A curvilinear method for large scale optimization problems

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Abstract

We consider the unconstrained optimization problem

$$\min_{x \in \mathbb{R}^n} f(x), \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and the dimension n is more than one thousand. This makes the computation of the Hessian matrix not possible or prohibitively expensive. Throughout we assume that the gradient $g(x) = \nabla f(x)$ of the function f is available . It is known that second order methods are more reliable and usually require fewer iterations and function evaluations than methods which use only function or gradient information. The main disadvantage of these methods is the calculation and the storage of the Hessian matrix which requires an additional $n(n+1)/2$ locations of core.

In this work, we concentrate on the effective use of the second order information contained in the Hessian matrix, without calculate or store it. For this purpose, we utilize a specific limited memory BFGS method with $m = 2$, for updating the approximate Hessian. The resulting approximation, called 2-BFGS, give us the ability to determine analytically the eigenvalues via simple formulas. The proposed method uses a curvilinear path in order to find a solution of (1). Our curvilinear path is based on the computation of two descent directions, the 2-BFGS direction and a direction of *negative curvature* which is taking into account the local nonconvexity of the objective function. The second direction is related to the eigenvector that corresponds to the most negative eigenvalue of the approximated matrix. The computation of this direction is accomplished by applying a single step of the inverse power method. Our numerical results show that the proposed approach is promising especially for very large problems ($n \geq 10^3$).

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