

# Customer Poaching, Coupon Trading and Consumer Arbitrage\*

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## Abstract

The price discrimination literature typically makes the assumption of no consumer arbitrage. This assumption is increasingly violated when firms distribute coupons to poach customers from rival firms and those coupons can be easily traded among consumers. We relax the no-arbitrage assumption and study the impacts of coupon trading on equilibrium prices, promotion intensities (frequency and depth) and profits. We find that a larger fraction of coupon traders among consumers or higher distribution costs reduce the attractiveness of couponing, and firms respond by lowering their promotion efforts. This leads to higher equilibrium prices and profits, since coupon targeting intensifies competition. These results are robust to the possibility of defensive couponing as an additional strategy to prevent ones own loyal customers from switching.

**Keywords:** Customer Poaching; Coupon Trading; Consumer Arbitrage.

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# 1 Introduction

There is a large literature on price discrimination.<sup>1</sup> A common assumption made in these studies (with few exceptions) is that consumers cannot engage in arbitrage.<sup>2</sup> In traditional markets, it is costly for a consumer to locate another potential buyer of the same product and then trade. In that sense, the no arbitrage assumption, while not entirely true, may not be unrealistic. However, this assumption is increasingly violated in the digital economy. For one reason, it is getting easier to buy products cheap and resell for a profit online (e.g. ebay.com), since the direct “consumer-to-consumer” markets are more developed and information can be easily exchanged on the internet. For another reason, one of the commonly used methods to achieve price discrimination is to target consumers with coupons, and coupons can be easily traded. In the case of online shopping, all that consumers need is a coupon code. Not surprisingly, more and more coupons are traded online. For a simple example, go to Ebay.com and search under “coupon,” you will find that there are over 20,000 (not counting multiple coupons in one listing) of them for sale.<sup>3</sup>

In this paper we relax the no consumer arbitrage assumption by allowing coupons to be traded. Specifically, we assume that *some* consumers have low hassle cost of selling or buying coupons, and we call them coupon traders.<sup>4</sup> Other consumers have prohibitively high cost of trading coupons and are called non-traders. We assume that firms have information (e.g. purchase history) to differentiate between their own loyal customers and their rivals’, and thus can price discriminate between them by sending coupons to their rivals’ loyal customers. We develop a location model of oligopolistic third-degree price discrimination to study how prices, promotion intensities and profits change as the fraction of coupon traders increases.

Depending on the method of distribution, coupons can be divided into two types: *mass media coupons* and *targeted coupons*. Mass media coupons are distributed randomly by the

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<sup>1</sup>A search in EconLit results in 521 papers with “price discrimination” in the titles of the papers.

<sup>2</sup>If a firm offers a menu of various qualities of products available to all consumers, and relies on consumers to self-select based on their characteristics, then consumers have no incentive to engage in arbitrage and the assumption is not needed. However, if the menu includes offers of different quantities, or if the firm give different options to different consumers, then the no arbitrage assumption is needed.

<sup>3</sup>These are only listings of coupons for auction or sale. Not all of them are sold. To get a sense of how many are actually sold, we searched for a specific coupon (Staples coupon), and checked the 10 listings with the earliest expiration times. We found that 5 of them had bids.

<sup>4</sup>For example, some consumers may be familiar with Ebay and have various accounts already set up for transactions there, so the incremental transaction cost of trading coupons online is minimal.

firms, and consumers, based on their characteristics, self-select as to whether to collect and use the coupons (Narasimhan 1984). However, with the availability of more and more data on actual consumer transactions, and better technology to utilize such data, firms do not need to rely exclusively on consumer self-selection. Instead, they can select shoppers with specific characteristics, and send targeted coupons (Shaffer and Zhang 1995). A popular form of targeted coupon is an offensive poaching coupon. Firms send offensive poaching coupons to poach rival firm's loyal customers, i.e., those who will purchase from the rival firm if prices are the same.<sup>5</sup> Some poaching coupons carry no restrictions in terms of who can use the coupons. For example, when a consumer purchases a a specific brand of product at a supermarket, he may receive a coupon for a rival brand during checkout. This coupon usually does not carry restriction on who can use it, and thus can be traded and used by any one. However, there are coupons which can be used only by specific types of customers. For example, if you receive an offer of a one time bonus by switching long distance call service to AT&T, you qualify for the offer only if you are not currently with AT&T.

In this paper we focus on offensive poaching coupons which carry no restriction on who can use them.<sup>6</sup> With this type of coupon, when a coupon trader receives a poaching coupon, he will always sell this coupon back to the couponing firm's loyal customers, since the latter values the coupon more. In the symmetric pure strategy equilibrium of this model we find that when the fraction of coupon traders increases, firms will promote (send coupons) less frequently and with lower coupon face values. This reduces competition, leading to higher prices and profits. Coupon trading has similar effects as an increase in the cost of distributing coupons, in the sense that both discourage firms from sending coupons. Therefore, both firms will reduce couponing which increases prices and firm profits, since price discrimination with coupons constitutes a prisoners' dilemma game.

The rest of the paper is organized as follows. We review the literature in Section 2. Our model is presented in Section 3, and Section 4 contains our main results. In Section 5 we offer some extensions of our model and we conclude in Section 6. The proofs are provided in the Appendix.

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<sup>5</sup>This is somewhat similar to "reciprocal dumping" in the trade literature (e.g. Brander and Krugman 1983). In both cases, each firm has disadvantage in one market, whether it is due to weaker preferences of consumers there (our case) or higher transportation cost to serve consumers there (the reciprocal dumping case). Firms poach each other's strong markets, leading to lower profits for both firms.

<sup>6</sup>In Section 4.2, we allow firms to distribute *defensive* coupons. That is, a firm can issue coupons to its own loyal customers to retain them in the presence of its rival firm's poaching coupons. We find that firms have no incentives to distribute defensive coupons in the equilibrium.

## 2 Literature review

Early studies consider coupon as a device to create market segmentation due to consumers' self-selection (*mass media coupons*). For example, in Narasimhan (1984), couponing enables price discrimination, providing a lower price to a particular segment of consumers, while keeping the price high for others. With the availability of more consumer information, firms can rely less on consumer self-selection and more on targeted coupons (see Shaffer and Zhang (1995) for examples of such practices).<sup>7</sup> Targeted coupons are mostly modeled as offensive poaching coupons, i.e., to poach rival firms' loyal customers (Shaffer and Zhang (1995), Bester and Petrakis (1996), Fudenberg and Tirole (2000) etc.).

The study most relevant to our paper is Bester and Petrakis. They look at a duopoly model where consumers have preferences for one brand over the other. Each firm can send out coupons to consumers who prefer the other firm's product. They show that couponing intensifies competition between the firms, and the equilibrium prices and profits are lower than when no coupons are offered. Since couponing leads to a prisoners' dilemma game, an increase in the cost of coupon distribution would lead to higher prices and lower consumer surplus.

Fudenberg and Tirole look at a two-period game where in the second period, firms can separate consumers who bought from them in period 1 from those consumers who did not. Consequently, each firm can poach the customers of their competitor's by sending them coupons to induce them to switch. They also find that poaching leads to lower prices. They then investigate the efficiency of long-term contracts or short-term contracts.

In the previous two papers, a poaching firm sends the same coupons to all of its rival's customers. The coupons are different in Liu and Serfes (2004) and Shaffer and Zhang (1995, 2002). In Liu and Serfes, both firms send coupons of different face values. This is because firms have detailed information which enables them to segment consumers into various groups. In Shaffer and Zhang (1995), each firm offers only one type of coupons, but it can choose to send coupons to only a portion of the customers (partial couponing), since each firm has the ability to identify and target each individual consumer. The reason firms do partial couponing is because some consumers' preferences are so strong that the poaching firm can't attract them by sending coupons. The game is still a prisoners' dilemma game in

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<sup>7</sup>Coupons can also enable firms to reward repeat purchase customers. In particular, firms can issue coupons to consumers buying from them, and these coupons offer discounts when these consumers buy from the same firms later. See Fong and Liu (2008) for details.

which the net effect of coupon targeting is the coupon distribution cost plus the discounts given to redeemers. Only when firms are asymmetric, the game may not be a prisoners' dilemma with one-to-one promotions (Shaffer and Zhang 2002), since there is also a market share effect. The firm with the higher-quality product may gain from one-to-one promotions at the cost of the lower-quality product firm.

There have been few studies analyzing resale or consumer arbitrage, and they typically consider only monopoly. In Anderson and Ginsburgh (1999), consumers differ in two dimensions: willingness to pay and arbitrage cost. In their setup, a monopolist can sell its product in two countries. It may sell in the second country even if there is no local demand, with the sole purpose of discriminating across consumers with different arbitrage costs in the first country. Calzolari and Pavan (2006) consider a monopolist's problem of designing revenue-maximizing mechanisms when resale is possible. They consider two consumers with two types of valuations and find that the revenue-maximizing mechanism may require a stochastic selling procedure. Hafalir (2007) extends two consumer types to continuous types and considers two specific models. In the first model, optimal menu without resale involves quantity discounts, and resale hurts the monopolist's revenue. In the second model, there are quantity premia. In this case, the monopolist can offer the same quantities as in the without resale optimal menu, but demand higher prices from the consumers. Therefore, its revenue increases when resale is allowed.

The auction literature has also considered how resale affects bidding. In particular, the information revealed in a primary auction market and changes in bidder participation patterns can create inefficiencies that affect the revenue ranking of standard auction formats when there is an option to resale in a secondary market. Zheng (2002) investigates the design of seller-optimal auctions when winning bidders can attempt to resell the good, and characterizes the sufficient and necessary condition for sincere bidding with resale. Haile (2003) considers how resale opportunities affect bidders' valuations and finds that the secondary market can benefit the initial seller if the resale seller can extract a sufficient share of the resale surplus. In our case, there is no pricing decision that is conditioned on the bidding outcome of the auction. The efficient re-allocation of coupons results via English auctions held with no incentive distortions.

### 3 The description of the model

Two firms – 1 and 2 – produce competing goods with constant marginal cost of  $c$ . For simplicity we normalize  $c = 0$ . Each consumer buys at most one unit of the good and is willing to pay at most  $V$ . We assume that  $V$  is sufficiently high and therefore the market is always covered. Consumers are heterogeneous with respect to the premium they are willing to pay for their favorite brand. This heterogeneity is captured by a parameter  $l$ , which represents the consumer's degree of loyalty. Specifically, a consumer located at  $l$  is indifferent between buying from the two firms if and only if  $l = p_1 - p_2$ . We assume that  $l$  is uniformly distributed on the interval  $[-L, L]$  with density 1.<sup>8</sup> When two firms charge the same prices, consumers located at  $l > 0$  prefer firm 1's product and are called firm 1's loyal customers. Similarly, customers with  $l < 0$  are firm 2's loyal customers.

The interval  $[-L, L]$  is partitioned into two segments:  $[-L, 0]$  and  $[0, L]$ , corresponding to firm 2's and firm 1's loyal customers respectively.<sup>9</sup> Firms know which segment each consumer is located in, but do not know exactly where in the corresponding segment. For example, for someone located at  $L/2$ , firms will know that she is located in the segment  $[0, L]$ , but not that she is located at  $L/2$ . There are two types of pricing strategies that a firm can adopt in our context: (i) Uniform pricing: Each consumer on the  $[-L, L]$  interval receives the same price. This price is also called the regular price, which is the price consumers pay without coupons. (ii) Segment couponing: A segment of consumers receives the same price. In particular, firm 1 will distribute coupons to consumers in  $[-L, 0]$ , and firm 2 will send coupons to those in  $[0, L]$ . Regular price net of the coupon face value is the actual price the consumers with coupons pay. Let  $p_1$  and  $p_2$  denote the uniform prices of the two firms, and let  $d_1$  and  $d_2$  denote the face values of the coupons offered by firm 1 and 2 respectively. Consumers who do not use coupons will pay either  $p_1$  or  $p_2$ , depending on which firm they buy from, but they will pay only  $p_1 - d_1$  or  $p_2 - d_2$  when using coupons (Following the literature, we consider cents-off coupons instead of percentage-off coupons).

Let  $\lambda_1 \in [0, 1]$  denote firm 1's promotion effort so that all consumers in the segment

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<sup>8</sup>A similar model has been used in Shaffer and Zhang (2002).

<sup>9</sup>We assume this exogenous information structure and do not investigate how this structure emerges. One can think of a two-period model where the information structure emerges endogenously after consumers make purchasing decisions in the first period (e.g. Fudenberg and Tirole). In a symmetric equilibrium of this two-period model, firms charge the same prices in the first period, and the information structure at the beginning of the second period will be what we assume here. In practice, firms can obtain such information from a number of different sources such as past transactions, credit card reports and marketing companies.

$[-L, 0]$  have  $\lambda_1$  probability of receiving firm 1's coupons. Similarly,  $\lambda_2 \in [0, 1]$  is the fraction of consumers in  $[0, L]$  who would receive coupons from firm 2. The cost of distributing coupons is increasing and convex in the promotion effort and the size of the segment. Specifically, we assume that the cost is  $k(\lambda_i L)^2$  for firm  $i$  ( $k > 0$ ).

All consumers incur no cost when using coupons.<sup>10</sup> However, they differ in whether they trade (buy/sell) coupons. A fraction  $\alpha$  of them have zero cost of trading coupons. We call them *coupon traders*. If these consumers receive coupons, they will either use the coupons or sell them to other consumers who value the coupons more. They may also buy coupons from other customers. The remaining  $1 - \alpha$  fraction have infinite cost of trading coupons and are called *coupon non-traders*.<sup>11</sup>

Our models are related to those in Bester and Petrakis, and Fudenberg and Tirole. If we set  $\alpha = 0$ , our model becomes the one in Bester and Petrakis (uniform distribution). If we set  $\alpha = 0$  and  $k = 0$ , our model becomes the second period of Fudenberg and Tirole (short-term contracts with uniform distribution).

The game we will study can be described as follows.

- Stage 1. Firms, simultaneously and independently, decide their regular prices ( $p_i$ ), promotion frequency ( $\lambda_i$ ) and depth ( $d_i$ ).<sup>12</sup>
- Stage 2. Coupon distribution is realized. Coupon trading then takes place.
- Stage 3. Consumers make purchasing decisions. If they use coupons, they will pay regular price minus the coupon face value.

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<sup>10</sup>We relax this assumption by introducing coupon non-users in Section 5.1. Our results do not change qualitatively with the introduction of coupon non-users.

<sup>11</sup>It's certainly more realistic to assume a smooth distribution of coupon trading costs. Consumers with trading costs below certain level are willing to trade coupons – coupon traders, and those with higher coupon trading costs will not – non-traders. The cutoff coupon trading cost will determine the fraction of coupon traders  $\alpha$ . Note that, in this setup  $\alpha$  and equilibrium prices and promotion strategies are interdependent. For tractability, we assume that  $\alpha$  is exogenous in this paper, and we reserve the endogenization of  $\alpha$  for future research.

<sup>12</sup>Similar to Bester and Petrakis, we model the price and promotion strategies as a simultaneous game. An alternative way of modeling is a sequential-move game where firms chooses one strategy (say price) before they choose the other strategy (say promotion strategy). However, it is unclear to us whether firms should choose price strategy or promotion strategy first. On the one hand, it is often viewed that regular price is a higher level managerial decision and is relatively slow to adjust in practice than promotions. On the other hand, we often observe regular price changes while promotion strategy (e.g. coupon face values) is relatively stable.

We assume that firms are risk neutral and maximize their expected profits. So firm  $i$ 's problem is to choose  $p_i, d_i$  and  $\lambda_i$ ,  $i = 1, 2$  to maximize its profit,

$$\max_{p_i, d_i, \lambda_i} \pi_i(p_i, d_i, \lambda_i, p_{-i}, d_{-i}, \lambda_{-i}), \quad i = 1, 2.$$

## 4 Analysis

We first provide a road map for how we solve the game. The consumers can be segmented into 3 groups, depending on whether they are coupon traders and whether they receive coupons. We calculate firms' profits from each group. Then we aggregate profits over all groups of consumers net of the coupon distribution cost. Solving the first order conditions, we obtain the equilibrium price, promotion frequency and depth.<sup>13</sup>

In particular, the three groups of consumers are:

Type (a): non-traders without coupon;

Type (b): non-traders with coupon;

Type (c): traders with or without coupon.

Based on firm  $i$ 's promotion effort  $\lambda_i$  ( $i = 1, 2$ ), each consumer targeted has an equal probability,  $\lambda_i$ , of receiving the coupon. Firm 1's coupons targeted consumers in  $[-L, 0]$ , while firm 2's target consumers in  $[0, L]$ .<sup>14</sup> Firms maximize their *expected* profits, thus we only need to consider, on average, how many consumers are of each type. We start by calculating each firm's demand and profit from each type of consumers, and then add them up to obtain each firm's overall demand and profit.

### (1) Demand and profit from type (a) consumers

We start with type (a) consumers, who are depicted in Figure 1. Consumer density can be different in  $[-L, 0]$  and in  $[0, L]$ , depending on their promotion frequency. In particular, there is a fraction  $(1 - \alpha)(1 - \lambda_1)$  of non-traders without coupon in the interval  $[-L, 0]$ , and a fraction  $(1 - \alpha)(1 - \lambda_2)$  of non-traders without coupon in the interval  $[0, L]$ . Without loss

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<sup>13</sup>First order conditions are necessary but not sufficient. We need to make sure that the solutions that we obtain constitute equilibrium strategies ( $p^*$ ,  $d^*$  and  $\lambda^*$ ). Instead of checking whether the Hessian is negative semidefinite (which is quite messy), we show that no firm has incentive to unilaterally deviate from this profile of strategies. Details are provided in the proof of Proposition 1 in the Appendix.

<sup>14</sup>Here we assume that firms can only distribute *offensive* coupons, i.e., coupons to poach rival firms' loyal customers. We relax this assumption in Section 4.2, and allow firms to distribute *defensive* coupons, i.e., coupons to their own loyal customers preventing them from switching.

of generality, assume that  $p_1 \geq p_2$ . Let  $l_a$  denote the location of the marginal consumer, which is defined by

$$l_a = p_1 - p_2 \geq 0.$$

Every consumer to the left of  $l_a$  will buy from firm 2 at price  $p_2$ , and those to the right will buy from firm 1 at price  $p_1$ .

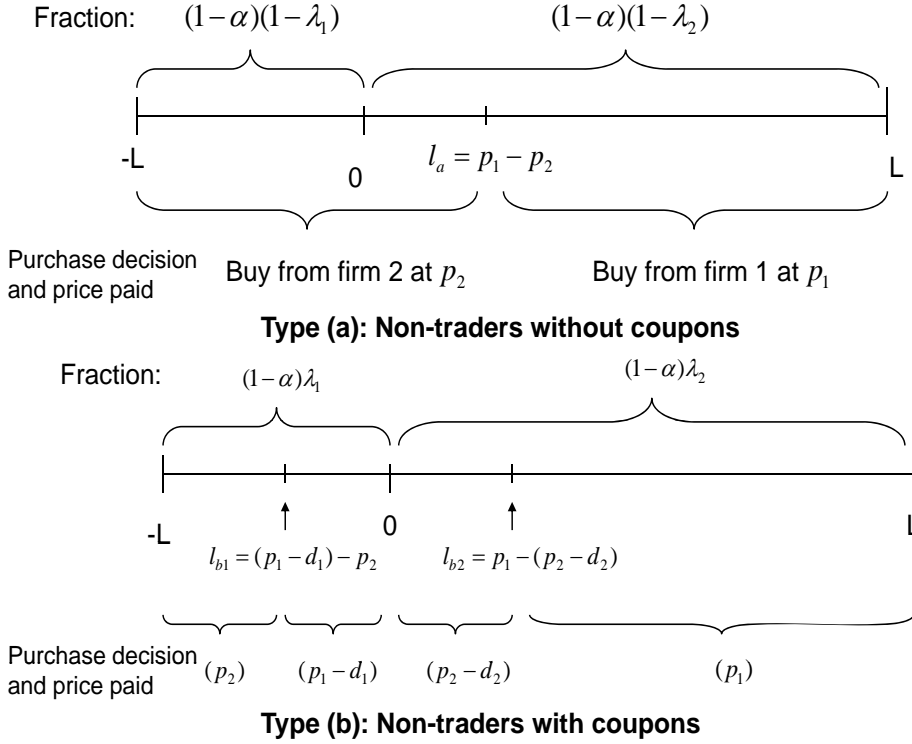


Figure 1: Type (a): Non-traders without coupons; Type (b): Non-traders with coupons.

Therefore, firms 1 and 2 make sales of<sup>15</sup>

$$\begin{aligned} d_{1a} &= (1-\alpha)(1-\lambda_2)(L-l_a) \\ &= (1-\alpha)(1-\lambda_2)(L-p_1+p_2) \end{aligned}$$

<sup>15</sup>We use  $d_i$ ,  $i = 1, 2$ , to denote firm  $i$ 's promotion depth (coupon face values), and  $d_{ij}$ ,  $i = 1, 2$ ,  $j = a, b, c$ , to denote firm  $i$ 's demand by type  $j$  consumers.

$$\begin{aligned}
d_{2a} &= (1 - \alpha)(1 - \lambda_1)L + (1 - \alpha)(1 - \lambda_2)l_a \\
&= (1 - \alpha)(1 - \lambda_1)L + (1 - \alpha)(1 - \lambda_2)(p_1 - p_2),
\end{aligned}$$

and profits of

$$\begin{aligned}
\pi_{1a} &= p_1 d_{1a} = p_1(1 - \alpha)(1 - \lambda_2)(L - p_1 + p_2), \\
\pi_{2a} &= p_2 d_{2a} = p_2(1 - \alpha)(1 - \lambda_1)L + p_2(1 - \alpha)(1 - \lambda_2)(p_1 - p_2).
\end{aligned}$$

## (2) Demand and profit from type (b) consumers

Next, we move on to type (b) consumers – non-traders with coupon – who are also depicted in Figure 1. Their densities are  $(1 - \alpha)\lambda_1$  in  $[-L, 0]$  and  $(1 - \alpha)\lambda_2$  in  $[0, L]$  respectively. There will be two marginal consumers.  $l_{b1} \in [-L, 0]$ , the left marginal consumer, is indifferent between buying from firm 1 with a coupon (thus paying  $p_1 - d_1$ ) and buying from firm 2 without a coupon (thus paying  $p_2$ ).<sup>16</sup> Similarly, the right marginal consumer located at  $l_{b2} \in [0, L]$  is indifferent between buying from firm 1 without a coupon (thus paying  $p_1$ ) and buying from firm 2 with a coupon (thus paying  $p_2 - d_2$ ). The exact locations of these two marginal consumers are given by

$$l_{b1} = (p_1 - d_1) - p_2, \quad l_{b2} = p_1 - (p_2 - d_2).$$

We make the assumptions that  $l_{b1} \in [-L, 0]$  and  $l_{b2} \in [0, L]$ , which obviously hold in a symmetric equilibrium ( $p_1 = p_2$ ). Consumers located in the interval  $[-L, l_{b1}]$  receive coupons from firm 1, but the coupon face value is not enough to compensate for their strong preferences for firm 2's product. They will not sell their coupons either, since they are non-traders. As a result, they will buy from firm 2 and pay  $p_2$ . However, for consumers located in  $(l_{b1}, 0]$ , they only have a weak preference for firm 2's product. With firm 1's coupons, they will choose to buy from firm 1 and pay  $p_1 - d_1$ . Similarly, consumers located in  $[0, l_{b2})$  will buy from firm 2 with coupons and pay  $p_2 - d_2$ , and consumers in  $[l_{b2}, L]$  will buy from firm 1 without coupons and pay  $p_1$ . Consequently, firms' profits are

$$\begin{aligned}
\pi_{1b} &= p_1(1 - \alpha)\lambda_2(L - l_{b2}) + (p_1 - d_1)(1 - \alpha)\lambda_1(0 - l_{b1}) \\
&= p_1(1 - \alpha)\lambda_2(L - p_2 + p_2 - d_2) + (p_1 - d_1)(1 - \alpha)\lambda_1(p_2 + d_1 - p_1),
\end{aligned}$$

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<sup>16</sup>We assume that firms do not match each other's coupons. If they do, firms would have no incentive to send poaching coupons unless some consumers do not request their loyal firm to honor the coupons they receive from the other firm. Similarly, if firms match each other's final sale price, then firms have no incentive to send poaching coupons either.

$$\begin{aligned}
\pi_{2b} &= p_2(1 - \alpha)\lambda_1(l_{b1} + L) + (p_2 - d_2)(1 - \alpha)\lambda_2(l_{b2} - 0) \\
&= p_2(1 - \alpha)\lambda_1(L + p_1 - d_1 - p_2) + (p_2 - d_2)(1 - \alpha)\lambda_2(p_1 - p_2 + d_2).
\end{aligned}$$

### (3) Demand and profit from type (c) consumers

The third type of consumers is traders, with or without coupon. Their density is  $\alpha$  in  $[-L, L]$ . If coupons are auctioned, they are expected to go to the bidders who value them the most. So we make the assumption that the outcomes of coupon trading are efficient. For example, firm 1's coupons sent to traders in  $[-L, 0]$  will be bought by coupon traders who are firm 1's most loyal customers, i.e., those close to  $L$ . These consumers will buy from firm 1 even without coupons. Consequently, they all value coupons at their face value ( $d_1$ ), which is the maximum value of coupons. We are not concerned with the exact transaction prices of coupons, i.e., how the surpluses will be divided between coupon buyers and sellers. Obviously, all traders who receive coupons (from their less preferred firm) will choose to sell them to the promoting firm's loyal customers. The intended objective of coupons is to poach a rival firm's loyal customers, but since these poached customers value coupons less than the promoting firm's loyal customers do, these coupons will end up in the hands of the promoting firm's loyal customers. Therefore, when coupons reach coupon traders, they lead to a pure loss of coupon face value and distribution cost for the firms. Intuitively, when  $\alpha$  is too large, no firm will choose to promote. We assume throughout the paper that  $\alpha < \frac{1}{2}$ , i.e., there are fewer coupon traders than non-traders.<sup>17</sup>  $\alpha\lambda_1L$  coupons are distributed to traders in  $[-L, 0]$  and  $\alpha\lambda_2(L)$  coupons are distributed to traders in  $[0, L]$ . After coupon trading takes place, all these coupons will be used by consumers who would not change their purchasing decisions even without the coupons. So, after the coupon distribution and trading, there is a marginal consumer  $l_c$  near the middle who does not have a coupon and is indifferent between both products at their regular prices,<sup>18</sup>

$$l_c = p_1 - p_2.$$

To the left of  $l_c$ , all consumers buy from firm 2. Those close to  $-L$  will use firm 2's

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<sup>17</sup>Our results show that when  $\alpha = \frac{1}{2}$ , firms stop sending coupons ( $\lambda_i = d_i = 0$ ,  $i = 1, 2$ ).

<sup>18</sup>This requires that there is more demand than supply for each firm's coupons, and the consumers in the neighborhood of  $l_c$  will not have coupons. For firm 1's coupons, we need  $\alpha\lambda_1L \leq \alpha[L - (p_1 - p_2)]$ . For firm 2's coupons, we need  $\alpha\lambda_2L \leq \alpha[L + (p_1 - p_2)]$ . In a symmetric equilibrium ( $p_1 = p_2$ ), clearly both constraints are satisfied since  $\lambda_i \leq 1$ ,  $i = 1, 2$ . Thus all coupons reaching traders will be traded back to the loyal customers and used.

coupon whether they receive it or buy it. Those to the right of  $l_c$  all buy from firm 1, with consumers close to  $L$  using firm 1's coupon. Consumers in the neighborhood of  $l_c$  will not have coupons to use, since there is more demand than supply for coupons.

Firms' profits from the traders are,

$$\begin{aligned}\pi_{1c} &= p_1\alpha(L - l_c) - d_1\alpha\lambda_1L \\ &= p_1\alpha(L - p_1 + p_2) - d_1\alpha\lambda_1L, \\ \pi_{2c} &= p_2\alpha(l_c + L) - d_2\alpha\lambda_2L \\ &= p_2\alpha(L + p_1 - p_2) - d_2\alpha\lambda_2L.\end{aligned}$$

#### (4) Firms' overall demand and profits

Now, aggregating firms' profits over all three types of consumers, and subtracting the cost of distributing coupons, we have

$$\begin{aligned}\pi_1 &= p_1(1 - \alpha)(1 - \lambda_2)(L - p_1 + p_2) + p_1(1 - \alpha)\lambda_2(L - p_2 + p_2 - d_2) \\ &\quad + (p_1 - d_1)(1 - \alpha)\lambda_1(p_2 + d_1 - p_1) + p_1\alpha(L - p_1 + p_2) - d_1\alpha\lambda_1L - k(\lambda_1L)^2\end{aligned}\quad (1)$$

$$\begin{aligned}\pi_2 &= p_2(1 - \alpha)(1 - \lambda_1)L + p_2(1 - \alpha)(1 - \lambda_2)(p_1 - p_2) - p_2(1 - \alpha)\lambda_1(L + p_1 - d_1 - p_2) \\ &\quad + (p_2 - d_2)(1 - \alpha)\lambda_2(p_1 - p_2 + d_2) + p_2\alpha(L + p_1 - p_2) - d_2\alpha\lambda_2L - k(\lambda_2L)^2\end{aligned}\quad (2)$$

Firm  $i$ 's problem is

$$\max_{p_i, d_i, \lambda_i} \pi_i(p_i, d_i, \lambda_i, p_{-i}, d_{-i}, \lambda_{-i}), i = 1, 2.$$

The next proposition summarizes the solution to this problem.

**Proposition 1.** *In the symmetric pure strategy equilibrium,*

1. *Each firm will choose*

(i) *a regular price of*

$$p^* = \frac{\left(\frac{2}{3}A - \frac{3}{2}\frac{-\frac{4}{9}\alpha^2 + \frac{16}{3}k}{A} - \frac{1}{3}\alpha\right)L}{-1 + \alpha},\quad (3)$$

where

$$A = \left(\alpha^3 + 36k\alpha - 27k + 3\sqrt{12\alpha^4k + 96\alpha^2k^2 + 192k^3 - 6k\alpha^3 - 216k^2\alpha + 81k^2}\right)^{\frac{1}{3}}.$$

(ii) a promotion depth of

$$d^* = \frac{1}{2} \left( p^* - \frac{\alpha L}{1 - \alpha} \right). \quad (4)$$

(iii) a promotion frequency of

$$\lambda^* = \frac{(\alpha L - p^* + \alpha p^*)^2}{8(1 - \alpha)kL^2}. \quad (5)$$

2. The corresponding equilibrium profit of each firm is

$$\pi^* = \frac{\alpha^4 L^4 + 64(1 - \alpha)^2 k L^3 p^* - 6(1 - \alpha)^2 \alpha^2 L^2 (p^*)^2 + 8(1 - \alpha)^3 \alpha L (p^*)^3 - 3(1 - \alpha)^4 (p^*)^4}{64(1 - \alpha)^2 k L^2}. \quad (6)$$

3. When  $k$  is sufficiently small, our  $\lambda^*$  formula leads to  $\lambda^* > 1$ , which should be replaced by  $\lambda^* = 1$ . The corresponding equilibrium is characterized by,

$$\lambda^* = 1, \quad p^* = \frac{2 - \alpha}{3 - \alpha} L, \quad d^* = \frac{1}{2} \left( p^* - \frac{\alpha L}{1 - \alpha} \right),$$

$$\pi^* = \frac{(k\alpha^3 - \alpha^3 - 7k\alpha^2 + 5\alpha^2 + 15k\alpha - 8\alpha + 5 - 9k)L^2}{(1 - \alpha)(3 - \alpha)^2}.$$

**Proof.** See Appendix. ■

#### A numerical example

From equations (3), (10), (11) and (6), we can see that  $p^*$  and  $d^*$  are linear in  $L$ ,  $\pi^*$  is quadratic in  $L$ , and  $\lambda^*$  is independent of  $L$ . Thus we normalize  $L = 1$ . When  $k$  is very small, using our  $\lambda^*$  expression, we would get  $\lambda^* > 1$ , which should be replaced by  $\lambda^* = 1$ . So we need  $k$  to be not too small to guarantee  $\lambda^* \leq 1$ .<sup>19</sup> In particular, we choose  $k = 1/2$ . We further set  $\alpha = 1/5$ , i.e., 20% of the consumers are traders. Then the equilibrium is

$$p^* = 0.9525, \quad d^* = 0.3512, \quad \lambda^* = 0.0987, \quad \pi^* = 0.9309.$$

Recall that the coupon distribution cost is  $k(\lambda L)^2$ , which is about 0.005 here, or about 0.5% of the regular price.

Prisoners' dilemma: The model without coupons is essentially a standard Hotelling model (with the measure of consumers being 2). It can be easily verified that the equilibrium price is  $p = 1$ . Each firm takes half of the market and enjoys a profit of  $\pi = 1$ .

<sup>19</sup>The exact cutoff value of  $k$  depends on  $\alpha$ . When  $\alpha = 0$ , we find that  $\lambda^* < 1$  when  $k > 0.056$ .

Sending coupons to consumers first reduces firms' regular prices (seeing now that  $p^* < 1$ ). This is because, when a firm's loyal customers are poached by the rival firm, it responds by lowering its regular price to try to retain these loyal customers. Lower regular prices lead to lower profits. The discounts which some consumers get by using coupons and the coupon distribution cost will lower firms' profits even further ( $\pi^* < p^*$ ).

## 4.1 Comparative statics

Proposition 1 provides the expressions of the equilibrium price and promotion variables ( $p, d, \lambda$  and  $\pi$ ). If we normalize  $L = 1$ , these variables are only functions of  $k$  and  $\alpha$ . Therefore, we can analyze how they vary when we change either  $\alpha$  or  $k$ , one at a time. The expressions for the relevant partial derivatives are very lengthy for reporting.<sup>20</sup> We tried various parameter values of  $\alpha$  and  $k$ , and found that the qualitative comparative statics results do not depend on the choice of values. Below, we assign parameter values and report the results in graphs.

### Fix $k$ and vary $\alpha$

As long as  $k$  is sufficiently large so that  $\lambda \leq 1$  in our formula, our results are robust to the choice of  $k$ . Results for  $k = \frac{1}{2}$  are reported in Figure 2.

From the figure we can see that, when the fraction of coupon traders ( $\alpha$ ) increases, firms promote less frequently ( $\lambda \downarrow$ ) and with lower promotion depth ( $d \downarrow$ ). They set higher prices and their profits increase. The intuition is as follows. Optimal promotion effort balances the following:

$$\begin{aligned} \text{benefit of couponing} &= (1 - \alpha)\lambda d(p - d) \\ \text{loss of couponing} &= \alpha\lambda Ld \\ \text{coupon distribution cost} &= k(\lambda L)^2 \end{aligned}$$

A firm reaps benefit when its coupons reaches non-traders, and the benefit is given by  $(1 - \alpha)\lambda d(p - d)$ .  $(1 - \alpha)\lambda d$  measures the extra consumers the firm can attract, at the discounted price of  $(p - d)$ . However, a loss is realized when the coupons reach traders in the form of  $\alpha\lambda Ld$ .  $\alpha\lambda L$  is the proportion of consumers affected, and  $d$  is the loss of

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<sup>20</sup>The Maple file which contains all the expressions is available upon request.

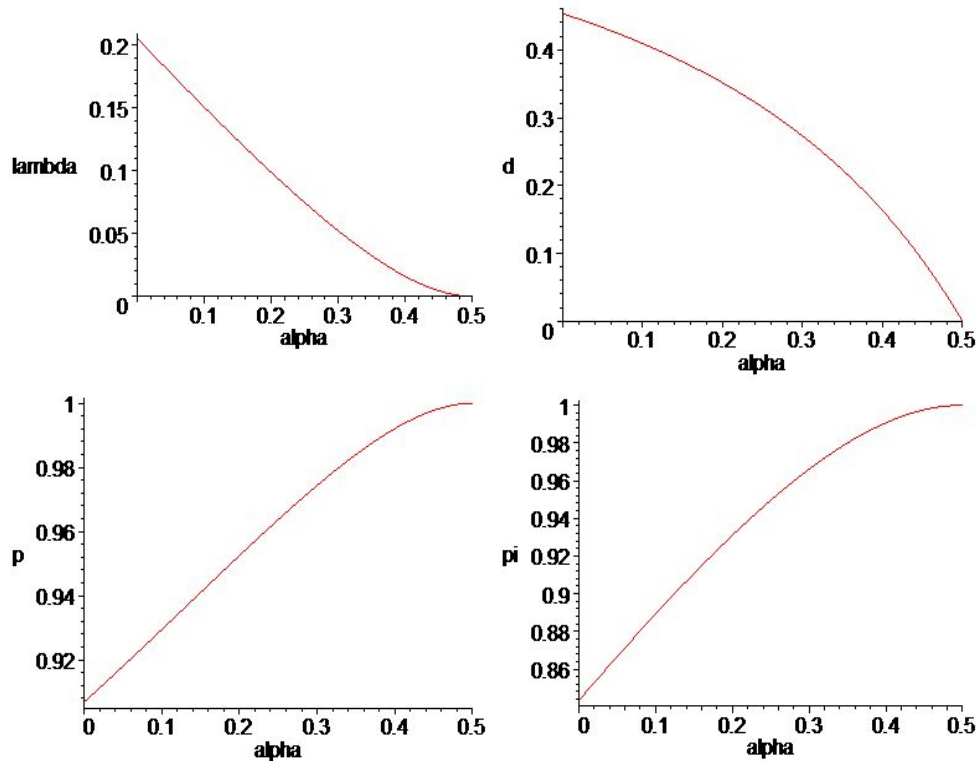


Figure 2: Comparative statics when  $\alpha$  varies.

revenue for each of these consumers. There is also a cost of distributing coupons in the form of  $k(\lambda L)^2$ . An increase in  $\alpha$  lowers the benefit and increases the loss. To re-balance the benefit, loss, and distribution cost ( $\lambda$  affects all three terms),  $\lambda$  needs to go down. This is because, the benefit and loss are linear in  $\lambda$ , while the cost of distributing coupons is quadratic in  $\lambda$ .

Now let's see why an increase in  $\alpha$  also puts downward pressure on coupon face value  $d$ . When  $\alpha$  increases, the benefit decreases and the loss increases. To re-balance the benefit and loss ( $d$  affects only the benefit and loss directly),  $d$  needs to decrease. While  $d$  does not enter into the distribution cost term, there is an indirect effect on promotion depth, which is the tradeoff between promotion frequency and depth. That is, a firm can poach more of a rival's customers by either sending more coupons with the same face values or sending the same number of coupons but with larger face values. This indirect effect implies that, when a firm reduces its promotion frequency, it increases its promotion depth. Our result suggests that, this indirect tradeoff effect is dominated by the direct effect of downward pressure on promotion depth. With fewer poaching coupons of less value there is less competition; thus price and profits go up. Obviously, consumers become worse off.

In a model with covered market and inelastic demand like ours, welfare analysis is not very informative. Nevertheless, we would like to point out an effect which coupon trading has on efficiency. Customer poaching leads to inefficient brand switching (consumers buy products they like less). If we fix firms' promotion intensities, allowing coupons to be traded implies less brand switching, thus improves efficiency. On top of this, coupon trading also reduces firms' promotion intensities, which leads to even less brand switching.

Fix  $\alpha$  and vary  $k$

We tried various values of  $\alpha \in [0, 1/2)$ , and the results do not change qualitatively. Results when  $\alpha = 0.2$  are plotted in Figure 3. These results are similar to those in Bester and Petrakis. When  $k$  is small, our analytical solution leads to  $\lambda > 1$ , and should be replaced by 1. To avoid this situation, we only pick  $k$  values that are not too small ( $k > 0.05$ ).

From the figure, we can see that when  $k$  increases, firms respond by promoting less frequently ( $\lambda \downarrow$ ) but with higher promotion depth ( $d \uparrow$ ). Prices (even net of coupon face value) and profits go up. These results are quite similar to the results when we fix  $k$  and vary  $\alpha$ , and so is the intuition. Both coupon trading ( $\alpha$ ) and distribution cost ( $k$ ) work against sending coupons, and firms have fewer incentives to promote. However, the implications on promotion depth are different. When firms promote less frequently due to larger cost of distributing coupons, they respond by increasing the promotion depth (tradeoff effect). This is because, unlike  $\alpha$ ,  $k$  does not directly affect the benefit and loss of promotion, but only indirectly through  $\lambda$  and  $d$ . Thus, when  $k$  increases, only the indirect tradeoff effect (higher promotion depth to go with lower promotion frequency) exists. An increase in  $k$  does not apply a direct downward pressure on  $d$  as an increase in  $\alpha$  does. This implies that promotion depth increases with  $k$ . Since sending coupons constitutes a prisoners' dilemma game, less promotion reduces competition intensity, which leads to higher prices (including prices net of coupons). There are two opposite effects governing the effects of an increase in  $k$  on profits. First, the cost of distributing coupons increases, affecting profits negatively. Second, when  $k$  increases, competition is less intense which will improve profits. Our results show that the second effect dominates the first one.

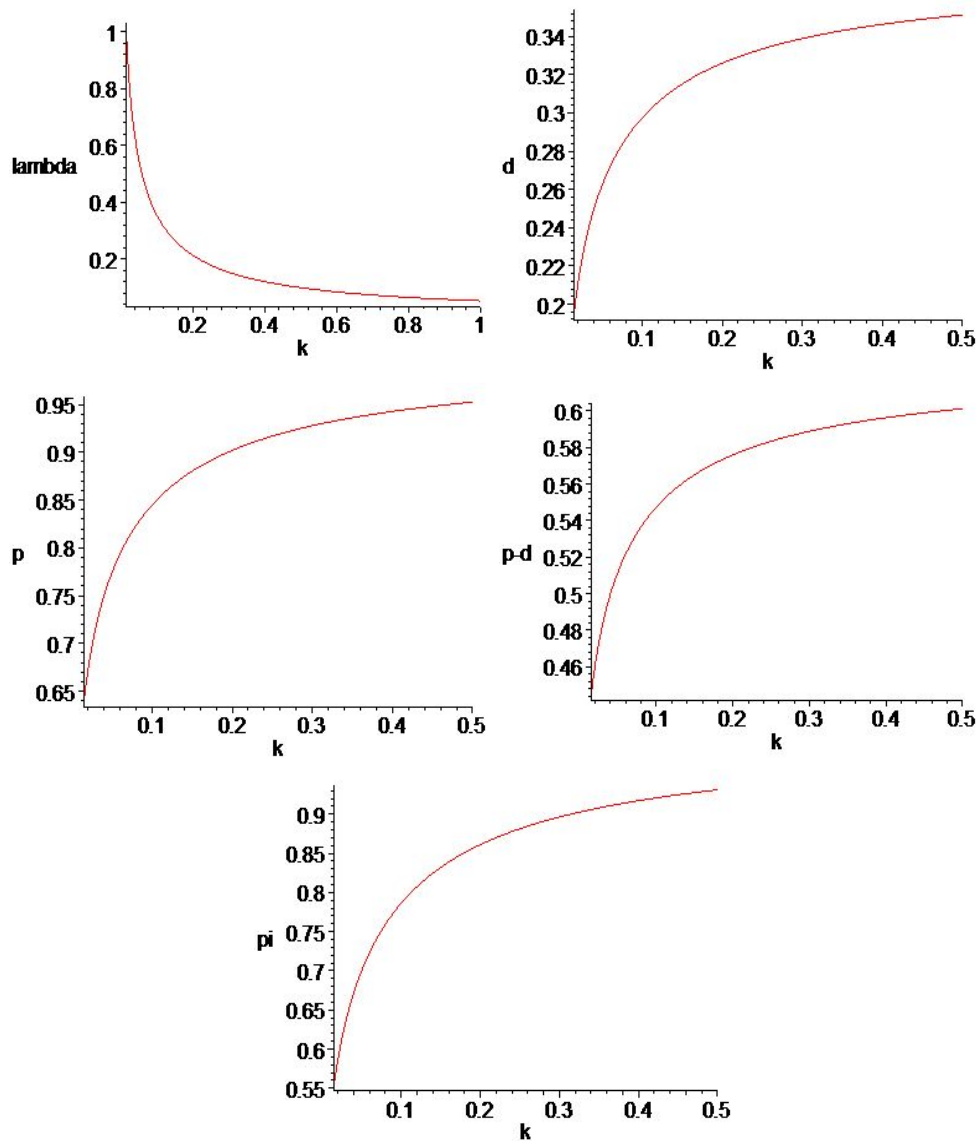


Figure 3: Comparative statics when  $k$  varies.

## 4.2 Defensive couponing

So far we have only considered offensive couponing. That is, firms send coupons to poach rivals' loyal customers, inducing them to switch. However, firms can also distribute defensive coupons to their own loyal customers, to prevent them from switching due to rival's offensive poaching coupons. In this section, we introduce defensive coupons, and we allow a firm to distribute defensive coupons alone or both defensive and offensive coupons. We find that this possibility does not alter our results, since no firm will distribute defensive

coupons. The results are presented in two lemmas. In Lemma 1, we show that the equilibrium in Proposition 1 remains an equilibrium even when we allow defensive coupons, since no firm has incentive to unilaterally deviate and distribute defensive coupons. In Lemma 2, we show that there exists no symmetric pure strategy equilibrium where both firms distribute defensive coupons.

**Lemma 1** *The symmetric pure strategy equilibrium in Proposition 1, where firms distribute offensive coupons only, remains to be an equilibrium when firms can distribute defensive coupons.*

**Proof.** See Appendix. ■

While Lemma 1 shows that our previous equilibrium remains an equilibrium when firms can distribute defensive coupons, it does not rule out the possibility that firms distributing both offensive and defensive coupons may also be an equilibrium. That is, there are multiple equilibria. The next Lemma deals with this issue.

**Lemma 2** *There exists no symmetric pure strategy where firms distribute defensive coupons, with or without offensive coupons.*

**Proof.** See Appendix. ■

## 5 Extensions

To check the robustness of our results, we extend our model in the following directions.

### 5.1 Introducing coupon non-users

In our model, we have assumed that all consumers are coupon users. This is unrealistic and the sole purpose of this is to simplify the analysis. Nevertheless, we show that our results do not change qualitatively if we introduce consumers who do not use coupons. In particular, assume that there is a fraction,  $1 - \gamma$ , of consumers who do not use coupons. They are also uniformly distributed on the interval  $[-L, L]$ , but they are allowed to have different price sensitivity than the coupon users do. Specifically, we assume that a coupon

non-user located at  $l$  is indifferent between buying from either firm if and only if  $l = \frac{p_1 - p_2}{t}$ .<sup>21</sup> The remaining  $\gamma$  fraction of consumers are the same as in our model. We then analyze two setups, depending on whether the firms can distribute mass media coupons, in addition to the poaching coupons or not. Specifically, they can in the second setup but not in the first one. We find that in both setups, our comparative statics results stay qualitatively the same as those in the main model.<sup>22</sup>

In the first one, firms cannot distribute mass media coupons. Following similar analysis, we look for a symmetric equilibrium ( $p_2 = p_1$ ,  $d_2 = d_1$ ,  $\lambda_2 = \lambda_1$ ), and solve for  $d_1$  and  $\lambda_1$  as functions of  $p_1$  first. The  $d_1(p_1)$  expression here is the same as the  $d_1(p_1)$  when there are no coupon non-users (see equation (10) in proof of Proposition 1).  $\lambda_1(p_1)$  here is  $\gamma$  times the  $\lambda_1(p_1)$  when there are no coupon non-users (see equation (11)). With  $d_1(p_1)$  and  $\lambda_1(p_1)$ , we can then solve for equilibrium price. Substitute the equilibrium price into  $d_1(p_1)$  and  $\lambda_1(p_1)$ , we obtain promotion depth and promotion frequency. We can then study how the equilibrium price, promotion intensity and profit change with respect to  $\alpha$  or  $k$ . The comparative statics results are the same as those in Section 4.1.

With the introduction of coupon non-users, especially when they are less price sensitive than the coupon users are, it is natural to consider not just poaching coupons, but mass media coupons as well. In the second setup, following Shaffer and Zhang (1995), we assume that firms can distribute mass media coupons to all consumers costlessly. We further assume that mass media coupons and poaching coupons can be combined. In the end, coupon non-users will not use any coupon. Coupon users will always use the mass media coupons. Usage of poaching coupons is similar to what we have analyzed in the main model. Once again, we find that the comparative statics results are qualitatively the same.

## 5.2 Non-tradable coupons

We assumed that coupons are tradable in our model. But why would firms allow their coupons to be traded?<sup>23</sup> With online coupons, firms can certainly tie coupon codes to the consumers they are targeting, and refuse to honor the coupons when they are used by others. To analyze the issue of non-tradable coupons, we introduce another stage to our

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<sup>21</sup>If we want coupon non-users to be less price sensitive, then we need  $t \geq 1$ . Recall that for coupon users the marginal consumer is  $l = p_1 - p_2$ .

<sup>22</sup>Details and Maple files for these two setups are available upon request.

<sup>23</sup>We thank Patrick Greenlee for bringing this to our attention.

three-stage game. In particular, in stage 0, firms first simultaneously and independently decide whether they want their coupons to be tradable. Whether coupons are tradable or not does not affect the cost of promotion. Once decisions of coupon types are made, the rest of the game proceeds in the same fashion as in Section 3. There are a total of four subgames after the coupon type decisions are made, depending on whether the coupons are tradable or not. In the first subgame  $(T, T)$ , both firms' coupons are tradable. This is as in our main model. In the second subgame, neither firm's coupons are tradable  $(NT, NT)$ . This is similar to Bester and Petrakis or  $\alpha = 0$  in our main model. Subgame 3  $(T, NT)$  and 4  $(NT, T)$  are symmetric to each other, where one firm's coupons are tradable but not the other firm's. We find that in general, firms want to mimic each other's decisions on coupon types (I want to make my coupons tradable if you make yours tradable).<sup>24</sup> Both subgame 1 and 2 can be supported in subgame perfect Nash equilibria. Equilibrium profits are higher in subgame 1, justifying our use of tradable coupons.<sup>25</sup>

## 6 Conclusion

There is a large literature on price discrimination, which has typically maintained the assumption that consumer arbitrage is not feasible. This assumption is increasingly violated when price discrimination is achieved through couponing, and coupons are increasingly traded online. We relax the no-arbitrage assumption by allowing coupons to be traded to and used by consumers not initially targeted by the firm. In particular, we assume that a fraction of consumers are coupon traders who can buy/sell coupons without incurring any cost. We then analyze the impact of coupon trading on firms' decisions to promote (in terms of frequency and depth of coupon promotions), the equilibrium prices and profits. We find that when the fraction of coupon traders increases, firms respond by promoting less frequently (sending fewer coupons out) and reducing the face value of coupons. This reduces competition and leads to higher equilibrium prices and profits. When the cost of distributing coupons increases, the results on promotion frequency, prices and profits are similar to the results when the fraction of coupon traders increases. This is because both coupon trading and distribution costs work against coupon promotions, reducing a

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<sup>24</sup>An exception is that when both  $\alpha$  and  $k$  are small, we have asymmetric equilibria where only one firm chooses to issue tradable coupons.

<sup>25</sup>The  $(NT, NT)$  subgame is the same as the  $(T, T)$  subgame but with  $\alpha = 0$ . From our comparative statics results, we have shown that equilibrium profits increases with  $\alpha$ . Thus  $(T, T)$  leads to higher profits for firms than  $(NT, NT)$  does.

firm's incentives to promote. The only difference is that while coupon face value decreases with the fraction of coupon traders, it increases with coupon distribution cost. In both cases, consumers are all worse off since prices increase. Our results are robust to several extensions including the introduction of coupon non-users and non-tradable coupons.

# Appendix

## Proof of Proposition 1.

From equations (1) and (2), firms' profits are,

$$\begin{aligned}\pi_1 &= p_1(1-\alpha)(1-\lambda_2)(L-p_1+p_2) + p_1(1-\alpha)\lambda_2(L-p_2+p_2-d_2) \\ &+ (p_1-d_1)(1-\alpha)\lambda_1(p_2+d_1-p_1) + p_1\alpha(L-p_1+p_2) - d_1\alpha\lambda_1L - k(\lambda_1L)^2\end{aligned}$$

$$\begin{aligned}\pi_2 &= p_2(1-\alpha)(1-\lambda_1)L + p_2(1-\alpha)(1-\lambda_2)(p_1-p_2) - p_2(1-\alpha)\lambda_1(L+p_1-d_1-p_2) \\ &+ (p_2-d_2)(1-\alpha)\lambda_2(p_1-p_2+d_2) + p_2\alpha(L+p_1-p_2) - d_2\alpha\lambda_2L - k(\lambda_2L)^2\end{aligned}$$

We can use the FOCs for both firms, or use FOC for firm  $A$  then impose symmetry conditions. Both lead to the same solutions. We will report the latter method here.

Taking derivative of  $\pi_1$  with respect to  $p_1$ ,  $d_1$  and  $\lambda_1$  respectively, then imposing the symmetry conditions ( $p_2 = p_1$ ,  $d_2 = d_1$  and  $\lambda_2 = \lambda_1$ ), we can obtain

$$\frac{\partial\pi_1}{\partial d_1} = -\lambda_1(\alpha L - p_1 - 2d_1\alpha + 2d_1 + \alpha p_1) = 0. \quad (7)$$

$$\frac{\partial\pi_1}{\partial\lambda_1} = -2k\lambda_1L^2 - d_1\alpha L - \alpha p_1d_1 + d_1^2\alpha + p_1d_1 - d_1^2 = 0. \quad (8)$$

$$\frac{\partial\pi_1}{\partial p_1} = L - p_1 - \lambda_1d_1\alpha + \lambda_1\alpha p_1 + d_1\lambda_1 - \lambda_1p_1 = 0. \quad (9)$$

Since the cost of coupon distribution is quadratic in  $\lambda$ , and the rest is roughly linear in  $\lambda$ , it is easy to see that the optimal  $\lambda_i > 0$ . Then, equation (7) implies,

$$d_1 = \frac{\alpha L - p_1 + \alpha p_1}{2(\alpha - 1)} = \frac{1}{2} \left( p_1 - \frac{\alpha L}{1 - \alpha} \right). \quad (10)$$

From this expression, obviously  $d_1 < \frac{p_1}{2}$ , thus  $p_1 > d_1$ .

Next, we substitute the expression of  $d_1$  into equation (8) and solve for  $\lambda_1$  to obtain,

$$\lambda_1 = \frac{(\alpha L - p_1 + \alpha p_1)^2}{8(1-\alpha)kL^2}. \quad (11)$$

Use  $d_1$  and  $\lambda_1$  in equation (9), we can solve for the equilibrium price,<sup>26</sup>

$$p_1 = \frac{\left( \frac{2}{3}A - \frac{3}{2} \frac{-\frac{4}{9}\alpha^2 + \frac{16}{3}k}{A} - \frac{1}{3}\alpha \right) L}{-1 + \alpha},$$

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<sup>26</sup>There are 3 solutions. We pick the one that is real and positive.

where

$$A = \left( \alpha^3 + 36k\alpha - 27k + 3\sqrt{12\alpha^4k + 96\alpha^2k^2 + 192k^3 - 6k\alpha^3 - 216k^2\alpha + 81k^2} \right)^{\frac{1}{3}}.$$

We can substitute this expression of  $p_1$  back into the expressions of  $d_1$  and  $\lambda_1$ . The final expressions are too lengthy to report. Since the solutions are symmetric across firms, and  $p_i > d_i$ , all our assumptions about the locations of marginal consumers in each of the three consumer groups are satisfied.

Note that, when  $k$  is sufficiently small, our  $\lambda_i$  formula leads to  $\lambda_i > 1$ . In this case, we need to replace it by  $\lambda_i = 1$ . Substituting it into equations (7) and (9), we can solve for the rest of the equilibrium variables. It can be shown that

$$p_i = \frac{2 - \alpha}{3 - \alpha}L, \quad d_i = \frac{1}{2} \left( p_i - \frac{\alpha L}{1 - \alpha} \right),$$

$$\pi_i = \frac{(k\alpha^3 - \alpha^3 - 7k\alpha^2 + 5\alpha^2 + 15k\alpha - 8\alpha + 5 - 9k)L^2}{(1 - \alpha)(3 - \alpha)^2}.$$

So far, we have used first order conditions to solve for the optimal choices of prices and promotion intensities. However, first order conditions are necessary but not sufficient. We need to make sure that the solutions we obtained indeed constitute an equilibrium. Instead of checking whether the Hessian matrix is negative semidefinite (which is quite messy), we show that this is an equilibrium by verifying that neither firm has an incentive to unilaterally deviate from this pair of strategies (Bester and Petrakis use similar method). Without loss of generality, we fix firm 2's price and promotion strategies at  $p^*$ ,  $d^*$  and  $\lambda^*$  as given in Proposition 1, and allow firm 1 to deviate from these strategies. Firm 1's deviation profit is given by equation (1), with  $p_2 = p^*$ ,  $d_2 = d^*$  and  $\lambda_2 = \lambda^*$ . We normalize  $L = 1$ , and optimal choice requires that

$$\frac{\partial \pi_1^{dev}}{\partial d_1} = \frac{\partial \pi_1^{dev}}{\partial \lambda_1} = 0.$$

Solving the first order conditions, we obtain

$$d_1^* = \frac{2(1 - \alpha)p_1 - (1 - \alpha)p^* - \alpha}{2(1 - \alpha)},$$

$$\lambda_1^* = \frac{\alpha^2 + \alpha^2(p^*)^2 + 4\alpha^2p_1 - 2p^*\alpha^2 + 2\alpha p^* - 4\alpha p_1 - 2\alpha(p^*)^2 + (p^*)^2}{(1 - \alpha)k}.$$

The first order conditions are necessary and sufficient (of course  $\lambda_1^* \leq 1$  has to hold). We leave the  $p^*$  term in the expression, as clearly the expressions will be too lengthy to

report if we substitute the value of  $p^*$ . Now firm 1's deviation profit depends only on  $p_1$ ,  $\alpha$  and  $k$ . We want to check whether firm 1 can increase its profit by deviating the value of  $p_1$  from  $p^*$ , i.e., to have

$$\pi_1^{dev}(p_1) > \pi^*, \quad \forall \alpha, k.$$

We tried various combinations of  $\alpha$  and  $k$ , and we found that firm 1 can never increase its profit by choosing a price different than  $p^*$ . Therefore, the solutions to the FOCs which we obtained before constitute an equilibrium. ■

### **Proof of Lemma 1.**

Without loss of generality, assume that firm 1 is the deviating firm. We fix firm 2's behavior as described in Proposition 1. Let region 1 denote the interval  $[-L, 0]$ , and region 2 the interval  $[0, L]$ . Let  $(\lambda_{11}, d_{11})$  denote firm 1's couponing intensity in region 1, i.e., firm 1's offensive couponing intensity. Similarly, let  $(\lambda_{12}, d_{12})$  denote firm 1's defensive couponing intensity in region 2. Note that, if firm 1 picks  $\lambda_{11} = 0$  and  $\lambda_{12} > 0$ , then it is sending defensive coupons alone.

There are various cases to consider depending on whether  $p_1 \geq p_2$ , and the size of  $p_1 - d_{12}$ . We will start by assuming that  $p_1 \geq p_2$  and  $p_1 - d_{12} > p_2$ . We divide the proof into three steps. In step 1, we calculate firm 1's deviation profit from each type of consumers, and its overall deviation profit. In step 2, we show that firm 1 has no incentive to deviate. We summarize the results for other cases in step 3.

#### Step 1: Calculate deviation profits

First consider non-traders in  $[-L, 0]$ . Since firm 2 does not distribute defensive coupons, the structure is the same as in the main model. The non-traders may buy from firm 1 only if they receive offensive coupons from firm 1, since  $p_1 \geq p_2$  and they like firm 2's product more. Here we are only concerned with firm 1, in particular, whether it has incentive to deviate. Therefore, we consider only the types of consumers from whom firm 1 earns positive profits.

#### Type (a) Non-traders in $[-L, 0]$ with firm 1's coupons

Firm 1's profit from these consumers is

$$\pi_{1a} = (1 - \alpha)\lambda_{11}(p_1 - d_{11})(0 - (p_1 - d_{11} - p_2))$$

Next, we consider non-traders in  $[0, L]$ . There are four types of consumers, including those who receive (1) both firms' coupons, (2) and (3) either firm's coupon, and (4) no coupon.

Type (b) Non-traders in  $[0, L]$  without coupons

Firm 1's profit from these consumers is,

$$\pi_{1b} = (1 - \alpha)(1 - \lambda_2)(1 - \lambda_{12})p_1(L - (p_1 - p_2)).$$

Type (c) Non-traders in  $[0, L]$  with firm 1's but not firm 2's coupons

$$\pi_{1c} = (1 - \alpha)(1 - \lambda_2)\lambda_{12}(p_1 - d_{12})(L - ((p_1 - d_{12}) - p_2)).$$

Type (d) Non-traders in  $[0, L]$  with firm 2's but not firm 1's coupons

$$\pi_{1d} = (1 - \alpha)\lambda_2(1 - \lambda_{12})p_1(L - (p_1 - (p_2 - d_2))).$$

Type (e) Non-traders with both firms' coupons

$$\pi_{1e} = (1 - \alpha)\lambda_2\lambda_{12}(p_1 - d_{12})(L - ((p_1 - d_{12}) - (p_2 - d_2))).$$

We then consider the traders, with or without coupons. We assume that there is more demand than supply of coupons, so all coupons will be traded to loyal consumers who value them at their face values.

Type (f) Traders with or without coupons

$$\pi_{1f} = \alpha p_1(L - (p_1 - p_2)) - \alpha \lambda_{11} L d_{11} - \alpha \lambda_{12} L d_{12}.$$

Aggregating firm 1's profits from all types of consumers, and subtracting the coupon distribution costs, we can obtain firm 1's overall deviation profit as the following,

$$\pi_1^{dev} = \pi_{1a} + \pi_{1b} + \pi_{1c} + \pi_{1d} + \pi_{1e} + \pi_{1f} - k(\lambda_{11}L)^2 - k(\lambda_{12}L)^2.$$

Step 2: Checking for incentive to deviate

We then solve for the optimal promotion intensities ( $\lambda_{11}$ ,  $\lambda_{12}$ ,  $d_{11}$  and  $d_{12}$ ), which are the solutions to the following first order conditions,

$$\frac{\partial \pi_1^{dev}}{\partial \lambda_{11}} = \frac{\partial \pi_1^{dev}}{\partial \lambda_{12}} = \frac{\partial \pi_1^{dev}}{\partial d_{11}} = \frac{\partial \pi_1^{dev}}{\partial d_{12}} = 0.$$

After this, we obtain  $\pi_1^{dev}$  as a function of  $p_1$ . We then impose the constraint  $p_1 \geq p_2$  and numerically solve for  $p_1$  as a solution (multiple solutions possible) to the first order condition

$$\frac{\partial \pi_1^{dev}}{\partial p_1} = 0.$$

The expressions are lengthy and we have to assign values for  $\alpha$  and  $k$  (after we normalize  $L = 1$ ). We tried various values of  $\alpha$  and  $k$ . Depending on their parameter values, we either (1) get the same results as that in Proposition 1 ( $\lambda_{11} = d_{11} = 0$ ), or (2) have the resulting  $\pi_1^{dev}$  lower than the  $\pi_1$  as given in Proposition 1 or (3) see violations (e.g.  $d_{12} < 0$ ).

This implies that when  $p_1 \geq p_2$ , and  $p_1 - d_{12} > p_2$ , firm 1 has no incentive to unilaterally deviate and distribute defensive coupons.

### Step 3: Other cases

There are various other cases depending on the size of  $p_1 - d_{12}$  and whether  $p_1 \geq p_2$ , including (1)  $p_1 \geq p_2$  and

(1a)  $p_2 - d_2 < p_1 - d_{12} < p_2$ ;

(1b)  $p_1 - d_{12} < p_2 - d_2$ .

(1c)  $p_1 - d_{12} = p_2$  (corner solution)

(1d)  $p_1 - d_{12} = p_2 - d_2$  (corner solution)

(2)  $p_1 < p_2$  with its own various sub-cases.

In all these cases, we get qualitatively the same results as the case above, and we skip the details here.<sup>27</sup> We either (1) get the same results as that in Proposition 1 (implying that  $\lambda_{11} = d_{11} = 0$ ), or (2) have the resulting  $\pi_1^{dev}$  lower than the  $\pi_1$  as given in Proposition 1 or (3) see violations (e.g.  $\lambda_{12} < 0$  or  $d_{12} < 0$ ).■

### **Proof of Lemma 2.**

Now both firms distribute defensive coupons as well as offensive coupons. Let region 1 and 2 denote the intervals  $[-L, 0]$  and  $[0, L]$  respectively. Let  $(\lambda_{ij}, d_{ij})$  denote firm  $i$ 's ( $i = 1, 2$ ) promotion intensity in region  $j$  ( $j = 1, 2$ ). Consumers' choices depend on prices and the face values of various coupons. Without loss of generality, assume that  $p_1 \geq p_2$ . We further restrict attention to the case where  $p_1 - d_{12} > p_2 - d_{22}$  and  $p_2 - d_{21} > p_1 - d_{11}$ . This is intuitive, in the sense that firms charge higher final prices (net of coupons) in

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<sup>27</sup>A maple file containing all the aforementioned cases is available upon request.

their strong markets than their rival firms. Now we know the ranking of final prices when consumers get both firms' coupons. We also know the ranking when consumers receive no coupon, since  $p_1 \geq p_2$  is assumed. However, we do not know the ranking of final prices when consumers receive only one firm's coupons, for example, those who receive firm 1's defensive coupons but not firm 2's offensive coupons. There are various cases, and we start with the following:  $p_2 > p_1 - d_{12}$ . We divide the proof into three steps. In step 1, we calculate both firms' profits from each type of consumers, and aggregate them to obtain their overall profits. In step 2, we show that there is always a violation and firms do not distribute defensive coupons. Results for other cases are summarized in step 3.

Step 1: Calculating deviation profit

We first consider non-traders in the interval  $[-L, 0]$ .

Type (a) With neither firm's coupons

$$\pi_{1a} = 0, \quad \pi_{2a} = (1 - \alpha)(1 - \lambda_{11})(1 - \lambda_{21})p_2L.$$

Type (b) With firm 1's but not firm 2's coupons

Note that  $p_1 - d_{11} < p_2 - d_{21}$  is assumed, then  $p_1 - d_{11} < p_2$  must hold.

$$\pi_{1b} = (1 - \alpha)\lambda_{11}(1 - \lambda_{21})(p_1 - d_{11})(0 - (p_1 - d_{11} - p_2)),$$

$$\pi_{2b} = (1 - \alpha)\lambda_{11}(1 - \lambda_{21})p_2((p_1 - d_{11} - p_2) + L).$$

Type (c) With firm 2's but not firm 1's coupons

$$\pi_{1c} = 0, \quad \pi_{2c} = (1 - \alpha)(1 - \lambda_{11})\lambda_{21}(p_2 - d_{21})L.$$

Type (d) With both firms' coupons

$$\pi_{1d} = (1 - \alpha)\lambda_{11}\lambda_{21}(p_1 - d_{11})(0 - (p_1 - d_{11} - (p_2 - d_{21}))),$$

$$\pi_{2d} = (1 - \alpha)\lambda_{11}\lambda_{21}(p_2 - d_{21})((p_1 - d_{11} - (p_2 - d_{21})) + L).$$

Next we consider traders in the interval  $[0, L]$ .

Type (e) With neither firm's coupons

$$\pi_{1e} = (1 - \alpha)(1 - \lambda_{12})(1 - \lambda_{22})p_1(L - (p_1 - p_2)),$$

$$\pi_{2e} = (1 - \alpha)(1 - \lambda_{12})(1 - \lambda_{22})p_2(p_1 - p_2).$$

Type (f) With firm 1's but not firm 2's coupons

Note that  $p_2 > p_1 - d_{12}$  is assumed.

$$\pi_{1f} = (1 - \alpha)\lambda_{12}(1 - \lambda_{22})(p_1 - d_{12})L, \quad \pi_{2f} = 0.$$

Type (g) With firm 2's but not firm 1's coupons

$$\pi_{1g} = (1 - \alpha)(1 - \lambda_{12})\lambda_{22}p_1(L - (p_1 - (p_2 - d_{22}))),$$

$$\pi_{2g} = (1 - \alpha)(1 - \lambda_{12})\lambda_{22}(p_2 - d_{22})(p_1 - (p_2 - d_{22})).$$

Type (h) With both firms' coupons

$$\pi_{1h} = (1 - \alpha)\lambda_{12}\lambda_{22}(p_1 - d_{12})(L - (p_1 - d_{12} - (p_2 - d_{22}))),$$

$$\pi_{2h} = (1 - \alpha)\lambda_{12}\lambda_{22}(p_2 - d_{22})(p_1 - d_{12} - (p_2 - d_{22})).$$

Finally, we consider all traders in both regions, with or without coupons.

Type (i) Traders with or without coupons

$$\pi_{1i} = \alpha p_1(L - (p_1 - p_2)) - \alpha \lambda_{11}Ld_{11} - \alpha \lambda_{12}Ld_{12}, \quad \pi_{2i} = \alpha p_2((p_1 - p_2) + L) - \alpha \lambda_{21}Ld_{21} - \alpha \lambda_{22}Ld_{22}.$$

Aggregate firms' profits from each type of consumers, and subtract the coupon distribution cost, we can obtain firms' overall profits as the following,

$$\pi_1 = \sum_{i=a}^i \pi_{1i} - k(\lambda_{11}L)^2 - k(\lambda_{12}L)^2,$$

$$\pi_2 = \sum_{i=a}^i \pi_{2i} - k(\lambda_{21}L)^2 - k(\lambda_{22}L)^2.$$

Step 2: First order conditions lead to violations

For each firm, we first calculate the partial derivatives of its profit with respect to its various choice variables.

Next, we solve for  $d_{ij}$  ( $i, j = 1, 2$ ) as the solutions to the following first order conditions,

$$\frac{\partial \pi_i}{\partial d_{ij}} = 0, \quad i, j = 1, 2.$$

Then we solve for  $p_1$  and  $p_2$  from the following equations,

$$\frac{\partial \pi_i}{\partial p_i} = 0, \quad i = 1, 2.$$

If we impose symmetry constraints,

$$\lambda_{21} = \lambda_{12}, \quad \lambda_{22} = \lambda_{11}.$$

then we can obtain  $d_{12}$  as a function of  $\alpha$ ,  $\lambda_{11}$  and  $\lambda_{12}$  only (after normalizing  $L = 1$ ). We tried various values of  $\alpha$  and find that  $d_{12} < 0$ . For example, when  $\alpha = 0$ , we obtain

$$d_{12} = \frac{2(\lambda_{11} - 1)}{\lambda_{11}(\lambda_{11}\lambda_{12} - \lambda_{12} + 2\lambda_{11} + 4)} < 0.$$

When  $d_{12} < 0$ , we should replace it by  $d_{12} = 0$ , which also implies  $\lambda_{12} = 0$ . That is, firm 1 will not distribute defensive coupons. Similarly we can obtain  $d_{21} < 0$  and firm 2 will not distribute defensive coupons either.

### Step 3: Other cases

There are various other cases, including  $p_2 < p_1 - d_{12}$ , and the various subcases when firms charge lower final prices (net of coupon face values) in their strong markets than their rivals. In all these cases, we obtain qualitatively the same results as in the case above and we skip the details.<sup>28</sup> That is, firm 1 will never distribute defensive coupons. By symmetry, firm 2 will not distribute defensive coupons either. ■

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<sup>28</sup>A maple file containing all cases is available upon request.

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