

The conditioning of FD matrix sequences coming from semi-elliptic Differential Equations

D. Noutsos¹, S. Serra Capizzano², P. Vassalos³.

A B S T R A C T

We are concerned with the study of spectral properties of the sequence of matrices $\{A_n(a)\}$ coming from the discretization, using centered finite differences of minimal order, of elliptic (or semielliptic) differential operators $L(a, u)$ of the form

$$(1) \quad \begin{cases} -\frac{d}{dx} \left(a(x) \frac{d}{dx} u(x) \right) = f(x) & \text{on } \Omega = (0, 1), \\ \text{Dirichlet B.C. on } \partial\Omega, \end{cases}$$

where the nonnegative, bounded coefficient function $a(x)$ of the differential operator may have some isolated zeros in $\overline{\Omega} = \Omega \cup \partial\Omega$. More precisely, we state and prove the explicit form of the inverse of $\{A_n(a)\}$ and some formulas concerning the relations between the orders of zeros of $a(x)$ and the asymptotic behavior of the minimal eigenvalue (condition number) of the related matrices. As a conclusion, and in connection with our theoretical findings, first we extend the analysis to higher order (semi-elliptic) differential operators, and then we present various numerical experiments, showing that similar results must hold true in 2D as well.

¹ Department of Mathematics, University of Ioannina, Ioannina, Greece. Email: dnoutsos@cc.uoi.gr.

² Dipartimento di Fisica e Matematica, Università dell'Insubria - Sede di Como, Italy. Email: stefano.serrac@uninsubria.it, serra@mail.dm.unipi.it.

³ Department of Informatics, Athens University of Economics and Business, Athens, Greece. Email: pvassal@aueb.gr, pvassal@uoi.gr