Snaky is a winner with one handicap

Hiro Ito and Hiromitsu Miyagawa

Abstract

Harary's generalized ticktacktoe, a kind of achievement games, have been investigated in many literatures. All animals (polyominos) except for "Snaky" have been decided whether they are winners or losers, i.e., Snaky is a unique unsolved animal. Moreover the handicap number, which is the minimum number of stones preset by the first player before starting the play for winning the game, of the Snaky have been known only at most two. That the handicap number of an animal is zero means it is a winner. In this paper we show a winning strategy of Snaky with one handicap. From this result we can see that the handicap number of Snaky is zero or one.

I. Introduction

Achievement games for polyominoes or *square animals* have been introduced by Frank Harary [1] and investigated by many literatures [2]–[6]. Given a polyomino P with n cells, two players Alice and Bob alternatively put black and white stones, respectively, on the cells of the square tessellation of the plane as a game board. The player who achieves a copy of P consisting of his/her stones only first wins the game, where reflections and $k\pi/2$-rotations for integers k are allowed. The polyomino P itself is called a *winner* if there exists a strategy for Alice, the first player, to win. Otherwise, P is called a *loser*. For this game, it is claimed that only eleven polyominoes are winners, the polyomino Snaky of Fig. 1 is undecided, and all remaining polyominoes are losers.

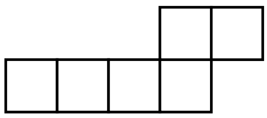


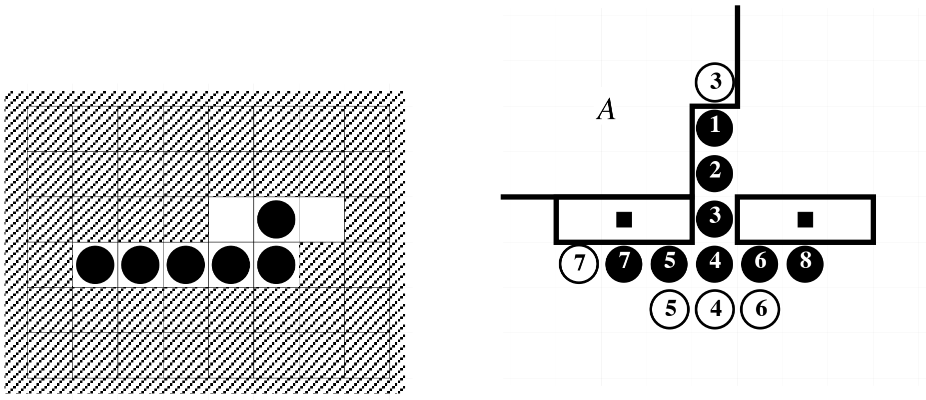
Fig. 1. Snaky.

A winning strategy for Bob cannot exist since every such strategy could be used by Alice one move earlier. Therefore that a polyomino P is a loser means that there exists a strategy for Bob to draw. Past studies have clarified that each polyominoe except Snaky is a winner or a loser. Only Snaky remains undecided and some papers have been given for examining Snaky [2], [4], [5], [6].

The more general *handicap achievement* have been introduced by Harary, *et al.* [2]. In handicap achievement games, Alice puts a given number of stones before the achievement game started. Now, for every polyomino P a minimum number of stones preset by Alice exists such that P is a winner for the remaining achievement game. The *handicap number* $h(P)$ is the corresponding minimum number of preset stones for the game. Clearly $0 \leq h(P) \leq n-1$ for every polyomino P consisting of n cells, and $h(P)=0$ if P is a winner for the original achievement game.

Harary, *et al.* [2] showed that the handicap number of Snaky is at most two by showing a winning strategy for Alice to achieve Snaky with two handicap stones. See Fig. 2, where black and white stones are of Alice and Bob, respectively. The number i on each stone means the stone is put in turn i. The two handicap stones are black stones 1 and 2, and thus white stones start from number 3. The blank squares mean really blank, i.e., there are no stones, and squares hatched by diagonal lines mean "don't care." If Alice achieves the construction of (a) of Fig. 2 then Alice wins, since putting a stone in one of the two blanks achieves a Snaky.

School of Informatics, Kyoto University. Mail: {itohiro@, miyagawa@lab2.}kuis.kyoto-u.ac.jp



(a) (b)

Fig. 2. The winning strategy of Snaky with two handicaps.

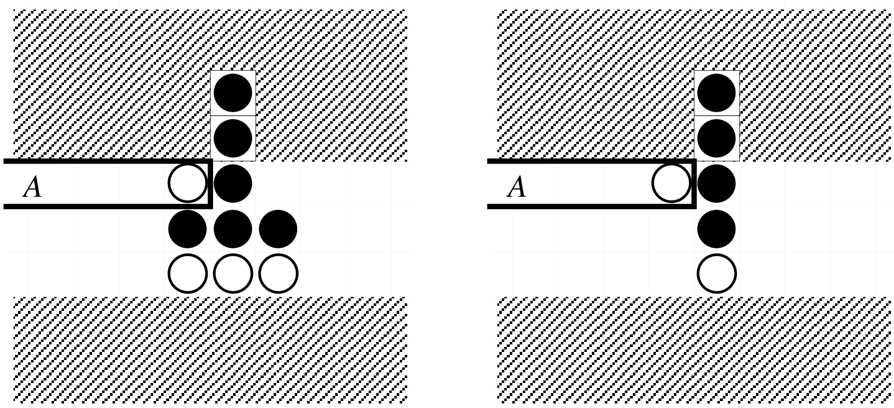
The winning strategy for handicap two is shown in (b) of Fig. 2. If Alice puts the first three stones as shown in (b), then Bob's the third movement (stone 3) is in the area A wlog. Thereafter each movement of Bob is forced as shown in the figure (otherwise Alice wins more quickly), and finally Bob cannot block two cells (marked by small black squares) in one movement and loses. From this strategy we can see $h(\text{Snaly}) \in \{0, 1, 2\}$. But so far no one has reduced the range of $h(\text{Snaly})$.

In this paper, we show a winning strategy for Alice to achieve Snaky with "one" handicap by sophisticating the above strategy. That is, we present the following result.

Theorem 1: The handicap number of Snaky is at most one, i.e., $h(\text{Snaly}) \in \{0, 1\}$.

II. Winning configurations

In this section we show three winning configurations, Type-I, II, and III. Type I is shown in Fig. 3 (a). The area A can include at most one white stone. (Note that all the other blank squares are really blank, and we don't care squares hatched by diagonal lines.) If such a configuration are achieved just after Bob's movement, i.e., the next movement is made by Alice, Alice can win. The proof is omitted in this paper since the space limitation. We also call the configuration of Fig. 3 (b) Type I (winning configuration), since if Alice achieves it, she can easily force Bob to make the configuration of (a).



(a) (b)

Fig. 3. Type I winning configurations.

Type II is shown in Fig. 4 (a). If such a configuration are achieved just after Bob's movement, Alice can win by using the strategy shown in (b) (Bob cannot block both of the small black squares).

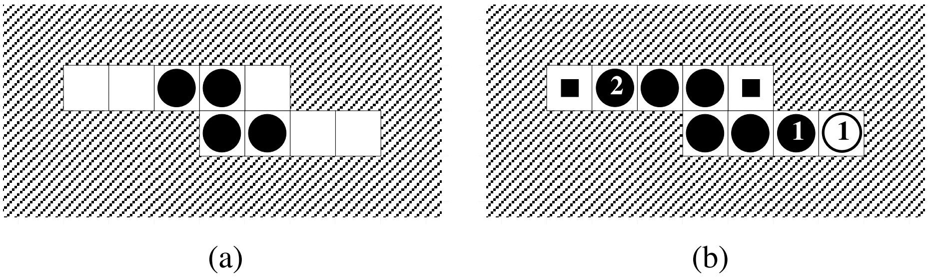


Fig. 4. Type II winning configuration and the proof.

Type III is slightly special: it can be used only just after Alice's sixth movement, i.e., Alice and Bob put six and four stones, respectively, and the next movement is Bob's. The configurations of (a), (b), and (c) of Fig. 5 are all Type III (winning configurations). They satisfy the following conditions:

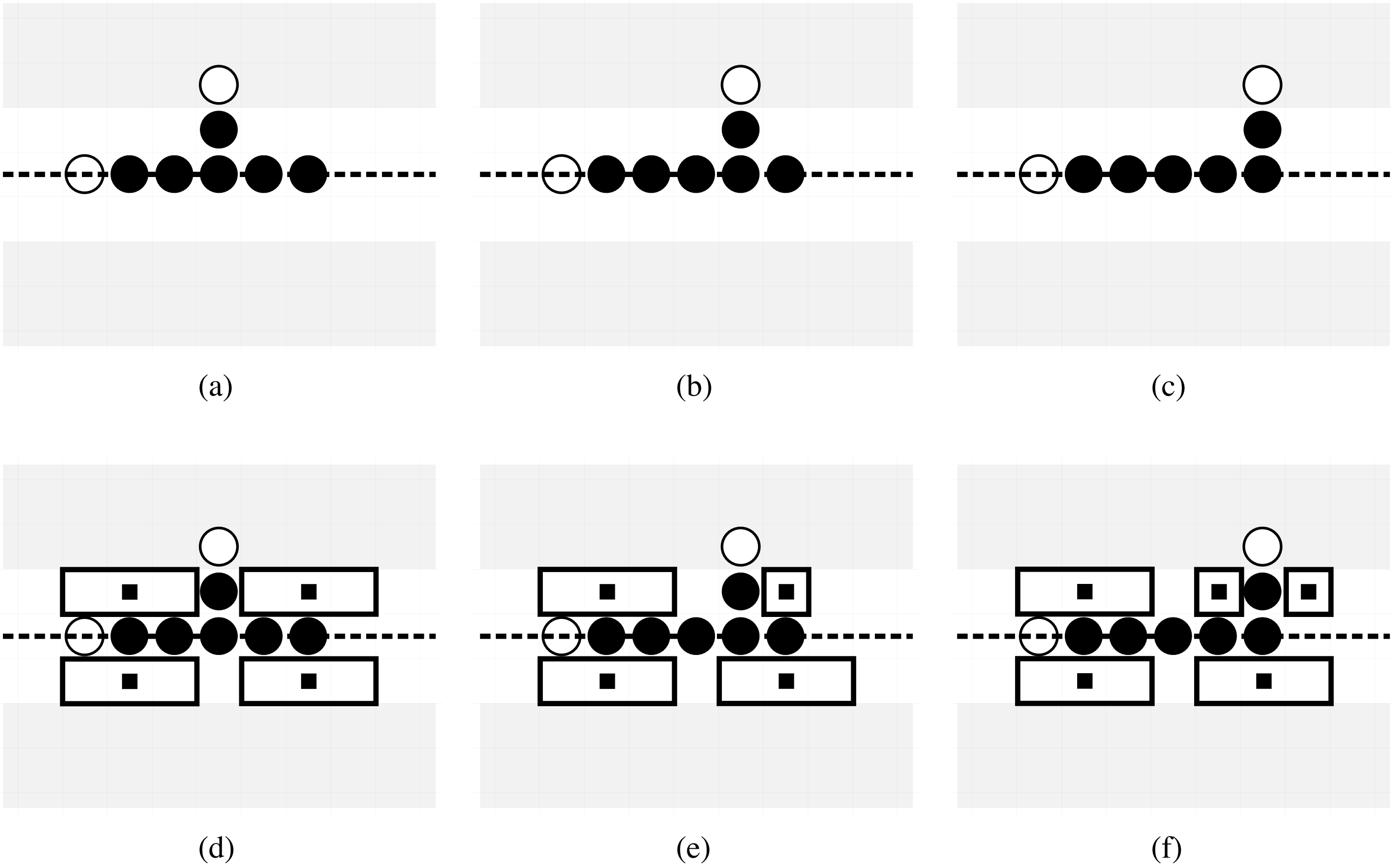


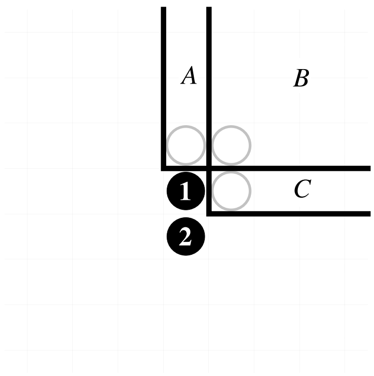
Fig. 5. Type III winning configurations and the proof.

- At least one white stone exists in the area hatched by gray mask.
- At least one white stone exists on the broken line or at least two white stones exist in the area hatched by gray mask.
- Five black stones are arranged successively on the broken line and the other stone is the next to one of the five stones.

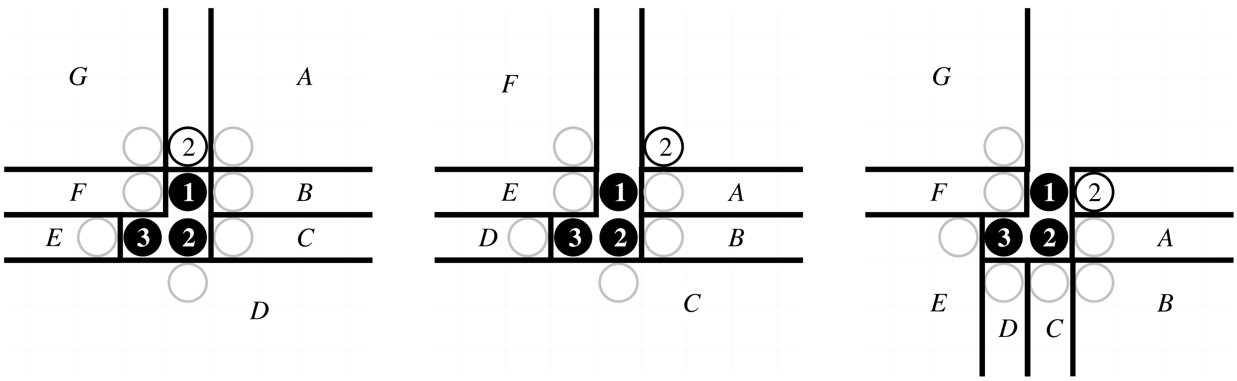
Note that there are at most two white stones on the blank squares, since there are at most four white stones and at least two of them are in the hatched area or on the broken line. Each configuration have at least four disjoint ares, which Bob must block (see (d)–(f)). He can block at most three of them by using existing two stones and one stone of the next movement, and hence at least one of the four areas remains and Alice wins.

III. Winning strategy for Alice

Alice's first two stones (including the handicap stone) are put in cells adjacent each other in the winning strategy. The position of Bob's second movement (note that Bob doesn't have a handicap stone and hence his movement starts from second one) has three possibility in areas A, B, or C of Fig. 6 (a), wlog. If Bob's second movement is in are A, B, or C, the Alice's third stone is put as Fig. 6 (b), (c), or (d), respectively. For each Alice's third movement, Bob's next movement has some possibility as shown in (b), (c), and (d), e.g., if Alice's third movement is in area A of (a), then Bob's third movement is divided into seven cases A–G of (b).



(a)



(b) (c) (d)

Fig. 6. Cases of Bob's third movement.

We analyze more these cases as follows. Three cases shown in Fig. 7 (a)–(c) are special cases, which are called **square types**. In (a), (b) and (c), Alice puts her fourth stone as shown in these figures. (Note that (a) and (c) are the same by using a rotation.) And after Bob's fourth move, the positions of white three stones are divided into four cases, (d)–(g). In (d), two white stones are in area A and the other one is in area B. In (e), two white stones are in area A and the other one is in area B. In (f), one white stone is in area A, and the other two are in areas B and C, respectively. In (g), three white stones are in areas A, B, and C, respectively. For each of them, Alice has a winning strategy. Because of the space limitation, we only show one in Fig. 8 for the case (d) of Fig. 7.

The cases except square types can be divided into two cases shown in Fig. 9 (a) and (b). We call them **nonsquare types**. Case (a) can be divided into four cases as (c)–(f). Case (b) can be divided into four

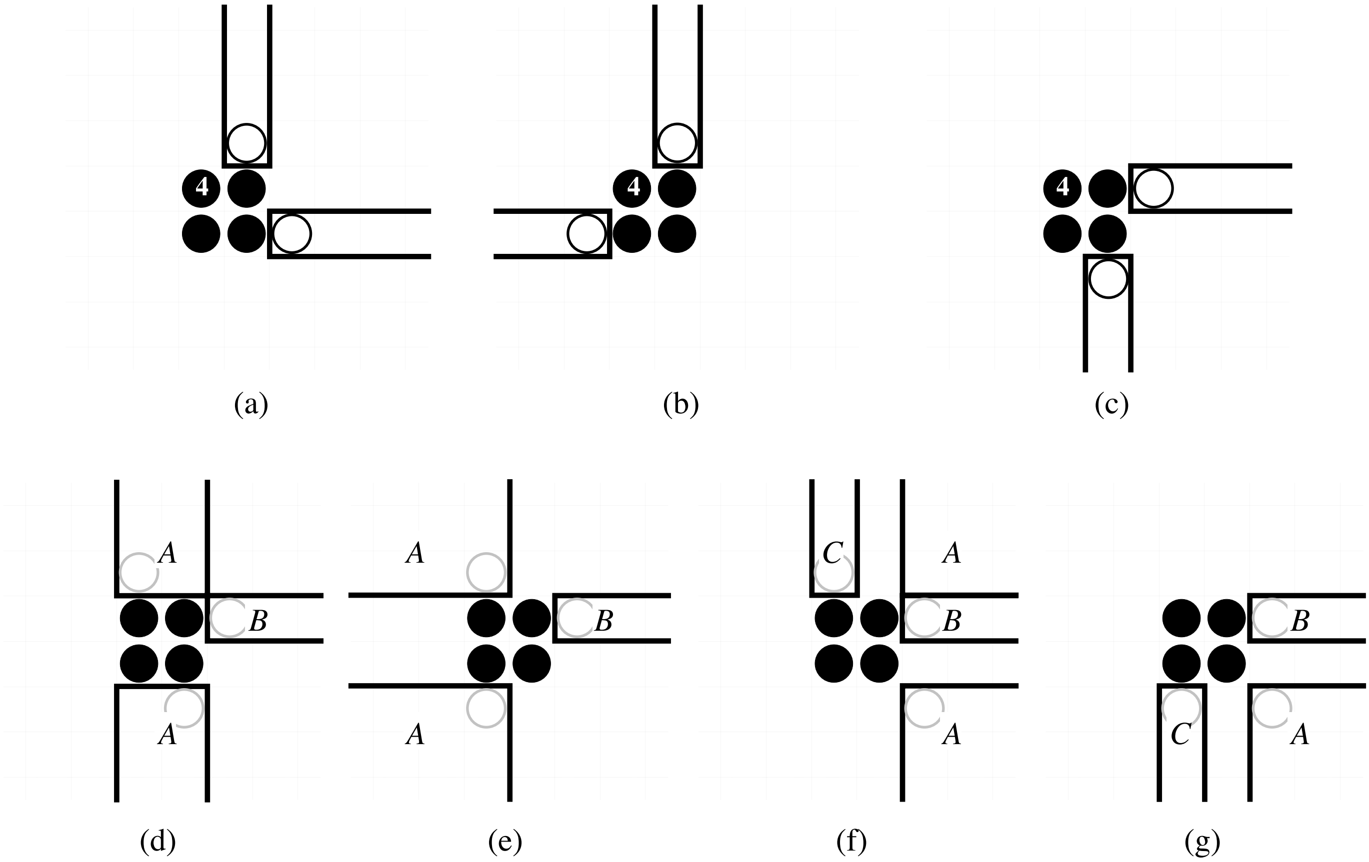


Fig. 7. Square types.

cases as (g)–(k). For each cases Alice has a winning strategy. But from the space limitation, only one for (d) is shown in Fig. 10.

IV. Conclusion and remaining problems

This paper showed a winning strategy of Snaky with one handicap. From this, we can see that the handicap number of Snaky is at most one (Theorem 1). The complete strategy will be shown on a technical report or a web page soon. Remaining problems whether the handicap number is one or zero. This problem seems difficult, since if it is one, we should show a winning strategy for Snaky, but it must be further complicated than the strategy for one handicap.

Acknowledgment

The authors would like to thank students took a lecture “discrete math, Adv.” of Department of Communications and Computer Engineering, Graduate School of Informatics, 2006–2007. This result bases the discussion in the lecture.

References

[1] F. Harary, Achieving the skinny animal, Eureka, **42**, 1982, pp. 8–14. (A revised version: Errata in Edition Number 42, **43**, 1983, pp. 5–6)
[2] F. Harary, H. Harborth, and M. Seemann, Handicap achievement for polyominoes, Congressus Numerantium, **145**, 2000, pp. 65–80.
[3] H. Harborth and M. Seemann, Handicap achievement for squares, J. Combin. Math. Combin. Comput., **46**, 2003, pp. 47–52.
[4] H. Harborth and M. Seemann, Snaky is an edge-to-edge loser, Geonbinatorics, **5**, 1996, pp. 132–136.
[5] H. Harborth and M. Seemann, Snaky is a paving winner, http://www.mathematik.tu-bs.de/preprints/199614.ps.
[6] N. Sieben, Snaky is 41-dimensional winner, Electronical J. of Combinatorial Number Theory, **4**, 2004, #G05.

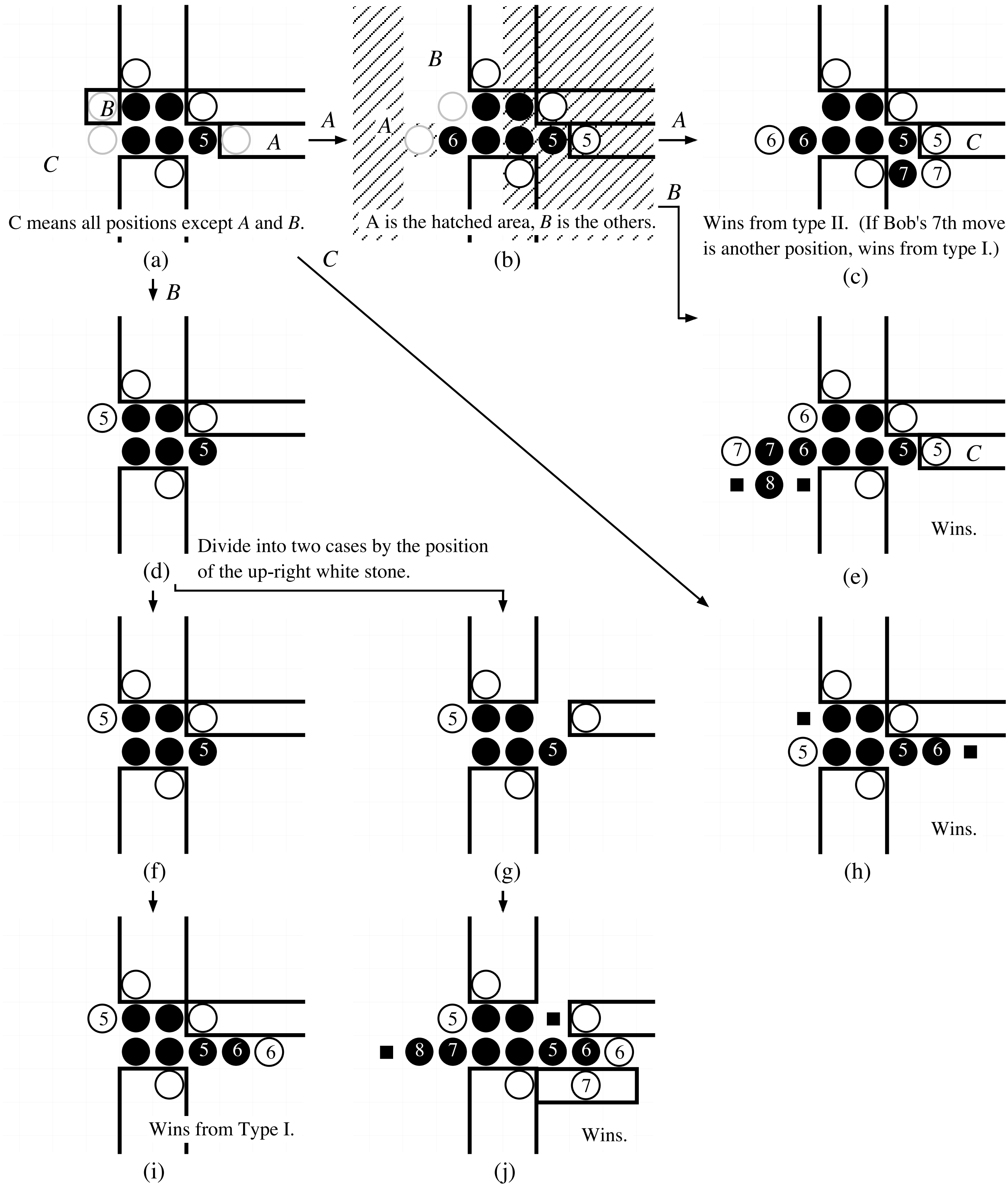


Fig. 8. Winning strategy for (a) of Fig. 7.

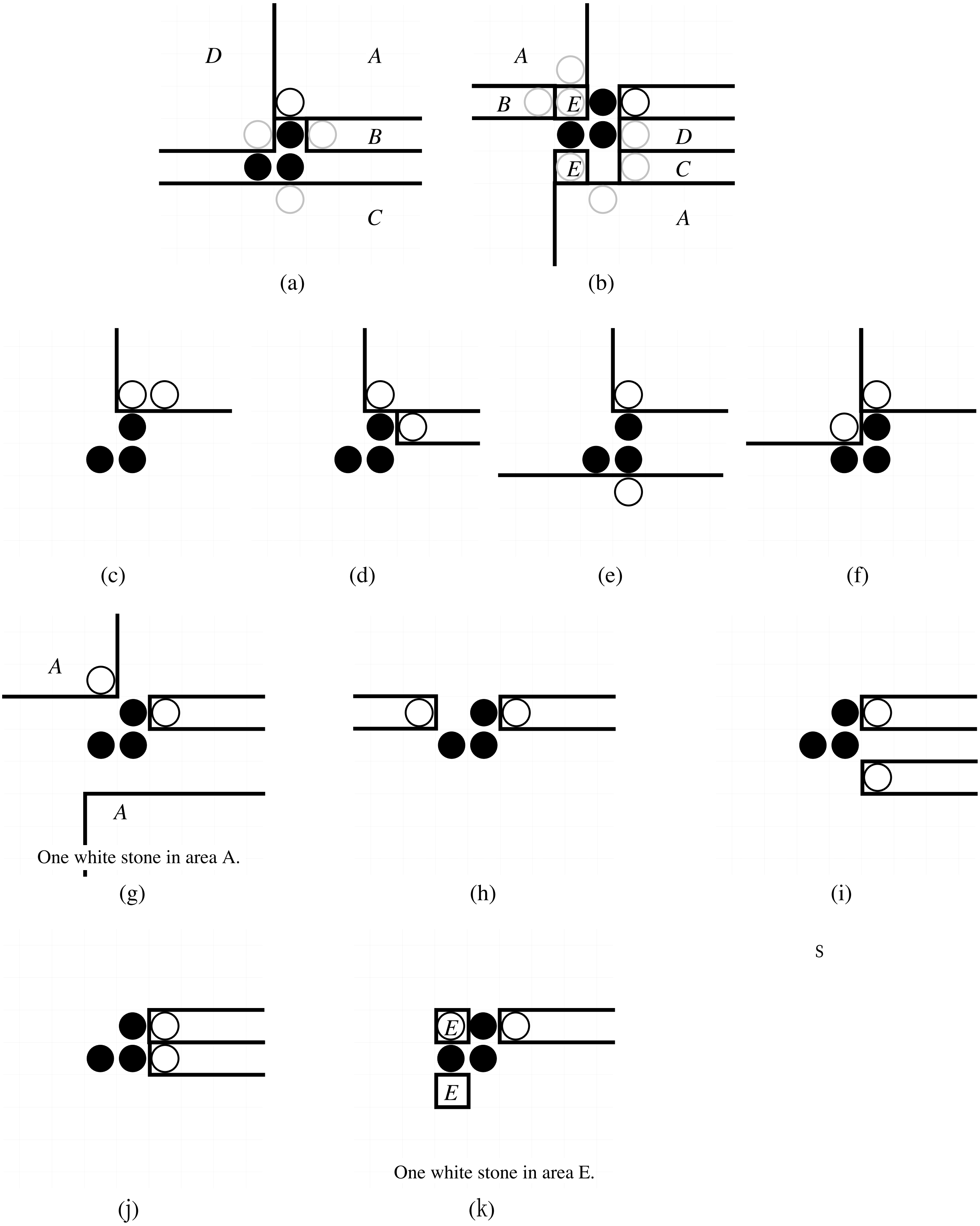


Fig. 9. Nonsquare types.

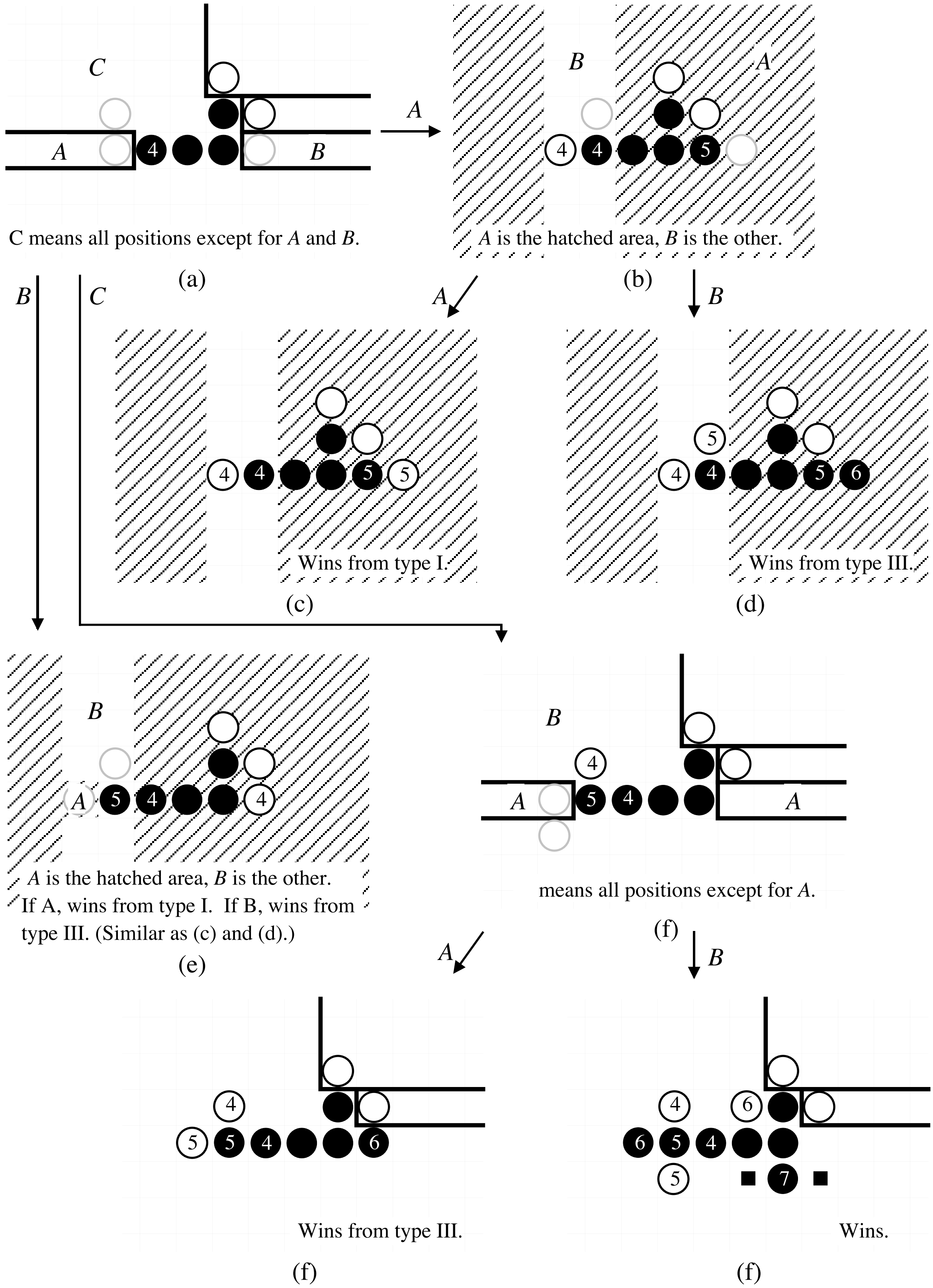


Fig. 10. Winning strategy for (d) of Fig. 9.