Monte Carlo evaluation of sensitivities in computational finance

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In computational finance, it is important to be able to compute not only the correct prices for financial derivative options, but also their sensitivity to changes in various input parameters, such as the current asset price, interest rate and level of volatility [1]. These first and second order derivatives are known collectively as the "Greeks", as many of them have associated Greek letters.

In this lecture I will discuss mathematical and computer science aspects of computing Greeks through Monte Carlo simulation. After a brief introduction to Monte Carlo simulation and the approximation of stochastic differential equations, I will present the three main approaches to computing Greeks: finite difference, likelihood ratio method (LRM) and pathwise sensitivity calculation.

The last of these leads very naturally to an adjoint implementation which makes it possible to compute the sensitivity to a large number of input parameters at a very low cost, little more than the cost of evaluating the price itself [2]. The practical development of adjoint codes is greatly assisted by using Automatic Differentiation (AD) tools [3], and I will discuss in particular the use of the FADBAD++ software package which is based on templates and operator overloading within C++.

The pathwise approach is not applicable when the financial payoff function is not differentiable. Even when the payoff is differentiable, the use of scripting in real-world implementations means it can be very difficult in practice to evaluate the derivative of very complex financial products. I will present a new idea to address these limitations by combining the adjoint pathwise approach for the stochastic path evolution with LRM for the payoff evaluation.

References

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