

# Numerical solution of some first order integro-differential equations arising in ultimate ruin theory

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**Abstract**—This paper presents models of ultimate ruin theory in the form of 1st order integro-differential equations and systems of such equations (cf. Albrecher and Boxma (2004), Dickson and Gray (1984), Lin, Willmot and Drekić (2003)), and gives some numerical results obtained from the direct solution of one of them (Albrecher and Boxma (2004)).

## I. INTRODUCTION

In a recent article (Makroglou (2004) ([13]), models of ruin theory in the form of second order integro-differential equations were presented together with the computational treatment of one of them by collocation methods.

This paper presents models applying to ultimate time ruin theory which are in the form of first order integro-differential equations and systems of such equations. For a brief bibliographical overview of methods of solution of integral and integro-differential equation models used in ruin

theory, we refer for example to Makroglou (2003) ([12]).

The notation used is that of Dickson and Waters (2002) ([6]) which was also followed in Makroglou (2004) ([13]). So we let:

$U(t)$	: surplus process, $t \geq 0$
$u$	: initial surplus
$X_i$	: amount of the $i$ -th claim
$N(t)$	: counting process for the number of claims up to time $t$
$S(t)$	: the accumulated claims process
$T_i$	: random variables for the claim inter-arrival times, $i = 1, \dots, \infty$
$T$	: time to ruin
$c$	: insurer's premium income/unit time assumed to be received continuously and such that $cE(T_i) > E(X_i), \forall i$
$F$	: distribution function of $X_i$ with density function $f$

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We also let:

$\psi(u) = Pr(T < \infty)$  the probability of ultimate ruin from initial reserve  $u$

$\phi(u) = 1 - \psi(u)$  the probability of ultimate non-ruin from initial reserve  $u$

$\delta$  : non negative parameter (interest force, or a dummy variable)

$m_k = E(X_i^k)$ .

For the classical risk model we have:

$$U(t) = u + ct - S(t) \quad (\text{I.1})$$

where

$$S(t) = \sum_{i=1}^{N(t)} X_i. \quad (\text{I.2})$$

The accumulated claims process  $\{S(t)\}_{t \geq 0}$  is a compound Poisson process with Poisson parameter  $\lambda$ . The claim sizes  $X_i, i = 1, 2, \dots, N(t)$  are assumed to be independent and identically distributed.

For continuous risk models where the risk process is a variation/extension of the classical compound Poisson process, see for example Dickson and dos Reis (1997)([4]), Klüppelberg and Stadtmüller (1998) ([9]), Stanford, Stroiński and Lee (2000) ([20]), Kalashnikov and Konstantinides (2000) ([8]), Lin, Willmot and Drekić (2003) ([10]).

Other types of continuous risk models include ones where the risk process is perturbed by Brownian motion (cf. Schlegel (1998) ([19])) and ones which allow for the insurance company to invest part of the surplus in bonds and part in the stock market (cf. Gaier and Grandits (2004) ([7])), or allow dependence between claim sizes and claim intervals (cf. Albrecher and Boxma (2004) ([1])).

For an introduction to models of the claims number process and the claim size distributions, see for example Mikosch (2004) ([16], chapters 2, 3).

Ruin theory models for the ultimate time case in the form of first order integro-differential

equations can be found for example in the papers by Dickson and Gray (1984) ([3]), Peters and Mangel (1990), Makroglou, Harper, Smith (2000) ([14]), Lin, Willmot and Drekić (2003) ([10]), Albrecher and Boxma (2004) ([1]), Dickson and Waters (1996) ([5]), Möller (1996) ([17]), Michaud (1996) ([15]).

The general form of a first order Volterra integro-differential equation (VIDE) is

$$y'(x) = G(x, y(x), \int_0^x K(x, s, y(s)) ds, \quad (\text{I.3})$$

$$0 \leq x \leq X, y(0) = y_0, \quad (\text{I.4})$$

where  $y(x)$  is the unknown function and  $G, K$  are given functions. Methods of numerical solution include quadrature methods, linear multistep methods, collocation methods, defect correction methods, Galerkin methods. For an introduction to the subject we refer for example to the books by Linz (1985) ([11]) and by Brunner (2004) ([2]).

This paper presents three examples of models of ultimate ruin theory in the form of first order integro-differential equations which were originally presented in the papers by Dickson and Gray (1984) ([3]), Lin, Willmot and Drekić (2003) ([10]), and Albrecher and Boxma (2004) ([1]) (section II), together with computational results obtained from the numerical solution of one of them, a model by Albrecher and Boxma (2004) ([1]) (section III). Conclusions may be found in section IV.

## II. DESCRIPTION OF SOME ULTIMATE RUIN MODELS IN THE FORM OF FIRST ORDER VIDES

In this section we present three models from the papers by Dickson and Gray (1984) ([3]) in section II.1, Lin, Willmot and Drekić (2003) ([10]) in section II.2 and by Albrecher and Boxma (2004) ([1]) in section II.3.

### A. The Dickson and Gray (1984) ([3]) example

Dickson and Gray (1984) ([3]) consider a surplus process of the form (I.1)-(I.2) in the presence of an absorbing upper barrier at some  $K > 0$ . It was assumed that the time interval  $T_1$  up to the first claim and the time lengths  $T_i$  between successive claims are independently and identically distributed with distribution function

$$A(t) = 1 - e^{-t}. \quad (\text{II.1})$$

Thus  $N(t)$  the number of claims up to time  $t$  is Poisson distributed with  $E(N(t)) = t$ .  $\{S(t)\}_{t \geq 0}$  the accumulated claims process is a compound Poisson process with parameter  $\lambda$ .  $c$ , the premium income per unit time was assumed to be constant,  $c = 1 + \theta$ , where  $\theta > 0$  is the constant premium loading. They let  $\xi(u, K)$  denote the probability of ruin from an initial reserve  $u$  in the presence of an absorbing upper barrier at  $K$ . The first order VIDE satisfied by  $\xi(u, K)$  is ([3], p. 175)

$$-(1 + \theta)\xi'(u, K) + \xi(u, K) = \int_0^u \xi(x, K)f(u-x)dx + 1 - F(u), \quad (\text{II.2})$$

$$\xi(K, K) = 0. \quad (\text{II.3})$$

The authors find the exact solution of (II.2)-(II.3) in three cases:

$$F(x) = 1 - e^{-x}, \quad (\text{II.4})$$

$$F(x) = \int_0^x \frac{y^{\alpha-1}e^{-y/\beta}}{\Gamma(\alpha)\beta^\alpha} dy, \quad (\text{II.5})$$

$$F(x) = 1 - \sum_{j=1}^n a_j e^{-\beta_j x}, \quad (\text{II.6})$$

where  $x \geq 0$ . In (II.5)  $F(x)$  is a Gamma distribution with  $\alpha$  assuming positive and integral values and  $\beta$  a scale parameter chosen equal to  $1/\alpha$  so that the distribution has unit mean. In (II.6),  $F(x)$  is a mixed exponential distribution provided that  $\sum_{j=1}^n a_j = 1$ ; the constants  $\beta_j$  in (II.6) are such that  $0 < \beta_1 < \beta_2 < \dots < \beta_n$ ,  $a_j > 0$ ,  $j = 1, 2, \dots, n$  and it is assumed that  $\sum_{j=1}^n (a_j/\beta_j) = 1$  so that the distribution has unit mean.

The VIDE (II.2)-(II.3) is solved by transformation to an ODE in  $\xi$  of order 2 in the case of the exponential distribution (II.4), of order  $\alpha + 1$  in the case of the Gamma distribution (II.5) and of order  $n + 1$  in the case of the mixed exponential distribution (II.6). Letting  $K \rightarrow \infty$ ,  $\xi(u, K) \rightarrow \psi(u)$ . So their results provide us also with formulas for  $\psi(u)$  the ruin probability with no upper barrier.

#### Exact solutions:

1. Case of the exponential distribution  $F(x)$  given by (II.4).

The exact solution  $\xi(u, K)$  of (II.2)-(II.3) is given by ([3], p. 175),

$$\xi(u, K) = b_0 + b_1 e^{-Ru} \quad (\text{II.7})$$

$$b_0 = -b_1 e^{-RK}, \quad (\text{II.8})$$

$$b_1 = \{(1 - R)^{-1} - e^{-RK}\}^{-1}, \quad (\text{II.9})$$

where  $R$  is obtained from the Lundberg's equation

$$1 + (1 + \theta)R = (1 - R)^{-1}. \quad (\text{II.10})$$

2. Case of the Gamma distribution  $F(x)$  given by (II.5).

- (i) Case of the characteristic equation of the corresponding ODE satisfied by  $\xi(u, K)$  having real roots only.

In this case the exact solution  $\xi(u, K)$  of (II.2)-(II.3) is given by ([3], p. 178),

$$\xi(u, K) = b_0 + \sum_{i=1}^{\alpha} b_i e^{-R_i u}, \quad (\text{II.11})$$

where ([3], p. 179)

$$b_0 = - \sum_{i=1}^{\alpha} b_i e^{-R_i K}, \quad (\text{II.12})$$

$$b_0 + \sum_{i=1}^{\alpha} \frac{b_i}{(1 - \beta R_i)^j} = 1, \quad (\text{II.13})$$

$$j = 1, 2, \dots, \alpha,$$

and the  $R_i$  are the (positive) roots of the equation

$$1 + (1 + \theta)R_i = (1 - \beta R_i)^{-\alpha}. \quad (\text{II.14})$$

- (ii) Case of the characteristic equation of the corresponding ODE having both real and complex roots ( $r$  real roots and  $2d$  complex roots  $-p_j \pm iq_j$ ).

In this case the solution  $\xi(u, K)$  of (II.2)-(II.3) is given by ([3], p. 181)

$$\begin{aligned} \xi(u, K) = & b_0 + \sum_{i=1}^r b_i e^{-R_i u} + \\ & \sum_{j=1}^d e^{-p_j u} (\gamma_j \cos q_j u + \delta_j \sin q_j u) \end{aligned} \quad (\text{II.15})$$

where the coefficients  $b_i, \gamma_j, \delta_j$  are found from ([3], p. 181)

$$\begin{aligned} 1 - b_0 = & \sum_{i=1}^r \frac{b_i}{(1 - \beta R_i)^n} \\ & + \sum_{j=1}^d \frac{\gamma_j}{g_j^n} \sum_{t=0}^{\lfloor n/2 \rfloor} \binom{n}{2t} \\ & (-1)^t (1 - \beta p_j)^{n-2t} (\beta q_j)^{2t} \\ & - \sum_{j=1}^d \frac{\delta_j}{g_j^n} \sum_{t=1}^{\lfloor n/2 \rfloor} \binom{n}{2t-1} \\ & (-1)^{t+1} (1 - \beta p_j)^{n-2t+1} (\beta q_j)^{2t-1}, \\ & n = 1, 2, \dots, \alpha. \end{aligned} \quad (\text{II.16})$$

and where  $b_0$  is determined by (II.15) using  $\xi(K, K) = 0$  and  $R_i, i = 1, \dots, r$  and  $p_j, q_j, j = 1, \dots, d$  satisfy ([3], p. 180)

$$1 + (1 + \theta)R_i = (1 - \beta R_i)^{-\alpha}, \quad i = 1, 2, \dots, r \quad (\text{II.17})$$

$$\begin{aligned} 1 + (1 + \theta)(p_j \pm iq_j) = & \\ (1 - \beta(p_j \pm iq_j))^{-\alpha}, & \\ j = 1, 2, \dots, d, & \end{aligned} \quad (\text{II.18})$$

and  $g_j = (1 - \beta p_j)^2 + (\beta q_j)^2$ .

3. Case of the mixed exponential distribution  $F(x)$  given by (II.6).

In this case the exact solution  $\xi(u, K)$  of the VIDE (II.2)-(II.3) is given by ([3], p. 182, 183)

$$\xi(u, K) = b_0 + \sum_{i=1}^n b_i e^{-R_i u}, \quad (\text{II.19})$$

where the  $b_i$ 's are found from

$$1 = b_0 + \sum_{i=1}^n \frac{b_i \beta_k}{\beta_k - R_i}, \quad k = 1, 2, \dots, n, \quad (\text{II.20})$$

the  $b_0$  is found from (II.19) using  $\xi(K, K) = 0$ , and the  $R_i$ 's (known to be distinct, real and positive in this case) are found from

$$1 + (1 + \theta)R_k = \sum_{j=1}^n \frac{\alpha_j \beta_j}{\beta_j - R_k}, \quad k = 1, 2, \dots, n. \quad (\text{II.21})$$

### B. The Lin, Willmot and Drekcic (2003) ([10]) example

Lin, Willmot and Drekcic (2003) ([10]) consider the classical compound Poisson risk model in the presence of a constant dividend barrier  $K$ , and they derive and solve an integro-differential equation for the Gerber-Shiu discounted penalty function

$$m_K(u) = E\{e^{-\delta T_K} \omega(U_K(T_K-), |U_K(T_K)|) \mathbf{1}(T_K < \infty)\} \quad (\text{II.22})$$

where  $\mathbf{1}(T_K < \infty) = 1$  if  $T_K < \infty$ , and equal to 0 otherwise,  $T_K = \inf\{t : U_K(t) < 0\}$ , the first time that the surplus becomes negative (time of ruin),  $\delta \geq 0$ ,  $\omega(x_1, x_2), 0 \leq x_1, x_2 < \infty$  a nonnegative function.

It is shown ([10], sections 3, 4) that  $m_K(u)$  is a linear combination of the Gerber-Shiu discounted

penalty function without barrier and a solution to an excessive or proper defective renewal equation. Results are given for several special cases of  $\omega(x_1, x_2)$ . Special cases of claim amount distributions considered are those of exponential and mixture of two exponential distributions.

The IDE for  $m_K(u)$  ([10], p. 553, equ. 2.6), is:

$$m'_K(u) = -\frac{\lambda}{c} \int_0^u m_K(u-y) dF(y) + \frac{\lambda + \delta}{c} m_K(u) - \frac{\lambda}{c} \zeta(u), 0 \leq u \leq K, \quad (\text{II.23})$$

$$\zeta(t) = \int_t^\infty \omega(t, y-t) dF(y) \quad (\text{II.24})$$

$$m_K(K) = \frac{\lambda}{\lambda + \delta} \gamma_K(K), m'_K(K) = 0 \quad (\text{II.25})$$

where  $c = \lambda p_1(1 + \theta)$  is the premium rate per unit time,  $p_1 = \int_0^\infty y dF(y)$ , and  $\theta$  is the relative security loading.

Extensions to the stationary renewal model are also given in [10], section 7, p. 563-565, where a system of an integral and integro-differential equation for  $m_{K,r}(u)$ , the Gerber-Shiu discounted penalty function in the renewal risk model and for  $m_{K,e}(u)$ , the Gerber-Shiu discounted penalty function in the equilibrium renewal risk model respectively is given. In addition, the  $m_{K,e}(u)$  function is expressed in terms of  $m_{K,r}(u)$ .

### C. The Albrecher and Boxma ([1]) example

Albrecher and Boxma (2004) ([1]) consider a generalization of the classical risk model, where the distribution of the time between two claim occurrences depends on the size of the previous claim (claim sizes - claim inter-occurrence times dependence risk model). They derive exact solutions for the probability of survival using Laplace-Stieltjes transforms.

In one of their models (Model 1), the claim occurrence process is assumed to be such that if a claim  $X_i$  is larger than a certain threshold  $H_i$ ,

then the time until the next claim is exponentially distributed with rate  $\lambda_1$ , otherwise it is exponentially distributed with rate  $\lambda_2$ . It is also assumed that  $\{H_i\}$  are independent and identically distributed random variables with distribution function  $H(\cdot)$ .

The system of integro-differential equations given for  $\phi_i(u), i = 1, 2$ , the probability of survival with initial reserve  $u$ , given that the first claim occurs according to the exponential distribution with rate  $\lambda_i$ , are ([1], p. 246)

$$c\phi'_1(u) - \lambda_1\phi_1(u) + \lambda_1 \int_0^u \mathbb{P}(H \leq y) \phi_1(u-y) dF(y) + \lambda_1 \int_0^u \mathbb{P}(H > y) \phi_2(u-y) dF(y) = 0, \quad (\text{II.26})$$

$$c\phi'_2(u) - \lambda_2\phi_2(u) + \lambda_2 \int_0^u \mathbb{P}(H \leq y) \phi_1(u-y) dF(y) + \lambda_2 \int_0^u \mathbb{P}(H > y) \phi_2(u-y) dF(y) = 0. \quad (\text{II.27})$$

A number of variations of this model are considered together with several examples in the Albrecher and Boxma (2004) ([1]) paper.

## III. NUMERICAL RESULTS

Numerical results are presented for one example, that of the system of VIDEs (II.26)-(II.27) from the paper of Albrecher and Boxma (2004) ([1], p. 246), and in particular

$$\phi'_1(u) = \frac{\lambda_1}{c} \phi_1(u) - \frac{\lambda_1}{c} \int_0^u [\mathbb{P}(H \leq y) \phi_1(u-y) + \mathbb{P}(H > y) \phi_2(u-y)] dF(y), \quad (\text{III.1})$$

$$\phi'_2(u) = \frac{\lambda_2}{c} \phi_2(u) - \frac{\lambda_2}{c} \int_0^u [\mathbb{P}(H \leq y) \phi_1(u-y) + \mathbb{P}(H > y) \phi_2(u-y)] dF(y). \quad (\text{III.2})$$

Using  $H(y) = 1 - e^{-\mu y}$  and  $F(y) = 1 - e^{-\nu y}$  and setting  $w = u - y$ , we obtain

$$\begin{aligned}
& \phi_1'(u) = \frac{\lambda_1}{c} \phi_1(u) \\
& - \frac{\lambda_1 v}{c} \int_0^u [e^{-v(u-w)} \phi_1(w) \\
& - e^{-(\mu+v)(u-w)} \phi_1(w) \\
& + e^{-(\mu+v)(u-w)} \phi_2(w)] dw, \quad (\text{III.3})
\end{aligned}
\quad \left( cs + \frac{\lambda_1 \mu v}{(v+s)(v+\mu+s)} - \lambda_1 \right) \\
\left( cs + \frac{\lambda_2 v}{v+\mu+s} - \lambda_2 \right) \\
- \frac{\lambda_1 \lambda_2 \mu v^2}{(v+\mu+s)^2(v+s)} = 0.$$

$$\begin{aligned}
& \phi_2'(u) = \frac{\lambda_2}{c} \phi_2(u) \\
& - \frac{\lambda_2 v}{c} \int_0^u [e^{-v(u-w)} \phi_1(w) \\
& - e^{-(\mu+v)(u-w)} \phi_1(w) \\
& + e^{-(\mu+v)(u-w)} \phi_2(w)] dw. \quad (\text{III.4})
\end{aligned}$$

(1)

$$c = 2, \lambda_1 = 3, \lambda_2 = 1, \mu = 2, v = 1.$$

$$\begin{aligned}
& \phi_1(x) = 1 - 0.007e^{-3.161x} \\
& - 0.938e^{-0.065x} \quad (\text{III.5})
\end{aligned}$$

$$\begin{aligned}
& \phi_2(x) = 1 - 0.003e^{-3.161x} \\
& - 0.867e^{-0.065x} \quad (\text{III.6})
\end{aligned}$$

([1], Example 3, p. 252).

(2)

$$c = 2, \lambda_1 = 1, \lambda_2 = 2, \mu = 1, v = 1.$$

$$\begin{aligned}
& \phi_1(x) = 1 - 0.632e^{-0.355x} \\
& + 0.017e^{-1.889x} \quad (\text{III.7})
\end{aligned}$$

$$\begin{aligned}
& \phi_2(x) = 1 - 0.798e^{-0.355x} \\
& + 0.028e^{-1.889x} \quad (\text{III.8})
\end{aligned}$$

([1], Example 4, p. 252).

The true solution of (III.3)-(III.4) was used to find  $\phi_1(0), \phi_2(0)$ . In the Albrecher and Boxma (2004) ([1]) paper, formulae for  $\phi_1(0), \phi_2(0)$  are given in terms of  $c, \lambda_1, \lambda_2, \chi_1(s) = \int_0^\infty e^{-sx} H(x) dF(x), \chi_2(s) = \int_0^\infty e^{-sx} (1 - H(x)) dF(x)$  and  $\sigma$ , the unique zero with  $\Re(\sigma) > 0$  of

The numerical results in the following tables show computed values of  $\phi_1(x)$  and  $\phi_2(x)$  in the first and the second row respectively, for 3 step sizes ( $h = 0.1, h = 0.05, h = 0.001$ ). They were obtained by using collocation (COL) methods with  $m = 2$ , and collocation parameters  $c_1 = 0, c_{m=2} = 1$  which makes the method equivalent to the trapezoidal method.

Table III.1 shows results for example (1), and Table III.2 results for example (2).

Absolute errors were computed for  $h = 0.1$  and  $h = 0.05$  using the results for  $h = 0.001$  in place of the true solution; using these results, the expected order of convergence from the application of the trapezoidal method ( $O(h^2)$ ) was verified. To use the actual true solution for testing the order of convergence of the method, recalculations are needed to find the coefficients involved in more decimal places than the three (3) given in the Albrecher and Boxma (2004) paper.

TABLE III.1  
EXAMPLE (1)

$x$	$h = 0.1$	$h = 0.05$	$h = 0.001$	true sol
0.1	0.06293918	0.06293005	0.06292703	0.063
	0.13635682	0.13635288	0.13635158	0.136
0.5	0.09047265	0.09050087	0.09051027	0.091
	0.15976651	0.15977251	0.15977455	0.160
1.0	0.12075395	0.12098261	0.12105873	0.121
	0.18675515	0.18680568	0.18682259	0.187
1.5	0.14942784	0.15008769	0.15030708	0.149
	0.21244199	0.21255107	0.21258751	0.214
2.0	0.17743935	0.17891589	0.17940610	0.176
	0.23712850	0.23728741	0.23734045	0.239

TABLE III.2  
EXAMPLE (2)

$x$	$h = 0.1$	$h = 0.05$	$h = 0.001$	true sol
0.1	0.40413536	0.40413536	0.40414260	0.404
	0.25296205	0.25296205	0.25297556	0.253
0.5	0.47750479	0.47750479	0.47754610	0.477
	0.34243390	0.34243390	0.34252417	0.343
1.0	0.55964211	0.55964211	0.55974850	0.559
	0.44413253	0.44413253	0.4440607	0.445
1.5	0.63024798	0.63024798	0.63044422	0.630
	0.53201578	0.53201578	0.53260885	0.533
2.0	0.69012211	0.69012211	0.69041780	0.690
	0.60638524	0.60638524	0.60747044	0.608

#### IV. CONCLUSIONS

Some models of ultimate ruin theory in the form of first order Volterra integro-differential equations and systems of such equations were briefly presented. Numerical solutions for one such model (Model 1, in Albrecher and Boxma (2004) ([1])), which was in the form of a system of two (linear) Volterra integro-differential equations were given, using polynomial collocation methods. The results were in good agreement with these found by analytical methods in [1]. The distribution functions used in the model were of exponential type. It will be interesting to extend the ideas of the paper for use with other types of claims distribution functions.

**Keywords:** Numerical solution, Volterra integro-differential equations, actuarial risk management, ultimate time ruin theory.

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