Extrapolation Methods for a Volterra Integral Equation with Weak Singularity

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Abstract

We are considering a class of integral equations of the form

$$u(t) = g(t) + \int_0^t \frac{s^{\mu-1}}{t^\mu} u(s) ds \tag{1}$$

which occur in connection with a problem in heat conduction. The particular class of equations we are investigating includes a parameter μ, which may take any prescribed positive value. When $\mu > 1$, it has been shown that there is a unique solution, but for $\mu < 1$ an infinite set of solutions exists. One of these is the 'smooth' solution, having a quantifiable gradient at $t = 0$.

When $\mu < 1$, the integral term is unbounded at the lower limit; the choice of numerical methods is severely restricted, and away from the origin this becomes expensive in computer time. We aim to construct a split-interval algorithm, with a primary method over an initial interval to cope with the singularity at $t = 0$, linked to a standard method over the 'well-behaved' section of the time interval, without loss of accuracy. This takes into account the length of the initial interval, as well as the step size.

The use of low-order product integration over the initial interval, is followed by a standard approximation method away from the origin; a carefully constructed extrapolation process at the end of the first interval results in raising of the overall order of the combined scheme to match that of the second-interval method.

The resulting split-interval algorithm is reliable and flexible, capable of achieving good accuracy, with convergence to the one particular smooth solution.