may affect the results' accuracy.

Index Terms

Time series forecasting, Lipschitz coefficient, portfolio optimization, unconstrained optimization.

I. INTRODUCTION

TIME series predictability in relation to the issue of portfolio optimization, has promoted into a major subject of study for numerous research and various statistical methodologies have implied when investigating the best set of assets [1], [2].

In financial terms, the time series theory focuses on the prediction of the stock market prices or on applications regarding the investigation of currency levels [3], [4]. Transactions, that follow the supply and demand forces, follow a continuous mode, and the value known as the "closing value" is the one used to indicate a day's yield. In practice, however, what any investor seeks for includes the points where the maximization of profit (or the minimization of loss), could be achieved; that in the mathematical spectrum is known as finding local minima and local maxima, respectively [5].

The general optimization problem is

$$\begin{aligned} \min_{x} \quad & f(x) \\ & g_i(x) = 0, i \in \mathcal{E}, \\ & g_i(x) \geq 0, i \in \mathcal{I}, \end{aligned} \tag{1}$$

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where f and g_i are functions of $\mathbb{R}^n \to \mathbb{R}$, and $\mathcal{E}$ and $\mathcal{I}$ are index sets for the equality and inequality constraints respectively. A well known class of algorithms for unconstrained minimization of functions $f(x)$ in n real variables

$$f\colon \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}, \tag{2}$$

having Lipschitz continuous first partial derivatives whose gradient $\nabla f(x)$ is available, is the steepest descent methods [6], [7] first proposed by Cauchy in 1847 [8]. The iterations are made according to the following equation:

$$x^{k+1} = x^k - \lambda^k \nabla f(x^k), \qquad k = 0, 1, 2, \ldots, \tag{3}$$

where λ^k is the smallest non-negative value of λ that locally minimizes f along the direction $-\nabla f(x^k)$ starting from x^k.

In this paper we propose an application of the SDAS algorithm - that belongs to the steepest descent algorithms - in combination with the backtracking technique proposed by Androulakis et al. [5]. The backtracking technique allows any optimization algorithm that obtains “memory”, to be applied towards finding future local optima.

Section II includes two parts: a brief description of the theoretical methodologies for portfolio optimization is presented in II-A while in II-B the applied algorithm is outlined. The methodology proposed in III is applied on the data described in section IV; section V includes the results obtained and promotes further research interests.

II. Literature review

A. Time series prediction for Portfolio Optimization

The earliest attempts towards forecasting future values, including stock market prices were mainly focused on macro economical methodologies based on theoretical models. What H. Markowitz proposed in 1952, [9], was afterwards used as the cornerstone of the portfolio selection research. Such a model assumes that if $Y_1, Y_2, \ldots, Y_n$ forms the time series, at time n for $n \geq 1$, interest is focused in predicting the next value Y_{n+1} based on the observed realizations of $Y_1, Y_2, \ldots, Y_n$, [10] and it could be characterized as a forward model, since the aim focuses on future values. The pioneer contribution of Markowitz’s mean-variance model is that the future asset returns can be detected through the historical information provided by past data, by assuming that the means, variances and covariances on both cases are similar. Despite of the practical aspect of this assumption in the dynamic stock markets, most of the existing portfolio selection models are based on probability theory; [11]–[19].

The distinguished linear time series models that have applied include the exponential smoothing model introduced by Brown [20] and the Box and Jenkins [21] one. This applies autoregressive integrated moving average (ARIMA) models to find the best fit of a time series to past values of this time series, in order to make forecasts; boths effectiveness though is a rather controversial issue [22]. Further, the non-linear models used to improve linear models’ inefficiencies [23], [24] that use both linear and non-linear time series models with a common set of objective, still put aside the issue of the best time to formulate or deal a stock portfolio.

[25] and [26] have included the concept of the stock market timing in theoretical means, involving financial trends and macroeconomic policies. Apart from the theoretical contributions, a different point introduced by [27] and [28] who incorporated with the best stock market timing by using Artificial Intelligence applications in order to predict future stock price movements including not only past, known prices but also price and market volatility factors that affect the stock price under certain different volumes of weight, as well.

B. Optimization techniques and SDAS algorithm

Lots of the unconstrained minimization methods are iterative in nature and hence they start from an initial random solution and proceed towards the minimum point in a sequential manner.

Algorithm 2.1: General iterative scheme of optimization.
Step 1. Start with a random point X^0.
Step 2. Set $i = 1$
Step 3. Generate a new approximation of local optimum X^1.
Step 4. If convergence rules satisfied go to Step 6.
Step 5. Set $i = i + 1$. Go to Step 3.
Step 6. Set $X^{opt} = X^i$.

Armijo provided in 1966 [29] a modification of the steepest descent method which automatically adapts the stepsize λ of the iterative scheme (3). Armijo supposed that if ∇f is Lipschitz continuous on the bounded level set $\mathcal{S}(x^0) = \{x : f(x) \leq f(x^0)\}$, i.e. there exists a Lipschitz constant $K > 0$, such that

$$\|\nabla f(x) - \nabla f(y)\| \leq K\|x - y\|, \tag{4}$$

for every pair $x, y \in S(x^0)$. The value of the stepsize λ in (3) has been related to the value of the Lipschitz constant K, [29]. In this case, the well known *Cauchy's method* or *the steepest descent algorithm* [8] states that the sequence $\{x^k\}_{k=0}^{\infty}$, defined by

$$x^{k+1} = x^k - \frac{1}{2K}\nabla f(x^k), \qquad k = 0, 1, 2, \dots, \tag{5}$$

converges to the point x^* which minimizes f (see [29] for a proof).

In [30] is proposed the following iterative scheme, named *SDAS*:

$$x^{k+1} = x^k - \frac{1}{2\Lambda^k}\nabla f(x^k), \qquad k = 0, 1, \dots, \tag{6}$$

where Λ^k is given by the relation:

$$\Lambda^k = \frac{\left\|\nabla f(x^k) - \nabla f(x^{k-1})\right\|}{\|x^k - x^{k-1}\|}. \tag{7}$$

In this way, the stepsize $0.5/\Lambda^k$ would be sensitive to the local shape of the objective function. Clearly, the iterative scheme (6) is related to the iterative scheme (5) and it converges when $0 \leq 0.5/\Lambda^k \leq K^{-1}$. This can be easily justified following, for instance, the proof in [29]. Obviously,while applying equation (7) in more than one points better approximation of the Λ^k may be calculated.

III. Backtracking with SDAS

A time series Y as a sequence of data points t formulates the $Y = Y_t : t \in T$ function, where t data points only refer to time. Consider Y_t be the time series of an asset's closing values and let Y_n denote the time series' value. The $f(t)$ function interprets the continuity of this daily exchanging process and its value at the closing time (as considered in stock market terms) is equal to the Y_t.

Since investors concentrate on exchanging stocks when their prices are found to be the most profitable ones, what they actually aim is buying stocks as cheap as possible and selling stocks as expensive as possible. In mathematical means this could be translated as the local minimum and maximum of the function $f(t)$, respectively [5].

When applying the optimization techniques and SDAS algorithm described in equation (6) before, it is concluded that most of these techniques in order to generate the new point t_{n+k}, use prior knowledge collected from the process data including points, function values, gradient values, matrix approximations etc. Thus, the next estimate of local optimum is calculated as follows

$$t_{n+k} = t_n - \Phi(t_0, t_1, \ldots, t_n), \tag{8}$$

where $\Phi(\cdot)$ corresponds to the to process of the repetitive algorithm, (6). Thus, by applying equation (7)

$$\Phi(t_0, t_1, \ldots, t_n) = \frac{1}{2\Lambda^{max}} Y'_{t_n}, \tag{9}$$

where Λ^{max} is given by

$$\Lambda^{max} = \max \frac{\left\| Y'_{t_i} - Y'_{t_j} \right\|}{\| t_i - t_j \|}, \tag{10}$$

where $i = 1, \ldots, n$, $j = 1, \ldots, n$ and $i \neq j$.

Practically, we seek for future optima $Y_{t_{max}}$, where $max > n$, which is not possible to obtain without information regarding future values. Based on the available past data, we may approximate a past local minimum, denoted by t_{min}, using a "sequence" of m past points that starts at the last known asset value. This sequence can be formulated as $[t_n, t_{k_1}, t_{k_2}, \ldots, t_{k_{m-2}}, t_{min}]$, where $n > k_1 > k_2 > \cdots > k_{m-2} > min$. Moreover, the use of equation (10) for the calculation of all points of this sequence gives an excellent approximation of Λ^{max}.

If this "sequence" of points is viewed as a forward process, represents a "sequence" starting from the minimum past point t_{min}, crisscrosses the last known asset value t_n and it probably leads to a maximum future point t_{max}. By applying this backtrack technique, the constructed "sequence" of points provides us with the knowledge needed to estimate a future maximum. Thus, by applying a maximization technique

$$t_{n+k} = t_n + \Psi\left(t_{k_{m-2}}, \ldots, t_{k_1}, t_n\right) \tag{11}$$

that uses prior knowledge regarding points and since the "sequence" of points is known, it can lead to approximate the most appropriate $\Psi(\cdot)$ that finally converges to a future local maximum. In our case, $\Psi(\cdot) = 0.5/\Lambda^{max}$. In algorithm 3.1 the previously described process is presented step-by-step.

Algorithm 3.1: Backtrack with SDAS algorithm.

Step 1. Start with last known value t_n.
Step 2. Use Algorithm 2.1 to compute a sequence of points $[t_n, t_{k_1}, t_{k_2}, \ldots, t_{k_{m-2}}, t_{min}]$ leads to "past" local minimum.
Step 3. Calculate $\Lambda^{max} = max\{\Lambda^K\}$ using points $[t_{min}, t_{k_{m-2}}, \ldots, t_{k_2}, t_{k_1}, t_n]$, equation (10).
Step 4. Calculate "future" point t_{n+k} using equation (11).

IV. Practical Results

The proposed application is tested on the daily closing prices of the Athens' Stock Market. The data consists of the daily closing prices of 18 years - from 1985 until 2002. Our sample's characteristics are: (a) numerous stock prices, and indexes that are randomly selected, (b) different periods of time, and (c) different number of known closing prices and indexes. By applying the technique given by (11) on this data the results are quite satisfactory, in means that if the t_{n+k} is not equal to the t_{max}, then t_{n+k} not only is certainly greater than t_{max}, but it also represents a point of the "sequence" that leads to t_{max}.

In Figure 1 is presented an application of the SDAS backtrack algorithm towards predicting Athens' Stock Market general index for the randomly chosen date of July 13, 1998; then Algorithm 3.1 is applied to approximate a future local minima and maxima. The last 50 known values of the general index are used; i.e. in the case of July 13, 1998, the 50 last known indexes are from May 4, 1998 until July 13, 1998. These points are represented on Figure 1 with the square symbol. In Figure 1, again, the gray circles stand for the index's actual values for the exchanging period from July 14, 1998 till September

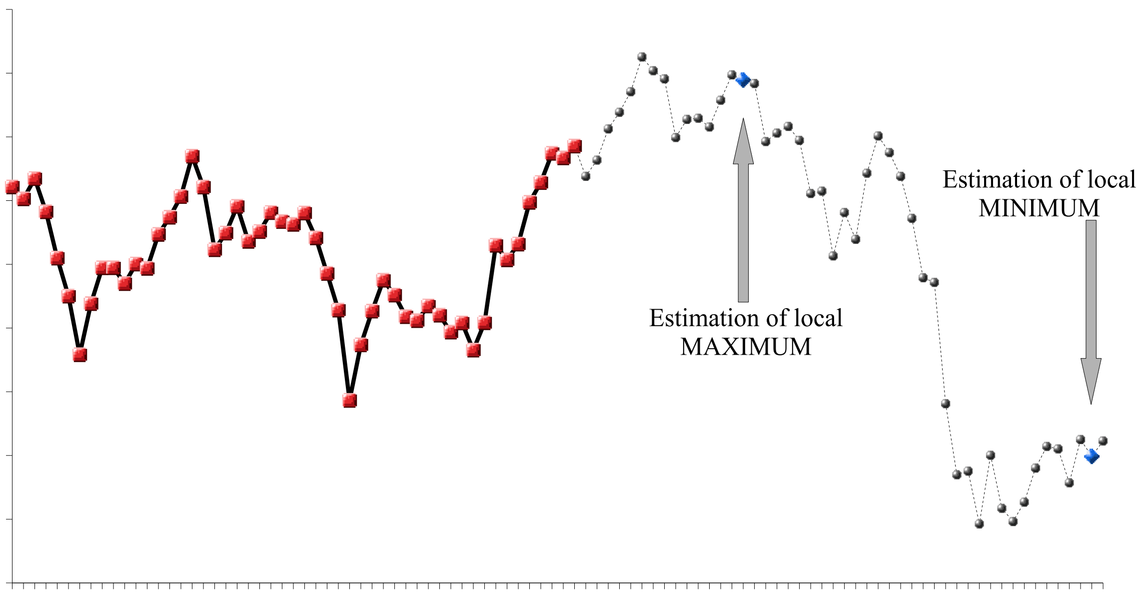

Fig. 1. An application of the SDAS backtrack algorithm on Athens Stock Market general index on July 13, 1998

16, 1998. Future values are connected together with the discontinuous line. The points depicted from the SDAS backtrack algorithm are symbolized with the rhomb symbol.

To investigate the SDAS backtrack algorithm's reliability, the algorithm was applied on 100 randomly chosen prices to approximate the dates t_{min} and t_{max} that produce a local minima and a local maxima, respectively. Since we assume that the investor seeks for assets that are relatively cheap to buy and their price increase, the table only includes the cases that the local minima emerges before the local maxima. The difference between the estimations for local minima and local maxima is extracted in terms of time, that is days of market activity. Furthermore, each asset's return was calculated based on the function

$$R = \frac{Y_{t_{max}} - Y_{t_{min}}}{Y_{t_{min}}} \tag{12}$$

On Table I are briefly reported the statistical indexes that characterize the variables days and return, equation (12). Specifically, the raw *days* includes indexes regarding the difference in market days between local minima and local optima, $t_{max} - t_{min}$; likewise, raw named *return* illustrates the indexes that result from variable R, equation (12). The abbreviations used as columns are referred to the local minimum, first quarter Q_1, mean value, median, third quarter Q_3 and standard deviation.

It is therefore, observed that the mean between the predicted local minima and local maxima is 15 days approximately. During this period the mean return is about 6.6%.

V. Conclusions and further research

In this paper we have applied the proposed backtrack technique using the Steepest Descent with Adaptive Stepsize (SDAS) algorithm incorporated in [30]. This method was selected because (a) uses prior knowledge (calculates an approximation of Lipschitz constant using all sequence points), and (b) as proposed by [31] the steepest descent methods are the most reliable as far as it concerns the convergence to the closest optimum; both characteristics could applied in portfolio optimization problems.

TABLE I
STATISTICAL INDEXES APPLIED ON RANDOMLY CHOSEN ASSETS

Quantity	**Min**	Q_1	**Median**	**Mean**	Q_3	**Max**	**s.d.**
Days	1.00	4.00	9.00	15.54	21.25	55.00	15.6548
Return	-0.23200	0.00200	0.05200	0.06621	0.11620	0.40200	0.13432

At this point, three directions could be proposed based on the backtrack technique: (a) applications of the backtrack technique in other minimization algorithms that use information collected from sequence points, such as quasi-Newton methods, conjugate gradient, etc, (b) using these techniques towards approximate a future minimum and a future maximum, as well, of an individual asset, so as to formulate an asset management technique for its behavior in an asset portfolio and (c) using backtrack technique with a multi-objective optimization method for managing a given portfolio Further research could be focused on these directions, due to the interesting results the technique provides; our research concentrates towards these directions, as well.

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