A new paradigm for construction of structured preconditioners

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(joint work with I. Oseledets and N. Zamarashkin)

Given a matrix A, we assume that a preconditioner C to A belongs to a class of structured matrices $\mathcal{S}$. A standard approach to construction of C suggests that we fix a mapping $A \mapsto C$ and then examine an approximate equation $A \approx C + R$ and estimate how the rank of R depends on the accuracy. A new paradigm suggests that this equation is set on the first place and a minimization of rank R is pursued in the very construction procedure for C (within a prescribed bound on the accuracy).

If A is a Toeplitz matrix and $\mathcal{S}$ is the space of circulants, then the latter problem can be efficiently solved by the "black dot algorithm" proposed in [1]. It was observed therein that this algorithm delivers a circulant with better properties than many other circulant preconditioners (Strang's, optimal, etc.) This is the case for a wide class of symbols described in [2]. In this talk we consider possible applications of the same paradigm to other classes of structured matrices, in particular two-level Toeplitz matrices and those with certain sparsity patterns.

References

[1] I.Oseledets, E.Tyrtyshnikov, A unifying approach to construction of circulant preconditioners, *Linear Algebra Appl.* 418 (2006), 435-449.

[2] N.L.Zamarashkin, I.V.Oseledets, E.E.Tyrtyshnikov, Approximation of Toeplitz matrices by sums of circulants and small-rank matrices, *Doklady Mathematics*, Vol. 73, No. 1 (2006), 100–101.