Numerical solution of a non-local elliptic problem modeling a thermistor with a finite element and a finite volume method

C. V. Nikolopoulos, G. Zouraris

Abstract

In this work we consider the non-local elliptic problem :

$$w'' + \lambda \frac{f(w)}{\left(\int_{-1}^{1} f(w)dx\right)^2} = 0, \quad -1 < x < 1,$$
$$\mathcal{B}_{\pm}(w) = w'(x) \pm aw(x) = 0, \quad x = \pm 1,$$

where $w = w(x) = w(x;\lambda)$, $\lambda > 0$ and f is a function with the properties $f(s) > 0$, $f'(s) < 0$, $f''(s) > 0$ for $s > 0$, $\int_0^\infty f(s)ds < \infty$. The solution of the equation represents the steady state of the thermistor device. The problem for a critical value of the parameter λ, λ^* has a unique solution, for $\lambda < \lambda^*$ at least two solutions and for $\lambda > \lambda^*$ has no solution. We apply a finite element and a finite volume method based on piecewise quadratic functions in order to find a numerical solution of the problem for the case that $\lambda < \lambda^*$ and for the stable branch of the bifurcation diagram. A comparison of these two methods is made regarding their order of convergence for $f = f(s) = e^{-s}$ and $f(s) = (1+s)^{-p}$ for $p = 2$, and for Robin boundary conditions. Also for the same problem but with Dirichlet boundary conditions, a situation where the solution is unique for $\lambda \leq \lambda^*$, an additional comparison is presented.

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