A Newton's method without direct evaluation of nonlinear function values

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Abstract

Consider the system of nonlinear equations

$$F(x) = 0 \tag{1}$$

where $F = (f_1, \ldots, f_n) : \mathcal{D} \subset \mathrm{I\!R}^n \to \mathrm{I\!R}^n$ is twice-continuously differentiable on an open neighborhood $\mathcal{D}^* \subset \mathcal{D}$ of a solution $x^* = (x_1^*, \ldots, x_n^*) \in \mathcal{D}$ of the system.

There are several methods for solving such nonlinear systems [1], [2]. Among them Newton's method is the most famous, because of its quadratic convergence when a good initial guess exists and the Jacobian matrix is nonsingular.

In many problems, especially in real life applications, the function values involved in the system are known only within some precision. Moreover, there are several problems which are defined by functions whose derivatives are unavailable or available at a prohibitive cost. Thus, it is very important to develop methods to face such cases.

For the second case there have been proposed derivative free methods to contribute in this area. But, for the case where the function values are known imprecisely, only one method has been proposed, according to our knowledge. This is referred to as "a Newton's method without evaluation of nonlinear function values", but it is applied only for polynomial equations [3]. For a system of algebraic and/or transcendental nonlinear equations, there have been proposed several methods for problems with imprecise function values [4], [5], [7]. Besides, Newton's method is problematic when the values of F are not accurately defined. Of course, this problem is common to all iterative methods which directly depend on function evaluations.

In this paper motivated by the target of [3] we attempt to achieve this target for all equations producing a new Newton's method. In particular, our goal is on one hand to remain on Newton's method because of its important quadratic convergence and on the other hand to make it suitable for problems with imprecise function values. Therefore, a new strategy is applied in Newton's method. The innovation of the proposed method is the replacement of the function F in Newton's method by a proper quantity which is directly free of accurate function values.

The proposed method consists of two main parts. In the first part, we take advantage of the proper pivot points $x_{pivot}^{p,i}$, proposed in an our previous work [6]. These points correspond to the functions $f_i(x), i = 1, 2, \ldots, n$ at an iteration $p = 1, 2, \ldots$. The important property of these points is that they lie on the zero hypersurface of the function $f_i(x)$ for some $i = 1, 2, \ldots, n$, i.e. on the plane $x_{n+1} = 0$. Thus, each function value is zero for the corresponding pivot point. At this point, it is worth noticing that the extraction of pivot points comes out via a componentwise technique which depends only on the signs of the functions [7], [8]. In the second part approximated expressions are defined, utilizing the pivot points, to replace the function values in Newton's method. Hence, the Newton's method is converted to one without direct evaluation of function values and therefore is applicable to problems with imprecise function values.

A comparison between the new method and Newton's one is given by using problems with different properties. Numerical applications show that the proposed method is efficient and in some cases it is much better than the original Newton's one.

Keywords: Newton's method, nonlinear systems, imprecise function values, pivot points, quadratic convergence.

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