What is the Fair Rent Thales Should Have Paid?

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Abstract. The first account of an option trade is found in Aristotle's Politics. Thales the Milesian is said to have purchased the right to rent the olive presses at a future point in time for a predetermined price. Although many authors in the Real Options literature make reference to Thale's option trade, no formal discussion has yet been made. This paper provides a more detailed treatment of this issue and calculates the ratio of the option value to the market rental price of presses.

Unlike common belief, derivative instruments are not recent inventions. Although it is not known exactly when the first option contract was traded, there is evidence that the Ancient Romans and Phoenicians used such contracts in shipping. Furthermore, account of an option trade is found in the writings of Aristotle. In book 1, Chapter 11 of Politics, Aristotle tells the story of Thales of Miletus (624-547 BC), one of the seven sages of the ancient world. People had been telling Thales that his philosophy was useless, since it had left him a poor man. "But he, deducing from his knowledge of stars that there would be a good crop of olives, while it was still winter raised a little capital and used it to pay deposits on all the oil-presses in Miletus and Chios, thus securing an option on their hire. This cost him only a small sum as there were no other bidders. Then the time of the harvest came and as there was a sudden and simultaneous demand for oil-presses, he hired them out at any price he liked to ask. He made a lot of money and so demonstrated that it is easy for philosophers to be rich, if they want to;

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but that is not their object in life. Such is the story of Thales, how he gave proof of his cleverness but, as we have said, the principle can be generally applied; the way to make money in business is to get, if you can, a monopoly for yourself. Hence we find governments also on certain occasions employing this method when they are short of money. They secure a sales monopoly for themselves."

The story of Thales in Aristotle's writings is often cited as the first published discussion of a derivative instrument (see Allende and Elías, 2004; Brach, 2003; Copeland and Antikarov, 2001; Cummins, Phillips, and Smith, 1998; Dimson and Mussavian, 1999; Ineichen, 2001; LeVeque, 1994). Thales actually purchased a call option. By giving deposits for the use of the olive presses, he purchased the right, but not the obligation, to hire the presses at a very low price since there were no other bidders. Had the harvest been poor, Thales would have chosen not to exercise his right to rent the presses and lost only the initial deposit, i.e. the option premium. Given that the olive harvest was bountiful Thales exercised the option and made a lot of money. In option pricing terminology the predetermined rental price, at which Thales contracted, is called the exercise price. If the market price is higher than the exercise price at maturity, the option is said to be "in-the-money" whereas otherwise "out-of-the-money". Note that Thale's option was issued "at-the-money", meaning that the exercise price of the contract was equal to the rental price at the time the contact was entered into. Thale's option expired deep in-the-money, which induced him to exercise it and, as a result he became rich. Whether the option would be of value or not at maturity depended on the market rental price of presses at that time. The latter was affected by the size of the olive harvest. Therefore, in the terminology of options, the underlying asset was the size of the olive harvest. The greatest the size of the harvest of olives to be pressed, the more valuable the option would be. Generally, the payoff from an option increases linearly with the price of the underlying variable at maturity. As Aristotle says, Thales managed to secure a high rental price by creating monopoly for himself.

In general, the value of an option depends on five variables. So far we have commented on two: the exercise price and the value of the underlying asset. The other three variables affecting the option price are the level of uncertainty of the underlying variable, the time to maturity of the option and finally the risk-free rate. The value of the option increases with the level of uncertainty. If there were no uncertainty over the size of the harvest and hence the rental price of presses, the option would be worthless. On the contrary, the greater the uncertainty, the greater the upside (profit) potential whereas the downside risk is limited. The reason is that in the case of a big upward move in the market rental price for presses, the owner of the option exercises it and receives the difference between the prevailing rental price at that time and the exercise price, while in the case of a downward move the option expires unexercised and the owner of the option loses only his initial investment, i.e. the option premium. The fourth variable is the time to maturity of the option. The value of an option contract increases with the time to maturity. Thales purchased his option six months before the harvest. The option would be less valuable if for example it had been purchased one day before expiration given that there would be less uncertainty over the final rental price of presses. Finally, the value of the option increases with the risk-free rate of interest because an increase in the risk-free rate decreases the present value of the exercise cost.

In most option-pricing models, the approach to valuation is based on arbitrage arguments. They involve setting up a riskless portfolio consisting of a position in the option and a position in the underlying asset. In absence of arbitrage opportunities, the return from the portfolio must equal the risk-free interest rate, r. The calculation of the option value is based on the construction of a risk-free portfolio that requires that one can continuously trade the underlying asset. However, in the present case, where one cannot short-sell the oil and the underlying asset is not traded, such an arbitrage technique is highly unrealistic. In the real options literature, where, the underlying asset is not traded in the market, the valuation is based on the use of spanning assets, i.e., it is assumed that there exists a dynamic portfolio of traded assets whose stochastic fluctuations are perfectly correlated with the stochastic process for the underlying asset. Further, if spanning does not hold, a dynamic programming approach could alternatively be used whereby the appropriate discount rate, ρ, should be defined exogenously as a part of the objective function. This objective function can simply reflect the decision maker's subjective valuation of risk. For simplicity, risk neutrality is assumed, or equivalently, that the risk of the underlying asset is uncorrelated with the overall economy. Under the assumption of risk neutrality the dynamic programming and the contingent claims valuation yield the exact same solution to the problem in-hand.

Making the above assumptions, allows us to calculate the value of the option using the Black-Scholes model, which yields a closed-form solution for the problem.

$$C = S_0 * N(d_1) - K * e^{-rT} * N(d_2)$$
$$d_1 = \frac{\ln(S_0 / K) + \left(r + \frac{1}{2} * \sigma^2 * T \right)}{\sigma * \sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma * \sqrt{T} \tag{1}$$

Where C is the value of the call option, S_0 is the initial price of the underlying asset (rental price), K is the exercise price (we assume that the option is issued at-the-money and hence S_0

= K), r is the risk-free rate, σ is the volatility of olive oil production (we assume that the market rental price of presses is perfectly correlated with the size of the olive oil production), $N(x)$ is the cumulative probability distribution function for a variable that is normally distributed with a mean of zero and a standard deviation of 1 and T is the time to maturity (life of the option).

Our data set includes the production of olive oil (in tons) in Greece for the period from 1961 to 2000 as well as the number of trees. We calculated the ratio of oil production to the number of trees and found an annual volatility of 43.37%. Concerning the risk-free rate in Ancient Greece, we rely on the writings of Xenophon. "But at present the citizens can acquire no gains so creditable as those from what they may contribute for this fund; for him whose contribution shall be ten minai[1], about the fifth part will return as interest from the fleet, as he will receive three obols a day; and to him whose contribution shall be five minai, there will be a return of more than a third. The most of the Athenians, assuredly, will receive annually more than they have contributed; for those who contribute a mina will have an income of almost two minai, and will have it in the city, being an income, too, that appears the safest and most durable of human things" (Poroi, chapter III).

Therefore, according to Xenophon's proposal, a loan of ten minai would be receiving interest of three obols a day, i.e., 360x3= 1080 obols or 1080:6 =180 drachmas annually. Given that one mina was equal to 100 drachmas, a loan of ten minai or 1000 drachmas would generate interest of 180 drachmas, thus the interest rate was around 18%. Actually, Xenophon, talking of the interest rate to be paid for the suggested fund, mentions that this rate will be almost the same as that for shipping loans. Therefore, here we get a connection between the degree of risk and the rate of interest. For shipping loans the rate would be expected to be "a fifth part" or 20%, but in the present case, which is considered very safe, 18% will suffice.

For σ = 43.37%, r = 18% and T = 0.5, we calculate the ratio C/S_0 = 16.46%. Figures 1, 2 and 3 plot the ratio C/S_0 for various values of the risk-free rate, volatility and time to maturity, respectively.

[1] It may be helpful at this point to comment on Attic money after Solon's reforms. The basic currency was the drachma coin and one drachma had 6 obols. The coins were made of silver and the content of an obol was 0.72 grams of silver, while that of the drachma 4.32 grams. Though Solon, on taking office in Athens, in 594 BC, did institute a partial debasement of the currency, for the next four centuries, until the absorption of Greece into the Roman empire in 146 BC, the Athenian drachma had an almost constant silver content (67 grains of fine silver until Alexander's days, 65 grains thereafter) and became the standard currency of trade in Greece and in much of Asia and Europe as well. Even after the Roman conquest, the drachma continued to be minted and widely used. In addition, there were two different sums of money; the mina, which had 100 drachmas and the talent, which had 60 minai or 6000 drachmas.

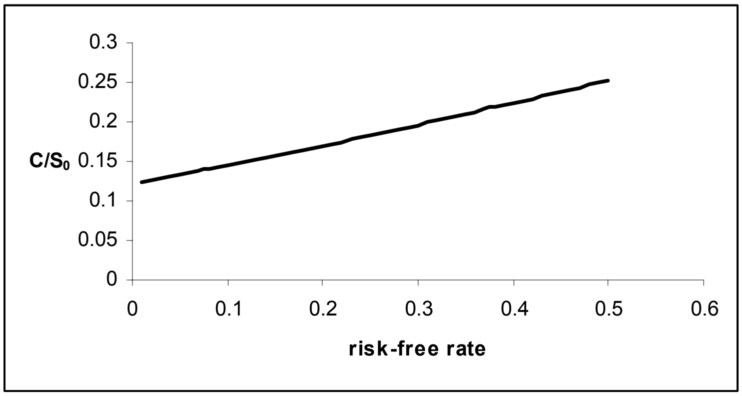

Figure 1: Ratio C/S_0 as a function of the risk-free rate of return.

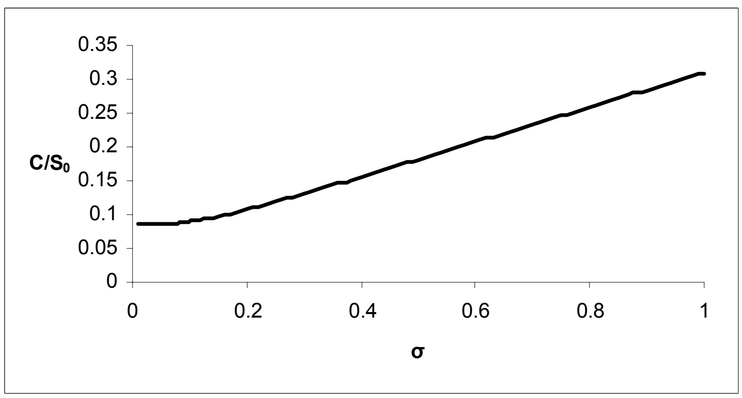

Figure 2: Ratio C/S_0 as a function of the volatility, σ.

Figure 3: Ratio C/S_0 as a function of the time to maturity.

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