

Location Decisions of Competing Platforms: The Principle of Maximum Differentiation May Not Hold*

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Abstract

We examine the spatial location decisions of two competing platforms in a two-sided market. The principle of maximum differentiation may fail to hold. In particular, when the cross-group network externalities are weak, we obtain maximum horizontal differentiation. For strong network externalities, however, the two platforms locate right next to each other (minimum differentiation), with one platform dominating the market (tipping). For intermediate network externalities, a location equilibrium in pure strategies does not exist. We then assume that platforms move sequentially. We show that in this case spatial differentiation is asymmetric and intermediate. The first mover locates in the middle, while the follower locates at an extreme point. Both platforms have positive market shares in equilibrium.

(Very Preliminary and Incomplete)

Keywords: Product Selection; Two-sided markets; Endogenous Locations; Cross-group Network Externalities.

JEL Classification Codes: D43, L13.

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1 Introduction

We examine the spatial location decisions of two competing platforms in a two-sided market. It is well-known that in one-sided markets firms choose maximum differentiation, d’Aspermont et. al. (1979). Maximum differentiation mitigates price competition. Most of the literature on two-sided markets has ignored the issue of product selection and usually assumes maximum horizontal differentiation when the market is characterized by spatial competition, e.g., Armstrong (2006).¹ This would be an innocuous assumption if maximum differentiation could extend in a two-sided market. However, as we argue, this may not be true.

We formulate a model with two competing platforms and two groups of agents that face quadratic transportation costs. Each agent receives higher benefit when more members from the other group join the platform. There is a trade-off between horizontal differentiation and market share. When a platform moves closer to the rival platform differentiation diminishes, but it becomes more likely for the platform that is moving closer to dominate the market (tipping). When the cross-group network externalities are weak the benefits from differentiation are more important and platforms differentiate maximally. When externalities are strong, the benefits from differentiation are relatively small and as a result differentiation is minimum in equilibrium. One platform emerges as the dominant medium of exchanges. More specifically, both platforms locate at the center, but only one platform is active and attracts all the agents. For intermediate values of the externalities, a location equilibrium in pure strategies does not exist. Then, we assume that platforms make their location decisions sequentially. The first-mover locates in the middle and the follower locates at an extreme point (*asymmetric location equilibrium*). Both platforms have positive but asymmetric market shares, with the first mover having a higher market share.²

In the absence of network externalities, even if firms move sequentially, the first mover will always locate at an extreme point to mitigate the ensuing price competition. Hence, the fact that the first mover in our model locates at the center, when the network externality is not weak, is a direct consequence of the network externalities.

There are examples of sequential entry into a market, where the first mover occupies a ‘central’

¹Exceptions are the papers by Gabszewicz et. al. (2002) and Kind et. al. (2007).

²We present our results in terms of the strength of the network externality, but, alternatively, we can vary the degree of differentiation (transportation cost) for any fixed positive externality. The two are inversely related, i.e., high externality is equivalent to low degree of differentiation and vice versa.

location, whereas the follower positions its product at a less central location. Consider, for example, the market for online video websites and in particular two important players in this market: YouTube and Hulu. YouTube, which was launched before Hulu, carries a huge number of diverse videos and clips, while Hulu is designed specifically for commercial movies and TV shows lovers. Our asymmetric location equilibrium can shed some light on these markets. One can view the platform that moves first, YouTube in this case, as a general video-sharing website that mostly caters to agents with intermediate preferences, while the follower, Hulu, can be viewed as a more specialized TV and commercial movie website. Consistent with our model predictions, YouTube has a higher market share than Hulu.³ Other examples include search engines on the Internet, where some are general, e.g., Google, Yahoo, while others, usually later entrants, are more specialized, e.g., Google Scholar.

Our duopoly model can also be used to understand the geographic locations of physical markets.⁴ When externalities are low (or equivalently transportation cost is high), markets will locate at the periphery, i.e., far apart from each other. For very strong externalities (or low transportation costs) only one market will be active and will locate at the center, while for intermediate externalities (transportation costs) there will be two active markets, one at the center and the other at the periphery. One implication of our model is that as the network externalities become stronger (or the transportation cost decreases) market shares will become more asymmetric and eventually one platform will dominate.

We would like, at this point, to highlight the role of ‘product’ selection. It is also true that even with fixed locations at the extremes the market tips when the externalities are strong enough (e.g., Armstrong (2006)). This is typical in models with cross-group network externalities. Nevertheless, when locations are fixed, we do not obtain asymmetric market structures where *both* platforms are active, which is the case when locations are endogenized. In that sense, a testable implication of our model is that market shares evolve ‘more continuously’ as the degree of network externalities (or the degree of differentiation) varies, starting from symmetric market structures when externalities are low, then moving to asymmetric market structures when externalities become stronger, and

³ According to “Disney’s Hulu Deal Raises Questions About YouTube Model,” Wall Street Journal, April 30, 2009, YouTube had 100 million viewers in March of 2009, while in the same time period Hulu had 41 million viewers.

⁴ Jin and Rysman (2009) investigate the pricing decisions of sportcard conventions. These conventions are two-sided markets since they try to attract both consumers and dealers. An important decision of these conventions is where to locate, given the competition they face from rival conventions.

eventually, for very strong externalities, the market is dominated by one platform.

Gabszewicz et. al. (2002) develop a model of media (newspapers) competition that features readers who have horizontal preferences with respect to the political ideology of a newspaper and advertisers. They show that when advertising revenue is important the two newspapers will choose the same political ideology (minimum differentiation). Without the advertising side the two newspapers differentiate maximally with respect to their political messages. An assumption that is made is that readers are indifferent to advertisements. Kind et. al. (2007) extend the Gabszewicz et. al. (2002) model by assuming that readers care about ads, either positively or negatively. They show that when the absolute value of this externality is high, newspapers will differentiate maximally. Our result is the opposite, as we show that strong externalities make minimum differentiation more likely. In addition, these papers do not obtain an asymmetric location equilibrium. We differ from the aforementioned two papers in that: i) both groups of agents in our model have horizontal preferences and as a consequence ii) we allow platforms to choose locations with respect to both groups of agents and not only with respect to one group (readers in those two papers).

The rest of the paper is organized as follows. We present the model in the next Section. In Section 3, we solve the model. We conclude in Section ???. The proofs can be found in the Appendix.

2 The description of the benchmark model

There are two groups of agents $\ell = 1, 2$ and two horizontally differentiated platforms $k = A, B$.⁵ We will denote the “other” group of agents by m . We capture platform differentiation as follows. There is a continuum of agents of group ℓ that is uniformly distributed on the $[0, 1]$ interval. Platform A is located at point a and platform B is located at point b , with $0 \leq a, b \leq 1$. We assume that transportation cost is quadratic in the distance d an agent has to ‘travel’ from his location to the location of the platform, td^2 , where the parameter $t > 0$ measures the per-unit cost of travel. We assume that each agent joins only one platform (single-homing). Each member of a group who joins a given platform cares about the number of members from the other group who join the same platform. Denote by $n_{\ell k}$ the number of participants from group ℓ that platform k attracts. The maximum willingness to pay for a member of group ℓ if he joins platform k is given by $V + \alpha_{\ell} n_{mk}$, where V is a stand-alone benefit each agent receives independent of the number of participants

⁵Our benchmark model follows closely the model in Armstrong (2006). We differ in that we endogenize the locations of the two platforms.

from the other group on platform k . The parameter $\alpha_\ell > 0$ measures the cross-group network externality for group ℓ participants. For simplicity, we assume that $\alpha_1 = \alpha_2 = \alpha$. The indirect utility of an agent from group ℓ who is located at point $x \in [0, 1]$ is given by,

$$U_\ell = \begin{cases} V + \alpha n_{mA}^e - t(a-x)^2 - p_{\ell A}, & \text{if he joins platform } A \\ V + \alpha n_{mB}^e - t(b-x)^2 - p_{\ell B}, & \text{if he joins platform } B \end{cases} \quad (1)$$

where $p_{\ell k}$ is platform k 's lump-sum charge to group ℓ participants and n_{mk}^e denotes the expectations agents from group ℓ have about how many agents from group m will join platform k . We assume that V is high enough which ensures that the market is covered. Prices cannot become negative and marginal cost is zero. As it is usual in these models (e.g., Armstrong (2006)) we assume that horizontal differentiation is more important than the cross-group network externality, $t > \alpha$.

The timing of the game is as follows. In stage 1, the two platforms make their location decisions, either simultaneously or sequentially. In stage 2, the platforms make their pricing decisions simultaneously. Finally, in stage 3, the agents decide which platform to join.

3 Analysis

We look for a subgame perfect Nash equilibrium. We solve the game backwards.

3.1 Stage 3: Agent decisions and market shares

The marginal agent in group 1 can be found as follows.

$$\begin{aligned} V + \alpha n_{2A}^e - t(a-x)^2 - p_{1A} - \left(V + \alpha n_{2B}^e - t(b-x)^2 - p_{1B} \right) &= 0 \\ \Rightarrow \hat{x}_1 &= \frac{p_{1B} - p_{1A} + t(b^2 - a^2) - \alpha(n_{2B}^e - n_{2A}^e)}{2t(b-a)}. \end{aligned} \quad (2)$$

The fraction of agents from group 1 that joins platform A is $n_{1A} = \hat{x}_1$ and the fraction from group 1 that joins platform B is $n_{1B} = 1 - \hat{x}_1$. Similarly, we can find the marginal agent in group 2.

$$\begin{aligned} V + \alpha n_{1A}^e - t(a-x)^2 - p_{2A} - \left(V + \alpha n_{1B}^e - t(b-x)^2 - p_{2B} \right) &= 0 \\ \Rightarrow \hat{x}_2 &= \frac{p_{2B} - p_{2A} + t(b^2 - a^2) - \alpha(n_{1B}^e - n_{1A}^e)}{2t(b-a)}. \end{aligned} \quad (3)$$

The fraction of agents from group 2 that joins platform A is $n_{2A} = \hat{x}_2$ and the fraction from group 2 that joins platform B is $n_{2B} = 1 - \hat{x}_2$.

In equilibrium, it must be that expectations are confirmed, that is, $n_{1A} = n_{1A}^e$, $n_{1B} = n_{1B}^e$, $n_{2A} = n_{2A}^e$ and $n_{2B} = n_{2B}^e$. Using (2) and (3), this defines a system of four equations in four unknowns, n_{1A} , n_{1B} , n_{2A} and n_{2B} . By solving the system we obtain the market shares as a function of prices and parameters.

$$n_{1A} = \frac{t(b-a)(p_{1B} - p_{1A} - \alpha) + \alpha(p_{2B} - p_{2A}) + D}{2[t^2(b-a)^2 - \alpha^2]} \quad (4)$$

$$n_{1B} = \frac{t(b-a)(p_{1A} - p_{1B} + \alpha) + \alpha(p_{2A} - p_{2B}) - t^2(b^3 + a^3) + F}{2[t^2(b-a)^2 - \alpha^2]} \quad (5)$$

$$n_{2A} = \frac{t(b-a)(p_{2B} - p_{2A} - \alpha) + \alpha(p_{1B} - p_{1A}) + D}{2[t^2(b-a)^2 - \alpha^2]} \quad (6)$$

$$n_{2B} = \frac{t(b-a)(p_{2A} - p_{2B} + \alpha) + \alpha(p_{1A} - p_{1B}) - t^2(b^3 + a^3) + F}{2[t^2(b-a)^2 - \alpha^2]} \quad (7)$$

where $D \equiv t\alpha(b^2 - a^2) - \alpha^2 + t^2(b^3 + a^3) - t^2ab(b+a)$ and $F \equiv -t\alpha(b^2 - a^2) + t^2ab(b+a) + 2t^2(b^2 + a^2) - \alpha^2 - 4abt^2$.

3.2 Stage 2: Platforms' pricing decisions

Platform k chooses p_{1k} and p_{2k} to maximize its profits

$$\pi_k = p_{1k}n_{1k} + p_{2k}n_{2k},$$

where $n_{\ell k}$, $\ell = 1, 2$ and $k = A, B$, are given by (4), (5), (6) and (7). The profit functions are strictly concave in a platform's own prices if

$$\alpha < t(b-a). \quad (8)$$

Alternatively, the above condition can be written as

$$a < b - \frac{\alpha}{t} \text{ or } b > a + \frac{\alpha}{t}.$$

If (8) is satisfied, then the first order conditions are also sufficient for profit maximization. The equilibrium prices then are given by

$$p_{1A} = p_{2A} = \frac{t(b-a)}{3}(2+b+a) - \alpha \text{ and } p_{1B} = p_{2B} = \frac{t(b-a)}{3}(4-b-a) - \alpha. \quad (9)$$

The equilibrium market shares, by substituting (9) into (4), (5), (6), and (12), are given by

$$n_{\ell A} = \frac{t(b-a)(2+b+a) - 3\alpha}{6(t(b-a) - \alpha)} \text{ and } n_{\ell B} = \frac{t(b-a)(4-b-a) - 3\alpha}{6(t(b-a) - \alpha)}, \ell = 1, 2.$$

For an interior equilibrium we need the market shares to be in $(0, 1)$. It turns out that $n_{\ell k} \in (0, 1)$ if and only if

$$\alpha < \min \left\{ \frac{t(b-a)}{3}(4-b-a), \frac{t(b-a)}{3}(2+b+a) \right\} \quad (10)$$

or equivalently, for $n_{\ell A}$ to be less than one (which implies that $n_{\ell B}$ is greater than zero) we must have

$$\alpha < \frac{t(b-a)}{3}(4-b-a) \Leftrightarrow a < \tilde{a} \equiv -\frac{1}{t} \left(-2t + \sqrt{3t\alpha + 4t^2 - 4bt^2 + b^2t^2} \right) \quad (11)$$

and for $n_{\ell A}$ to be greater than zero (which implies that $n_{\ell B}$ is less than one) we must have

$$\alpha < \frac{t(b-a)}{3}(2+b+a) \Leftrightarrow a < \hat{a} \equiv -\frac{1}{t} \left(t - \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2} \right). \quad (12)$$

For any given location b of platform B the market tips when platform A locates close enough to platform B , as the above thresholds indicate. The interior equilibrium profits as a function of the platforms' locations, after we substitute (9) into the profit functions, are⁶

$$\pi_A(a, b) = \frac{(t(b-a)(2+b+a) - 3\alpha)^2}{9(t(b-a) - \alpha)} \text{ and } \pi_B(a, b) = \frac{(t(b-a)(4-b-a) - 3\alpha)^2}{9(t(b-a) - \alpha)}. \quad (13)$$

It can be easily verified that when $\alpha = 0$ (no cross-group network externality) the equilibrium profits reduce to those in d'Aspermont et. al. (1979), where it is each platform's dominant strategy to locate at the extreme points (maximum differentiation). As we show next, this is not always the case when $\alpha > 0$.

3.3 Stage 1: Platforms' location decisions

Platforms choose their locations a and b to maximize profits as they are given by (13). We assume that they do so simultaneously. The thresholds (12) and (11) are very important at this stage.

⁶We will deal with the tipping solution next when we will analyze the location game.

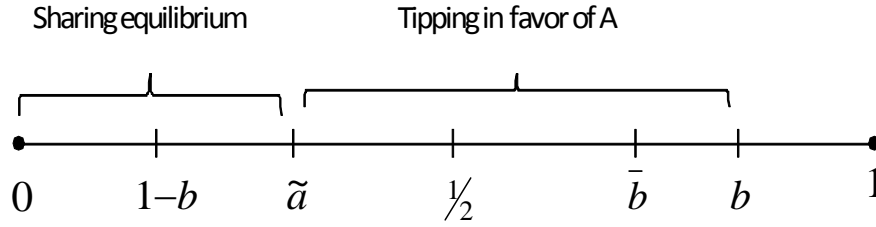


Figure 1: Type of equilibrium as the location a of platform A varies from 0 to b , with the location of platform B fixed at $b \geq \bar{b}$

It turns out that $\hat{a} \geq \tilde{a}$ if and only if $b \geq \bar{b} \equiv 1/2 + \alpha/(2t)$. Symmetrically, we can define the thresholds for platform B for a fixed location a of platform A . When $\hat{a} \geq \tilde{a}$ the binding threshold is the \tilde{a} . In this case, as it will become evident below, the other threshold is irrelevant. The opposite is true when $\hat{a} \leq \tilde{a}$.

We make the assumption that when the market tips it is the platform that is closer to the middle point that attracts all the agents. If they are equidistantly located from the middle, $a = 1 - b$, then all agents join platform A . In what follows, we assume that $a \leq b$. In the proofs of the Propositions, we allow $a > b$, but in this case the roles of the platforms are reversed and the profit functions and the thresholds can be obtained via a simple relabeling.

When $b \geq \bar{b}$, as platform A increases a it attracts all agents when $a = \tilde{a} \geq 1 - b$ (market tips).⁷ After this point, the market remains tipped in favor of A until $a = b$. Figure 1 depicts this case.

When $b \leq \bar{b}$, platform A 's market share becomes zero when $a = \hat{a} \leq 1 - b$.⁸ After this point, the market will tip in favor of A when $a = 1 - b$ (symmetric locations). The market will remain tipped in favor of A until $a = b$. Figure 2 depicts this case.

When $a > b$ the roles of the platforms are reversed (A becomes B and B becomes A). The analysis in this case will follow from the above two cases and it will be equivalent to fixing a and allowing b to vary. This in turn is equivalent to the above two cases after we set $b = 1 - a$.

The second order condition (8) is always satisfied when the market has not tipped, i.e., when

⁷It can be shown that $\tilde{a} \geq 1 - b$ if and only if $b \geq \bar{b}$.

⁸It can be shown that $\hat{a} \leq 1 - b$ if and only if $b \leq \bar{b}$. In addition, $\hat{a} = 0$, when

$$b \leq -1 + \frac{\sqrt{t^2 + 3t\alpha}}{t} < \bar{b}.$$

This suggests that when platform B moves closer to the middle after the above threshold platform A 's market share will be zero even when $a = 0$.

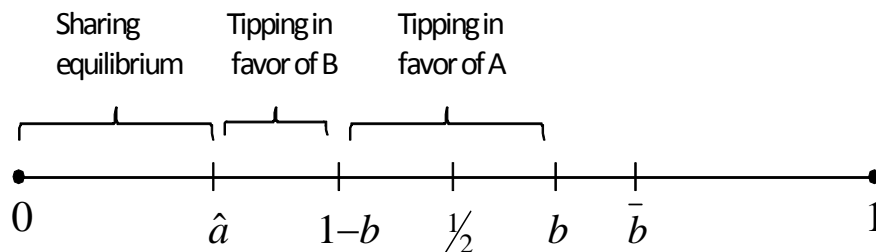


Figure 2: Type of equilibrium as the location a of platform A varies from 0 to b , with the location of platform B fixed at $b \leq \bar{b}$

$$a < \min \{\tilde{a}, \hat{a}\}.$$

The next Proposition shows that a symmetric location equilibrium that involves less than maximum differentiation (i.e., $a > 0$ and $b < 1$) does not exist.

Proposition 1 *A symmetric location equilibrium, $a = 1 - b$, with $0 < a < 1/2$, i.e., less than maximum horizontal differentiation does not exist.*

From any symmetric location configuration, platforms will either want to locate far apart from each other (maximum differentiation), or one platform wants to move close to the rival and dominate the whole market.

It will be helpful at this point to examine the properties of the profit function of platform A for a fixed location b of platform B . The derivative of platform A 's profit function (13) with respect to a is

$$\frac{\partial \pi_A(a, b)}{\partial a} = \frac{(2at - 2bt + \alpha - 4abt + 4a\alpha + 3a^2t + b^2t)(2bt - 2at - 3\alpha - a^2t + b^2t)t}{9(bt - \alpha - at)^2}.$$

Note that

$$\frac{\partial \pi_A(a = 0, b)}{\partial a} = \frac{(\alpha - 2bt + b^2t)(2bt - 3\alpha + b^2t)}{9(bt - \alpha)^2} < 0$$

if $b > \left(-t + \sqrt{3t\alpha + t^2}\right)/t$.⁹ So, the derivative of the profit function at $a = 0$ is always strictly negative when the market is shared.

⁹There are four roots when we solve

$$\frac{\partial \pi_A(a = 0, b)}{\partial a} = \frac{(\alpha - 2bt + b^2t)(2bt - 3\alpha + b^2t)}{9(bt - \alpha)^2} = 0$$

with respect to b . One is negative and the other is greater than one, so we rule them out. The remaining two are

$$r_1 \equiv \frac{1}{t} \left(-t + \sqrt{3t\alpha + t^2}\right) \text{ and } r_2 \equiv \frac{1}{t} \left(t - \sqrt{-t\alpha + t^2}\right).$$

For any a now, the derivative $\partial\pi_A(a, b)/\partial a$ becomes zero at

$$a = a_1 \equiv -\frac{1}{t} \left(t - \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2} \right) \quad (14)$$

$$a = a_2 \equiv \frac{1}{3t} \left(-t - 2\alpha + 2bt + \sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2} \right) \quad (15)$$

$$a = a_3 \equiv -\frac{1}{t} \left(t + \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2} \right)$$

$$a = a_4 \equiv \frac{1}{3t} \left(-t - 2\alpha + 2bt - \sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2} \right).$$

Roots a_3 and a_4 are negative, so we rule them out.¹⁰ Also note that $a_1 = \hat{a}$, where \hat{a} is given by (12). As we have mentioned before, we have $\hat{a} = a_1 \geq \tilde{a}$ if $b \geq \bar{b} \equiv 1/2 + \alpha/2t$ and $\tilde{a} \geq a_1 = \hat{a}$, if $b \leq \bar{b}$. This implies the following about the profit function of platform A for any fixed location b of platform B .

Case 1: $b \geq \bar{b} \equiv 1/2 + \alpha/(2t)$. Figure 3 depicts this case, where we have assumed that $a_2 \leq \tilde{a}$. Whether this holds or not depends, as we explain below, on the magnitude of α .

We have that $\hat{a} = a_1 \geq \tilde{a}$, so only root a_2 may be relevant. Given that it is the only relevant root coupled with the fact that the profit function of platform A is strictly decreasing at $a = 0$ when sharing takes place, the profit function must attain a local minimum at $a = a_2$, if $a_2 \leq \tilde{a}$. The two platforms share the agents (no tipping) when $a < \tilde{a}$. As platform A moves closer to platform B its prices fall (see (9)) but the market shares after a certain point may increase. This may happen after $a = 1 - b$ where platform A is closer to the middle point $1/2$ than B . That is why a minimum may be attained at $a = a_2$ and after this point the profit function increases. This is not always true as a_2 may be greater than \tilde{a} , in which case the profit function of platform A is decreasing until $a = \tilde{a}$. Platform B is losing market share and at $a = \tilde{a}$ tipping occurs in favor of A . Profits increase for A as it moves closer to b because its distance to the marginal agents, who are located at 1, decreases. When $a > b$ the platforms reverse roles. Overall, the profit function of platform A

It can be shown that $r_1 \geq r_2$ if and only if $t \geq \alpha$, which we have assumed holds. It can be computed that \hat{a} (from (12)) becomes zero when $b \leq r_1 \equiv (-t + \sqrt{3t\alpha + t^2})/t$. Therefore, the other feasible root, root r_2 , becomes irrelevant since when $b \leq r_1$ the market tips.

¹⁰Root a_3 is clearly negative. Root a_4 is negative because at $\alpha = 0$, a_4 becomes

$$\frac{-t(2-b)}{3t} < 0$$

and the derivative of a_4 with respect to α is

$$\frac{1}{t} \left(-\frac{1}{6} \frac{t + 8t\alpha - 8bt}{\sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2}} - \frac{2}{3} \right)$$

which is always negative for $b \in [0, 1]$.

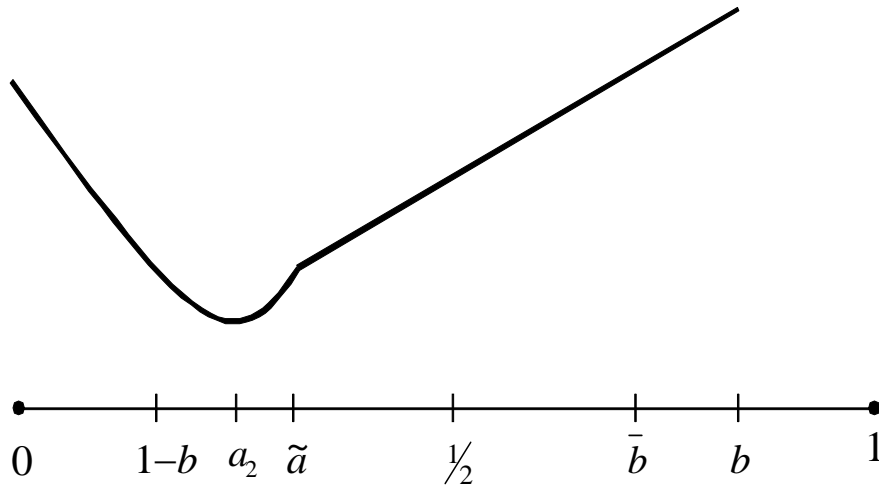


Figure 3: Profit function of platform A as its location a varies from 0 to b for a fixed location b of platform B at $b \geq \bar{b}$

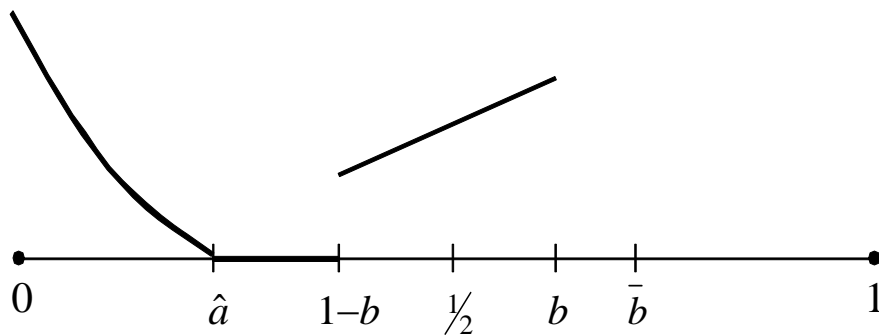


Figure 4: Profit function of platform A as its location a varies from 0 to b for a fixed location b of platform B at $b \leq \bar{b}$

as a function of its location a is U-shaped up to $a = b$ when $b \geq \bar{b}$.

Case 2: $b \leq \bar{b} \equiv 1/2 + \alpha/(2t)$. Figure 4 depicts this case.

Given that platform A 's profit function is strictly decreasing at $a = 0$ when the market is shared and that $\hat{a} = a_1$ is now a relevant root, root a_2 becomes irrelevant. This is because at $a = \hat{a} = a_1$ the market tips in favor of B and the slope of A 's profit function becomes zero. This implies that a_2 cannot be less than $\hat{a} = a_1$, since if that was the case there should be one more root in that range. Hence, in this case it must be that $a_2 > \hat{a} = a_1$ and therefore a_2 is irrelevant. The two platforms share the market when $a < \hat{a}$, unless $b \leq \left(-t + \sqrt{3t\alpha + t^2}\right)/t$, in which case platform A 's market share is zero at $a = 0$. As platform A moves closer to b both prices and market shares

fall (because now B is closer to the middle than in case 1 above) and at $a = \hat{a}$ the market tips in favor of B . Then, at $a = 1 - b$ the market tips in favor of A . As in case 1 above, the profit function is U-shaped up to $a = b$.

The above two cases will be used in the proofs of Propositions 2 and 3. These cases also suggest that the profit function will also be U-shaped when $a > b$. This follows because the case of $a > b$ is a simple relabeling of the above two cases.

Next, we examine whether an equilibrium with maximum differentiation, $a = 0$ and $b = 1$, exists. The next Proposition summarizes the result.

Proposition 2 (*Maximum differentiation*) *A maximum horizontal differentiation equilibrium, where $a = 0$ and $b = 1$, exists if and only if $\alpha \leq t/3$.*

When the cross-group network externalities are weak, a platform has no incentive to move close to the rival platform in order to become the dominant platform. The benefit from differentiation is minimized in this case and the dominant platform is only able to benefit from the larger market share. Since the externalities are weak, so is the benefit from the larger market share.

The next Proposition presents the result when the cross-group network externality is high.

Proposition 3 (*Minimum differentiation*) *Suppose $\alpha \in [5t/12, t)$. The location equilibrium entails minimum horizontal differentiation and tipping in favor of one platform. In particular, $a = b = 1/2$ and platform A attracts all the agents. If $\alpha \in (t/3, 5t/12)$ a location equilibrium in pure strategies does not exist.*

The intuition behind the minimum differentiation result is as follows. When the cross-group network externalities are important, less than maximum differentiation makes tipping more likely and benefits the platform that is able to attract all agents. The dominant platform has an incentive to locate in the middle for two reasons: i) there is no room for the rival platform to differentiate itself enough by moving to an extreme and attract some agents and ii) the rival platform cannot move closer to the middle point and attract all agents. Indeed, the rival platform makes zero profits regardless of where it chooses to locate and for the equilibrium to exist we assume that it also locates at the middle point. So, the two platforms are not differentiated and in this case we

assume that all agents coordinate and join one platform, platform A . Platform A charges prices equal to α and B 's prices are zero. Recall that we do not allow for negative prices.

3.3.1 Platforms locate sequentially

When an equilibrium in pure strategies does not exist, we restore existence by assuming that platforms move sequentially, with platform A moving first. The next Proposition summarizes the subgame perfect Nash equilibrium.

Proposition 4 (*Asymmetric location equilibrium*). *Suppose $\alpha \in (t/3, 5t/12)$. Platform A moves first and locates at $a = 1/2$. Platform B moves second and locates at $b = 1$. Both platforms attract agents in equilibrium (sharing equilibrium). Platform A attracts more agents than platform B . The equilibrium profits are*

$$\pi_A(a = 1/2, b = 1) = \frac{(7t - 12\alpha)^2}{72(t - 2\alpha)} \text{ and } \pi_B(a = 1/2, b = 1) = \frac{(5t - 12\alpha)^2}{72(t - 2\alpha)}.$$

Platform A moves first and locates in the middle. Given that the externality is not as strong as in Proposition 3, platform B can attract some agents and make strictly positive profits when it locates at the extreme. Platform A has no incentive to move away from the middle point, because in this case B has a profitable deviation to locate closer to the middle point than A and leave A with zero agents and profits.

A Appendix

A.1 Proof of Proposition 1

Differentiate $\pi_A(a, b)$ from (13) with respect to a and then impose symmetry, i.e., $a = 1 - b$. This yields

$$\left. \frac{\partial \pi_A(a, b)}{\partial a} \right|_{a=1-b} = -\frac{t(1+4a)}{3} < 0.$$

Hence, each platform has an incentive to move to the endpoints of the $[0, 1]$ interval. The above assumes that conditions (12) and (11) are satisfied, so the pricing equilibrium is interior (no tipping).

Now assume that the platforms move closer, so that tipping occurs. For symmetric locations, using conditions (12) and (11), this implies $b < \bar{b} \equiv 1/2 + \alpha/(2t)$ (and $a > \bar{a} \equiv 1/2 - \alpha/(2t)$). In this case, according to our assumption, platform A attracts all agents. At that point platform B 's profits are zero. Is this an equilibrium? The answer is negative. Platform B has a profitable deviation. It can move closer to the middle point than A and attract all the agents.

A.2 Proof of Proposition 2

We set $b = 1$ and examine how the profit function of platform A behaves as a ranges from 0 to 1. Because $b \geq \bar{b}$, only the \tilde{a} threshold is relevant. Overall, the profit function of A exhibits a U-shape pattern with respect to a (see also Figure 3). This implies that any interior location $a \in (0, 1)$ yields lower profits than the extreme locations. For an interior pricing equilibrium we need (11), $a < \tilde{a}$, to be satisfied. When $a = 0$, we have that $\partial \pi_A(a = 0, b = 1) / \partial a = -t/3 < 0$, which implies that platform A has no incentive to locally deviate from $a = 0$, when platform B is at $b = 1$. (This is the standard result from one-sided markets). But A can deviate globally and induce tipping in its favor.

Next, we examine the tipping case. This case arises when $a \geq \tilde{a}$. When $b = 1$, this amounts to

$$a \geq \tilde{a} \equiv -\frac{1}{t} \left(\sqrt{3t\alpha + t^2} - 2t \right).$$

When $a \geq \tilde{a}$, platform B 's market shares and prices are zero. The marginal agents from both groups are located at 1. Platform A has attracted all the agents. The indirect utility of the marginal agent from group ℓ if he joins platform B is V , while if he joins platform A instead his indirect utility becomes $V + \alpha - t(1 - a)^2 - p_{\ell A}$. So, platform A will set its price to keep the marginal agent

indifferent between the two platforms. This yields

$$p_{\ell A} = \alpha - t(1 - a)^2.$$

Prices and profits for platform A increase in a . Platform A then locates at $a = 1$, charges $p_{\ell A} = \alpha$ and dominates the market. Platform A 's profits are

$$\pi_A(a = 1, b = 1) = 2\alpha.$$

On the other hand when $a = 0$ the market is shared. Hence, from (13), platform A 's profits are

$$\pi_A(a = 0, b = 1) = t - \alpha.$$

Platform A has no incentive to deviate from $a = 0$ if and only if $t - \alpha \geq 2\alpha \Rightarrow \alpha \leq t/3$.

If $\alpha > t/3$, then platform A has an incentive to locate at $a = 1$ and dominate the market. Platform B in this case earns zero profits. Hence, a maximum horizontal differentiation equilibrium does not exist.

A.3 Proof of Proposition 3

In Proposition 2, we proved that platform A has an incentive to locate at $a = 1$, when B is located at $b = 1$ and $\alpha > t/3$. But this location configuration cannot be an equilibrium. Platform B 's profits are zero, and if platform B deviates to $b = 0$, and assumes the role of platform A , its profits will become strictly positive. This is because when differentiation is maximum the equilibrium is always sharing, given our assumption that $t > \alpha$. We also showed in Proposition 1 that a symmetric interior location equilibrium with $a < 1/2$ does not exist. What is left to be examined are interior asymmetric location equilibria.

To this end, we fix $b < 1$ and we examine the properties of $\pi_A(a, b < 1)$. We have the following two cases.

Case 1: $b \geq \bar{b}$. Figure 3 depicts this case.

Case 2: $b \leq \bar{b}$. Figure 4 depicts this case.

For any fixed b we either have sharing or tipping. The above two cases suggest that if the equilibrium involves sharing the optimal location of A is either at $a = 0$ or at $a = 1$, as the platform wants to move away from the rival. Without loss of generality let's assume that $b \geq 1/2$.

Hence, platform A will either locate at zero (given that $b \geq 1/2$, $a = 0$ is optimal when the equilibrium involves sharing) or it will induce tipping in its favor and locate at $a = b$. Note that if $a > b$ the market will tip in favor of B . If $a = 0$, platform B , as we showed in Proposition 2, will have an incentive to also locate at zero. If it is $a = b > 1/2$, platform B makes zero profits and can move to a location that is closer to the middle in order to attract all the agents and enjoy strictly positive profits. If it is $a = 1/2 < b$, A will have an incentive to move to $a = b$. Hence, the only possible equilibrium is $a = b = 1/2$, with platform A attracting all agents. To be an equilibrium, platform B must not be able to secure strictly positive profits even when it locates at the extreme. We had showed that $\hat{a} = 0$ (from (12)), when

$$b \leq -1 + \frac{\sqrt{t^2 + 3t\alpha}}{t} < \bar{b}.$$

If

$$-1 + \frac{\sqrt{t^2 + 3t\alpha}}{t} \geq \frac{1}{2}$$

then platform A cannot obtain positive profits regardless of where it moves if B is located at $1/2$. The above inequality holds if and only if $\alpha \geq 5t/12$. Here, the roles of A and B are reversed: A is located at $1/2$ and B chooses where to locate in order to make strictly positive profits. With a simple relabeling, the above discussion implies that platform B cannot obtain strictly positive profits regardless of where it locates.

If $\alpha < 5t/12$ a location equilibrium in pure strategies does not exist. Platform B will have an incentive to move to $b = 1$, but then platform A will have an incentive to locate at $a = 1$ and so on.

A.4 Proof of Proposition 4

Fix $a \leq 1/2$ and look at the optimal location of platform B . Platform A 's profit function is U-shaped with respect to a for any $b \geq a$, see Figures 3 and 4. If $a < 1/2$, platform B 's profit is decreasing starting from $b = 1$ until $b = 1 - a$. After that point, the market tips in favor of B and its profits tend to 2α as b tends to a , independent of where a is. These profits are higher than the profits platform B obtains if it stays at $b = 1$. This can be seen as follows. When $a = 0$, 2α is higher than the profits of platform B when $b = 1$ (which are $t - \alpha$) if and only if $\alpha > t/3$. Moreover, as a increases both the (interior) prices of platform B and its market shares decrease. Therefore, moving away from $b = 1$ and locating arbitrarily close to a will yield higher profits for B . This

suggests that a location configuration with $a < 1/2$ and $b = 1$ cannot be an equilibrium. Platform B has a profitable deviation and platform A makes zero profits. The subgame perfect equilibrium is for platform A to locate at $a = 1/2$ and B to locate at $b = 1$. Given $a = 1/2$, platform B has no profitable deviation. In addition, following from the proof of Proposition 3, platform B attracts agents (sharing equilibrium). Platform A has no incentive to locate to $a < 1/2$ because in that case platform B can locate closer to the middle point and attract all the agents, leaving A with zero profits.

The profits of platform A are

$$\pi_A(a = 1/2, b = 1) = \frac{(7t - 12\alpha)^2}{72(t - 2\alpha)}$$

and of platform B are

$$\pi_B(a = 1/2, b = 1) = \frac{(5t - 12\alpha)^2}{72(t - 2\alpha)}.$$

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