Asset pricing implications in a production economy with regimes*

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Abstract

Unless extreme forms of rigidity are allowed, standard Real Business Cycle models are well known to generate counterfactual asset pricing implications. This paper seeks to circumvent all such rigidities by providing a simple extension to the prior literature where we study an economy that follows a regimes process both in the mean and the volatility, in conjunction with Epstein-Zin preferences for the consumers.

We provide a detailed theoretical and numerical analysis on the model's predictions. We also show that a reasonable parametrization of our "rigidity-free" model conveys plausible financial figures that are in line with empirical observations over the U.S. postwar economy. Furthermore, we provide evidence to support the necessity of modelling the decline of macroeconomic risk in the class of models we are studying.

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1 Introduction

The role of asset prices in macroeconomics gained huge popularity after Mehra and Prescott (1985). Besides the well celebrated equity premium puzzle, the main implication of their seminal contribution is that, in a simple endowment economy where agents have a power utility function, by arbitrarly increasing the risk aversion parameter, one can make an agent sufficently risk averse to offset the observed low volatility of consumption and thus obtain a high risk premium. On the contrary, in a standard RBC model, an agent can smooth her consumption path by substituting between labor and leisure in response to productivity shocks. So, by just increasing her coefficient of risk aversion, we are not able to solve the equity premium puzzle[1].

This is one serious flaw of standard RBC models that enjoy a high capability of explaining the main features of business cycles, but need the introduction of restrictive assumptions to explain observed financial figures. Our paper seeks to overcome this problem by providing a simple extension to the prior RBC literature. We introduce an economy that switches between booms and busts where technological shocks follow a hidden two state Markov chain, in conjunction with recursive preferences for the consumers.

Assuming a different utility specification is not novel in the literature. In fact, alternatives to the power utility specification and their implications in the asset pricing literature have been studied since the late eighties. One

[1] for an exaustive analysis on the role of asset prices in RBC models see Lettau (2003).

particularly prominent line of research examines the asset-pricing implications of including habits in the utility function (e.g. Constantinides, 1990; Abel, 1990; Gali, 1994; Jermann, 1998, Campbell and Cochrane, 1999; and Boldrin et al., 2001.) Another strand of this research, which emanated from Epstein and Zin (1989), Epstein and Zin (1991), and Weil (1989), examines generalizations of the power utility function that unlink risk aversion and the elasticity of intertemporal substitution.

This paper adopts this utility function, referred to as Epstein-Zin preferences. Two reasons drive this choice. First, this form of utility function is widely used in the latest asset-pricing research (see Bansal and Yaron, 2004; Campbell and Viceira, 2001; Campbell et al., 2003; Brandt et al., 2004; Lettau et al., 2006; Guvenen, 2005 among others). Second, since Epstein-Zin preferences nest the power utility function as a special case, these are particurarly useful to provide a closer comparison with the standard models based on power utility specification.

In the RBC literature, the first analysis on asset prices, (Danthine et al., 1992, Rouwenhorst, 1995) while unsuccessful in explaining the behavior of returns over the business cycle, provided useful insights on what would be a necessary ingredient of a successful model. Along this line, to improve the capability in explaining financial figures, the main literature on asset pricing with a non-trivial production side (Jermann, 1998, Boldrin et al., 2001), introduced some form of rigidity in the model. While both Boldrin et al. (2001) and Jermann (1998) specify a habit utility for consumers, the

former contribution relies on a limited mobility of production's factors and the latter introduces capital adjustment costs[2].

The assumption of a recursive but time non-separable felicity function is not novel in this literature. Tallarini (2000), or more recently, Gomes and Michaelides (2008), assume Epstein-Zin utility function, and both document the inability to generate reasonable return's figures without introducing some production frictions in the model.

This paper takes a different approach on the production side of the model and instead of imposing any kind of restriction, provides a simple extension of a standard RBC model where the economy switches between booms and busts. This is accomplished by letting the economy follow a hidden markov chain. Most of the literature studies the implication of a Markov switching process in the conditional mean and in the volatility of the endowment process[3]. Differently, here the regimes are introduced via the production side by allowing the technology shocks to follow a latent two states process both in

[2]A different approach can be found in Cochrane (1991) who evaluates asset pricing implications from the producer's first order conditions.

[3]Regime switching is widely used in economics since the seminal contribution by Hamilton (1989). In particular, in the asset pricing literature, the implications of a Markov switching process in the conditional mean of the endowment process are analyzed by Cecchetti et al. (1990); Kandel and Stambaugh (1991); Cecchetti et al. (1993); Abel (1994); Abel (1999). Recently, the time series properties of the second moments gained popularity in this framework: by setting up an equilibrium economy where the endowment process follows a latent two state regime switching process, Veronesi (1999) shows a better explanatory power of volatility clustering than a model without regimes. In the same setting, Whitelaw (2000) introduces time-varying transition probabilities between regimes, finding a complex nonlinear relation between expected returns and stock market volatility. A recent contribution that studies the impact of regime switches in the volatility of the endowment process is in Lettau et al. (2006).

the mean and the volatility[4].

Our work is closely related with the main literature on asset pricing with a non-trivial production side cited above, and it contributes a novel theoretical framework where a sizeable equity premium can be obtained without imposing any kind of rigidity on the production side of the model and without the need of an implausibly high value for the risk aversion of the agents.

The remainder of the paper is organized as follows: Section 2 introduces the general model, derives equilibrium asset prices, and analyzes the determinants of the equity premium predicted by the model. Section 3 discusses model calibration and provides numerical analysis of asset returns' properties over the business cycle, while section 4 provides a sensitivity analysis of the results' determinants. Section 5 concludes. Proofs, algebraic derivations, and additional results are provided in Appendix A.

2 Model

A standard production economy with two actors is considered. Consumers are modeled via a representative, risk averse, agent which derives utility from consumption, while the production side is modeled through a standard representative firm that maximizes its shareholders' value. There are two securities in the economy: a riskless bond that agents can use for transferring their wealth to the future, and equity, which provides a claim on firm's profits.

[4]In a different setting Cagetti et al. (2002) model the tecnology shocks as a Markov switching model in the first moment.

2.1 Firms and technology

Each period the firm has to decide how much human capital to employ and how much capital to invest in physical assets.

In particular, there is only one traded good which is produced through a constant return to scale technology. Analytically, production can be described using a Cobb-Douglas production function with human and physical capital as factors:

$$Y_t = A_t K_t^{\theta} H_t^{1-\theta}, \tag{1}$$

where θ is the share of physical capital.

The human capital H evolves according to:

$$H_{t+1} = (1 - \delta_H) H_t + E_t, \tag{2}$$

where δ_H is its depreciation rate, and E_t is the investment in education.[5]

The capital stock's K evolution is governed by:

$$K_{t+1} = (1 - \delta_K) K_t + I_t, \tag{3}$$

where δ_K is its depreciation rate, and I_t indicates capital investment.

If we consider the productivity shock (A) in a regime switching model, we can express its law of motion as a process with stochastic parameters

[5] As in Barro and Sala-I-Martin (2004) we can think of human capital as the number of workers multiplied by the human capital of a typical worker.

depending on the state of the economy:

$$\Delta \log A_t = \mu(s_t) + \sigma(s_t)\varepsilon_t, \tag{4}$$

where μ and σ define the mean and the volatility of the process, and s_t indicates the state of the economy. We assume that s_t follows a hidden Markov chain with transition probabilities matrix P (see Hamilton (1989)). The evolution of the state of the economy in terms of state beliefs (ξ_{t+1}) can be expressed as realizations of the equation:

$$\xi_{t+1} = P\xi_t + \epsilon_t. \tag{5}$$

The agents cannot directly observe the state of the economy, s_t, and they have to rely on interpreting external signals. The agents update their belief according to the posterior probabilities computed as

$$\hat{\xi}_{t+1|t} = P\frac{\hat{\xi}_{t|t-1} \odot \zeta_t}{\mathbf{1}'\left(\hat{\xi}_{t|t-1} \odot \zeta_t\right)}, \tag{6}$$

where $\odot$ denotes the Hadamard product, ζ_t is a vector that stacks the conditional densities of the tecnological shocks' growth rates:

$$\zeta_t = \begin{bmatrix} f\left(\Delta \log A_t \mid s_t = 1, \Omega_{t-1}\right) \\ \vdots \\ f\left(\Delta \log A_t \mid s_t = n, \Omega_{t-1}\right) \end{bmatrix} \tag{7}$$

with the density of $\Delta \log A_t$ conditional on state s_t is defined as:

$$f\left(\Delta \log A_t \mid s_t = i, , \Omega_{t-1}\right) = \frac{1}{\sqrt{2\pi}\sigma(s_t)} \exp\left\{-\frac{\left(\Delta \log A_t - \mu(s_t)\right)^2}{2\sigma(s_t)^2}\right\}, \quad (8)$$

where Ω denotes the information set.

2.2 Consumers

The representative agent has preferences defined over current consumption and future utility. Following Epstein and Zin (1989, 1991), the utility function is defined recursively by:

$$U(C_t, \mathbb{E}_t(U_{t+1})) = \left[(1-\beta)C_t^{\frac{1-\gamma}{\alpha}} + \beta(\mathbb{E}_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{1-\gamma}}, \quad (9)$$

where C_t indicates aggregate consumption, β is the time preference parameter, and $\alpha \equiv (1-\gamma)/(1-1/\psi)$, with $\gamma > 0$.

The parameter γ is the coefficient of relative risk aversion (RA), while the elasticity of intertemporal substitution (EIS) is given by ψ [6].

[6] An interesting feature of this utility function specification is that it nests the power utility. In fact, when $\gamma = \frac{1}{\psi}$ equation 9 can be solved forward to get the standard power utility function.

2.3 Equilibrium

To begin, we set up a social planner problem where the resource constraint is defined as:[7]

$$C_t + I_t \leq Y_t. \tag{10}$$

Following the standard literature, we can characterize the equilibrium allocation by solving the social planner problem where the welfare function is defined as:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U_t \right]. \tag{11}$$

The solution can be found by maximizing 11 with respect to 1, 2, 3, 4 and 10.

2.3.1 Asset prices

Next, we derive the equilibrium asset prices implied by the model. We know from Hansen and Richard (1987) that optimality of the consumers' solution implies the existence of a unique pricing kernel (Q) for pricing all the available assets (i.e. the Euler equation holds true for any asset return (R) as $\mathbb{E}_t \left[Q_{t+1} R_{t+1} \right] = 1$).

[7] This implicitely implies that consumption includes education expenditures. That is once the optimal consumption is chosen, the education expenditure is determined by the budget constraint.

Epstein and Zin (1989) show that the stochastic discount factor for the case of utility over consumption is given by

$$Q_{t+1} = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}} (R_{c,t+1})^{\alpha-1}, \tag{12}$$

where $R_{c,t+1}$ is the equilibrium gross return to consumption claim between t and $t+1$.

We examine two different types of assets: a one period asset that yields one unit of consumption (i.e. a "risk-free" asset), and a claim to the physical capital (i.e. an "equity asset"). If firms only use retained earnings to finance their capital the equity holder gets a dividend $D_t = \frac{\partial Y_t}{\partial K_t} K_t - I_t = \theta Y_t - I_t$. This implies that the unlevered stock market of this economy represents the value of the capital stock.

Proposition 2.1. *When the economy meets a "balanced growth condition"* [8]*, then the gross dividends' growth rate can be expressed as:*

$$\frac{D_{t+1}}{D_t} = \frac{A_{t+1}}{A_t} \left[\frac{K_{t+1}}{K_t} \right]^{\theta} \left[\frac{H_{t+1}}{H_t} \right]^{1-\theta} = \frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta}, \tag{13}$$

where $\eta_t = \left(1 - \delta_H + \frac{E_t}{H_t}\right)$ *and* $\lambda_t = \left(1 - \delta_K + \frac{I_t}{K_t}\right)$.

Given proposition 2.1 we can express the price to consumption ratio (PC) of the consumption claim $R_{c,t+1} = \left(\frac{P^c_{t+1} + C_{t+1}}{P^c_t}\right)$ as:

[8] The same result follows if the ratio of investment to output is assumed to be constant between two periods (i.e. $\frac{I_{t+1}}{Y_{t+1}} = \frac{I_t}{Y_t}$).

$$PC_t^{\alpha} = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta} \right)^{1-\gamma} (PC_{t+1} + 1)^{\alpha} \right]. \tag{14}$$

In the same fashion we can express the price to dividend ratio (PD) of the dividend claim $R_{e,t+1} = \left(\frac{P_{t+1}^e + D_{t+1}}{P_t^e} \right)$ as

$$PD_t = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta} \right)^{1-\gamma} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} (PD_{t+1} + 1) \right] \tag{15}$$

Proof. see Appendix A □

Following the approach of Lettau et al. (2006), we solve Equations 14 and 15 numerically. Thus we can get the equity return from:

$$R_{e,t+1} = \frac{1 + PD_{t+1}}{PD_t} \frac{D_{t+1}}{D_t}, \tag{16}$$

and calculate its second moment as

$$\mathbb{E}_t \left[R_{e,t+1}^2 \right] - \mathbb{E}_t \left[R_{e,t+1} \right]^2. \tag{17}$$

Finally, the risk free rate can be expressed as: $R_{f,t+1} = (\mathbb{E}_t [Q_{t+1}])^{-1}$, from which we can calculate both its first and second moment.

2.4 The equity premium

In this section we provide a first grasp on the determinants of the equity premium implied by the model. We do that by log-linearizing the Euler equations for the equity asset and the risk free asset respectively, and solving them for the expected excess return. Thus, we can study the role played by the interplay of the utility function parameters, namely the risk aversion and the elasticity of intertemporal substitution.

2.4.1 The role of utility parameters

Following the analysis in Campbell et al. (1997), if consumption growth rates and asset returns are homoskedastic and jointly lognormal, the equity premium can be expressed as[9]

$$EP = \gamma\sigma_g^2 + \left(1 - \frac{1-\gamma}{1-1/\psi}\right)\sigma_\omega^2 + \left(1 - \frac{1-\gamma}{1-1/\psi} + \gamma\right)\sigma_{\omega,g} \qquad (18)$$

where σ_g^2 is the variance of the log consumption growth, σ_ω^2 is the variance of log $\frac{1+PDt+1}{PD_t}$, and $\sigma_{\omega,g}$ is their covariance[10].

The first component of equation 18, $\gamma\sigma_g^2$, is the determinant of the equity premium when an agent has a power utility function. Clearly, as has already been estabilshed in the literature, the only way to increase the equity premium through this term is to increase the coefficient of risk aversion, leading

[9]Even if the log-linearization does not strictly hold for our non-linear economy, the implications that follow are valuable.

[10]For a detailed loglinear analysis in a similar framework see Brevik and d'Addona (2006).

to the well known finance puzzles.

The second component links the variance of increases in the price-dividend ratio with the utility function parameters. The variance is always positive so, we can focus on the coefficient $\left(1 - \frac{1-\gamma}{1-1/\psi}\right)$ to analyze the contribution of this term to the equity premium. To have a positive contribution of the above coefficient we need the term $\left(\frac{1-\gamma}{1-1/\psi}\right)$ to be less than 1. When the elasticity of intertemporal substitution (EIS) is larger than 1, this implies a higher RA parameter relative to the inverse of the EIS, or a preference for early resolution of uncertainty, in the language of Kocherlakota, 1990.

It is important to note that an EIS larger than 1, implies procyclical prices (see Brevik and d'Addona (2006)). Thus, in order to have a positive contribution by the variance of prices to the equity premium we need an agent with a preference for the early resolution of uncertainty coupled with procyclical prices.

In the same fashion we can analyze the third component of equation 18. Focusing on the case when both RA and EIS are greater than 1, the prociclycality of prices leads to a positive covariance between consumption growth rates and prices themselves. So a positive contribution of this third term is assured if the numerator of its coefficient, rewritten as $\frac{\gamma(2-1/\psi)-1/\psi}{1-1/\psi}$ is positive. This turns out to be always true for the case we are focusing on.

The above analysis can be used to define the restrictions we can impose on the model to expect a better performance in explaining financial figures. That is: in order to have a positive contribution by all the terms in equation

18 to the equity premium, it is sufficient to assume preference for the early resolution of uncertainty for the representative agent and a EIS parameter stricly greater than 1.

Interestingly enough, both the preference for the early resolution of uncertainty and prociclycal prices are not necessary requirements for a positive contribution by the two second moments to the equity premium. In fact it is straighforward to see that, when ψ is less than 1, we still can have a positive value of the second component in equation 18, provided that agents have a preference for the late resolution of uncertainty (that is $\gamma < 1/\psi$). Moreover, the covariance of prices and consumption growth rates would be negative in this case. So a sufficient condition for having a positive contribution from the last term in equation 18 is to have a RA parameter greater than 1. Summarizing, if prices are countercyclical, then the equity premium is monotonically increasing in both the variance of prices and the (negative) covariance between consumption growth rates and prices themselves, if agents have preference for late resolution of uncertainty and have a RA parameter greater than 1.

3 Empirical analysis

3.1 Estimation

Having provided the theoretical implications given by our framework, we can now turn to the numerical analysis of the model.

The data used for the calibration span from the beginning of 1952 to the end of 2004. The dataset is expressed in real terms with a quarterly frequency. The financial series (prices and dividends) are on the S&P 500 composite, while the risk-free rate is the yield on 1 year treasury bills. These series are from Robert J. Shiller's webpage[11]. The main economic series are downloaded from the Bureau of Economic Analysis' website[12]. Consumption is quarterly real total personal consumption expenditures (NIPA table 2.3.6, line 1), GDP is quarterly real gross domestic income (NIPA table 1.1.6, line 1), investment in physical capital is quarterly non residential fixed investment (NIPA table 5.3.5, line 2), and education expenditures are personal education and research expenditures (NIPA table 2.5.5, line 104). Both the human capital and the physical capital series are constructed using the perpetual inventory method. Finally, we use the official recession dates as reported on the website of the National Bureau of Economic Research[13].

To estimate the technology shocks, a standard technique based on the growth accounting framework is applied. In particular, with constant return to scale, it is possible to decompose the output growth in two parts, and thus the change in technology shocks can be estimated as:

$$\Delta \log A = \Delta \log Y - \theta \Delta \log K - (1-\theta) \Delta \log H. \tag{19}$$

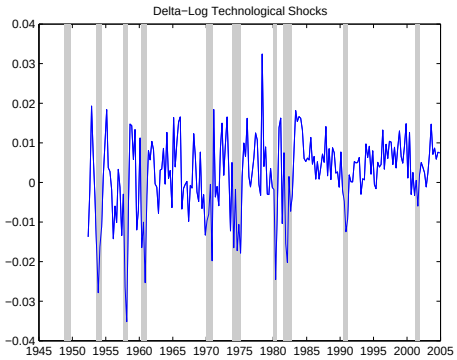
Figure 1 shows the time series of the estimated technological shocks. A

[11] http://www.econ.yale.edu/∼shiller/data.htm.
[12] http://www.bea.doc.gov/.
[13] http://www.nber.org/cycles.html/.

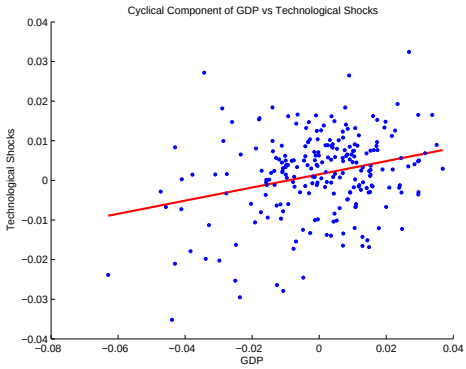
Figure 1: Estimated technological shocks

This figure plots the empirical estimate of technological shocks. Data, transformed with logarithms, are quarterly starting from I-1952 to IV-2004. Panel A plots the estimated series of technological shocks, coupled with the recession periods according to NBER (shadow areas in the graph). Panel B shows a scatter-plot of GDP cyclical component, estimated with a HP filter, versus the shocks.

Panel A



Panel B



procyclical behavior in the series is clearly evident.

The regime switching specification for the US economy with two possible states both for the mean and the volatility of the productivity shocks is also estimated. Parameter estimates for the model were computed using a Markov-Chain Monte-Carlo (MCMC) procedure following Kim and Nelson (1999).

The results from this analysis are given in Table 1. An important finding from our estimation is the high persistence of the states associated with the mean. In fact, the probabilities for the first moment of switching from the two states are 5.67% and 19.72%, respectively. This implies an average duration of more than four years (17.7 quarters) for high mean states (associated

Table 1: Estimation of regime switching economy

This table reports the estimated parameters of a two state Markov switching model for the US postwar economy. The estimates are based on a MCMC algorithm from Kim and Nelson (1999) with both the mean and the volatility of technological shocks being different in the two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2004; source: BEA). Standard errors are reported in parenthesis.

Technological shocks' process estimation				
State	$\mu(s)$	$\sigma(s)$	p_{ij}^{μ}	p_{ij}^{σ}
High (s=high)	0.0047	0.0101	0.0567	0.0191
	(0.0010)	(0.0009)	(0.0227)	(0.0164)
Low (s=low)	-0.0109	0.0046	0.1972	0.0209
	(0.0069)	(0.0005)	(0.0680)	(0.0207)

with booms), and more than one year (5.3 quarters) for low mean states (associated with busts). Hence, if we find ourselves in either of the two states, we expect to stay there for several periods. The results are more striking for the second moment. Looking at its switching probabilities, it is clear how the volatility state is extremely persistent, so if we find ourselves in either one of the two states for the volatility, it is very well the case that we will face that state for the majority of a sample period.

To get a final grasp on the estimation of the regime switching economy, we investigate the capability of the model in picking up the historical business cycles of the US postwar economy. Figure 2 reports this analysis by plotting the estimated posterior probability, associated with the mean of the productivity shock process, of being in the recession state. It is evident how the Markov switching model is able to capture fairly well the US recessions as chronicled by the official NBER business cycle dates (the gray areas in the graph).

Figure 2: Posterior probabilities of a recession

This figure shows the estimated posterior probabilities of being in a recession coupled with the official NBER recession dates (shadow area). Data employed in the estimation are quarterly starting from I-1952 to IV-2004.

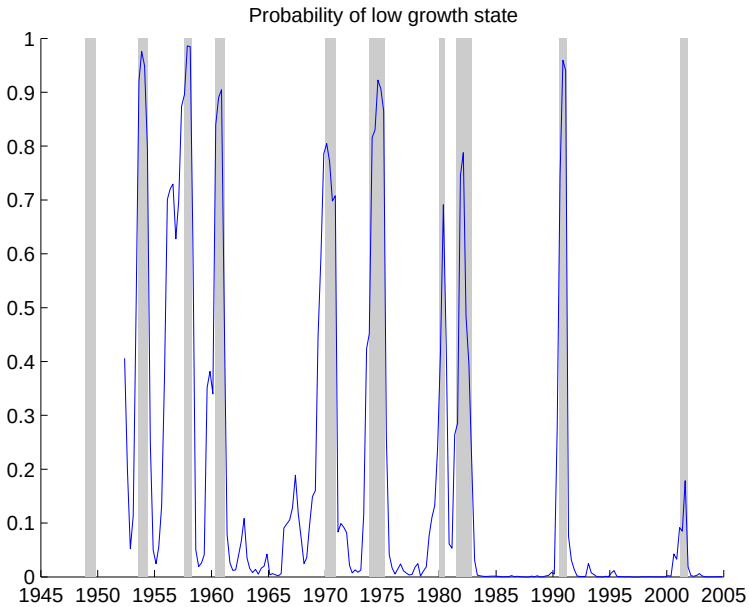
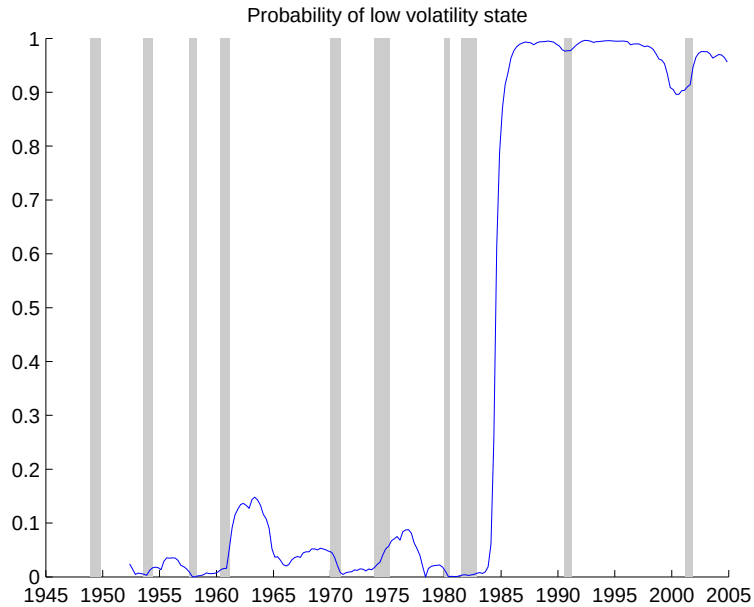


Figure 3 reports a similar analysis by plotting the estimated posterior probabilities, associated with the volatility of the productivity shock process, of being in the low state. The obtained graph is consistent with the declining macroeconomic volatility starting in the mid eighties and documented widely in the literature (see Blanchard and Simon, 2001, and Lettau et al. (2006) among others), also named as "the great moderation" by Stock and Watson (2002).

Figure 3: Posterior probabilities of the low volatility state

This figure shows the estimated posterior probabilities of being in a low volatility state coupled with the official NBER recession dates (shadow area). Data employed in the estimation are quarterly starting from I-1952 to IV-2004.



3.2 Calibration

The basic calibration sets the model's parameters as follows: the share of physical capital θ, is fixed to 0.36 which is the standard approach taken in the literature on business cycles. The depreciation rate for physical capital is fixed at 0.021 , which is also standard in the literature. For human capital, we follow Heckman (1976) by choosing a value of 0.009.

The ratios of investment to physical capital and the ratio of education expenses to human capital are set to replicate the difference in GDP growth

rates during the two states of the economy. This gives investment to capital ratios of 0.0365 and 0.002 during booms and during busts respectively. Similarly, education expenses to human capital ratios are set to 0.0145 and 0.001 respectively.

3.3 Results

In this subsection we present the results from solving the model numerically. As a first comparison, we set the utility function parameters as in Lettau et al. (2006): $\gamma = 30$, $\psi = 1.5$, and we fix $\beta = 0.9925$.

The basic results from this calibration are presented in table 2. It reports the set of estimates discarding the first five years of data, in order to address the well known critique to the perpetual inventory methodology used in the capital estimations.

As shown in this first set of results, the model overshoots in estimating the mean equity premium. This is mainly due to the poor performance in matching the risk free rate. In fact, with the proposed parametrization, we obtain a real risk free rate that is negative and big (almost -7%).

Again, we can use a loglinear approximation as in Campbell et al. (1997), to interpret this quite odd result. Loglinearizing the Euler equation, and solving it for the risk free return, gives us a simplified framework to analyze it. The expected risk free rate can be expressed as:

Table 2: Financial series of the US economy using Lettau et al. (2006) utility parameters

This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/ shiller/data.htm). The coefficient of risk aversion is set to 30, the EIS is set to 1.5.

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.084	0.074	0.030
Risk Free	0.013	-0.067	0.006	0.002
Equity premium	0.058	0.152	0.074	0.043
$\rho(r^f; r^e)$	0.047	-0.062	-	-
$\rho(r^e_{t+1}; r^e_t)$	0.105	0.594	-	-
$\rho(r^f_{t+1}; r^f_t)$	0.890	0.611	-	-

$$r_{f,t+1} = -\log\beta + \frac{1}{\psi}\,\mathrm{E}_t\left[g_{t+1}\right] - \frac{\alpha}{2}\left[\frac{1}{\psi^2}\sigma_g^2 + \left(\frac{1}{\alpha} - 1\right)\sigma_c^2\right] \tag{20}$$

where $\mathrm{E}_t\left[g_{t+1}\right]$ is the expected consumption growth rate, and σ_g and σ_c denote the volatility of the log consumption growth rate and the volatility of the log return to the consumption claim, respectively.

By inspection, we can see that the coefficient on the volatility of the consumption claim, which is $(.5(\alpha - 1))$ is negative with the parametrization adopted above. So, this is the term that is pulling down the required risk free in the model, given the positivity of volatilities.

Given the unsatisfactory performance of the previous calibration, especially regarding the risk free rate, we perform a simple exercise: we fix the

Table 3: Empirical series of US financial markets matching the risk free rate
This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/ shiller/data.htm). The coefficient of risk aversion is set to 10. The obtained EIS by matching the empirical mean of the risk free rate is 1.371

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.065	0.074	0.035
Risk Free	0.013	0.013	0.006	0.009
Equity premium	0.058	0.052	0.074	0.050
$\rho(r^f; r^e)$	0.047	0.061	-	-
$\rho(r^e_{t+1}; r^e_t)$	0.105	0.598	-	-
$\rho(r^f_{t+1}; r^f_t)$	0.890	0.610	-	-

risk aversion parameter to 10, and we let the elasticity of intertemporal substitution vary to match the mean risk free rate of the postwar US economy[14]. The results for this calibration, obtained discarding the first five years of data, are reported in table 3.

As shown in this table, the model performs fairly well in matching the mean equity premium and the first moment of the equity asset. In fact the model predicts a yearly equity return of 6.5%, and we match the real risk free return to a level of 1.3%, with a EIS parameter of 1.371. This leads to a predicted yearly equity premium of 5.2%. The matching of second

[14]It is well known that Mehra and Prescott (1985) indicate 10 as an upper bound for an acceptable RA parameter in their setting. But is is important to point out that even if a risk aversion parameter higher than 10 can be perceived as implausible in a standard power utility setting, the parameter's implications in a Epstein-Zin utility framework change dramatically with respect to the power utility case. For a detailed theoretical characterization of these implications see Campanale et al. (2007).

moments is also satisfactory. In particular, both the standard deviations of the risk free asset and the equity asset are of the same magnitude of their empirical counterparts. Regarding the correlation and the autocorrelation of the assets, the model performs well in matching the autocorrelation of the risk free asset as well as the correlation between equity and the risk free asset. Less satisfaction is derived from the performance in matching the equities' autocorrelation. This is probably due to the nature of the model in which prices have a strong persistence with respect to the states of the economy.

The drastic improvement in the model performance is due to the decrease of both the risk aversion and the elasticity of intertemporal substitution assumed for the representative agent. By lowering the agent risk aversion we directly influence the equity claim: a lower value of γ makes equities more appealing; this leads to higher prices and so decreases the equity return. Instead, the risk free claim is directly influenced by the elasticity of substitution: the lower is the EIS the lower is the willingness of an agent to transfer her wealth overtime; this leads to lower prices for a risk free asset and so to a higher risk free return.

Before moving ahead in our empirical analysis, it is worth analyzing the value obtained for the EIS parameter to match the risk free return, given that the empirical estimates in the literature vary considerably. One approach of the empirical research focuses on a representative agent setup and uses aggregate consumption data. This leads to estimates of the EIS coefficient below 1, and even close to 0 (see e.g. Hall, 1988; Campbell and Mankiw, 1989,

1991; Hahm, 1998; Yogo, 2004; and Zhang, 2006). Another strand of research relies on microeconomic survey data to avoid potential biases in the aggregate data. If a stockholder is considered, these studies find EIS parameters around or above 1. (See Beaudry and van Wincoop, 1996; Vissing-Jørgensen, 2002; Vissing-Jørgensen and Attanasio, 2003; and Guvenen, 2005). Even if there is this lack of consensus in the economic literature, the recent asset-pricing literature relies on the higher EIS estimates of the latter literature, in fact both Bansal and Yaron (2004) or Lettau et al. (2006) calibrate their models with an EIS greater than one. As discussed above (cf. section 2.4.1) this choice is mainly linked to the capability of generating procyclical prices when agents have a recursive utility function. Consequently, we can consider the value of 1.371 for the EIS, obtained by matching the risk free rate, in line with the latter literature, and theoretically well grounded.

3.4 The role of leverage

An important assumption in Lettau et al. (2006) is that dividends on equity are a power function of aggregate consumption. With this they can model leverage by specifing the dividends as[15] $D = C^{\kappa}$ and define $\nu = \frac{1}{\kappa}$ so that $D^{\nu} = C$.

An interesting comparison can then be made by embedding their assumption in our setting. Given their leverage specification we can express the price to consumption ratio (PC) of the consumption claim $R_{c,t+1} = \left(\frac{P^c_{t+1}+C_{t+1}}{P^c_t}\right)$

[15]They follow Abel (1999) in this assumption.

as:

$$PC_t^{\alpha} = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta} \right)^{-\frac{\nu\alpha}{\psi}+\nu\alpha} (PC_{t+1}+1)^{\alpha} \right]. \tag{21}$$

In the same fashion the price to dividend ratio (PD) of the dividend claim $R_{e,t+1} = \left(\frac{P_{t+1}^e + D_{t+1}}{P_t^e} \right)$ becomes:

$$PD_t = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta} \right)^{-\frac{\nu\alpha}{\psi}+\nu\alpha-\nu+1} (PC_{t+1}+1)^{\alpha-1} (PC_t)^{1-\alpha} (PD_{t+1}+1) \right] \tag{22}$$

Proof. see Appendix A □

The influence of leverage in our model can be analyzed pointing at the different responses of realized and required returns of an asset to a change in the volatility of the future cash flows of the asset itself.

From the definition of dividends ($D = C^{\kappa}$, $\kappa > 1$ for leverage) it is clear that the volatility of the dividends is augmented by a factor κ when they are modeled in such a way. So, by introducing leverage, we should be able to obtain a higher *required* risk premium, keeping the utility function parameters fixed. The intuition behind this is straightforward: the more volatile a stream of cash flows is, the less attractive would be a claim to such a stream for an agent who has a preference for a smooth consuption path. So, to have an agent hold such an asset, the *expected* equity premium has to increase in order to compensate her.

Table 4: Empirical series of US financial markets: Lettau et al. (2006) with leverage

This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/ shiller/data.htm). The coefficient of risk aversion is set to 30, the EIS is set to 1.5, and the leverage parameter is set to 4.5.

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.066	0.074	0.064
Risk Free	0.013	0.028	0.006	0.002
Equity premium	0.058	0.039	0.074	0.091
$\rho(r^f; r^e)$	0.047	-0.048	-	-
$\rho(r^e_{t+1}; r^e_t)$	0.105	0.604	-	-
$\rho(r^f_{t+1}; r^f_t)$	0.890	0.606	-	-

The *realized* equity premium, that is what we are looking at, should then act in the opposite way. As said above, a stream of cash flows that is (more) volatile is actually unappealing for an agent that prefers to smooth her consumption path. So introducing the leverage in the model, would shift the preferences of the agent towards the risk free asset, making its realized returns higher and reducing the realized returns on the equity asset.

Having pointed this out, we can now test the performance of our model, when a leveraged equity is considered. We re-calibrate the model using the full set of parameter from Lettau et al. (2006): $\gamma = 30$, $\psi = 1.5$, and $\kappa = 4.5$. The results of this calibration, obtained by discarding the first five years of data, are presented in table 4.

As shown in the table, introducing the leverage drastically changes the

model performance. In fact the model performs well in matching the mean equity return and we obtain a mean risk free that is positive and somehow in line with the empirics, even if it is 150 basis points higher than the rate observed on the US postwar data. This is driving the predicted equity premium down to a value of 3.9% versus an observed value of 5.8%. The matching of second moments is generally satisfactory, at least in term of the predicted sign, but the performance in matching the equities' autocorrelation.

3.5 The role of macroeconomic risk

Loosely speaking, the insight we can gather from Lettau et al. (2006) is that the reduction in macroeconomic volatility in the last twenty years can account for a good portion of assets' valuations in the recent past. So, it is natural to ask ourselves what is the role of the volatility regimes in the economy we are studying. We then re-estimate our regime switching economy, imposing the restiction of a single state for the volatility of the productivity shocks.

The parameters for the restricted regime switching model are again obtained using the same Markov-Chain Monte-Carlo (MCMC) algorithm used for the unrestricted model. The resulting estimates are given in table 5.

As expected the estimated value for the volatility is in between the values we obtained for the two states version of the regime switching process. The main departure from the unrestricted estimation is that the difference between the means in the two states is sharper. Furthermore, the estimated probabilities of switching from the two mean states are 6.30% and 22.08%,

Table 5: Estimated parameters for the restricted regime switching model

Reported are the estimated parameters for the restricted version of the model. The estimates are based on a MCMC algorithm from Kim and Nelson (1999) with the mean of technology shocks being different in two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2004; source: BEA). Standard errors are reported in parenthesis.

State	$\mu(s)$	σ	p_{ij}^{μ}
High (s=high)	0.0051		0.0630
	(0.0007)	0.0074	(0.0219)
Low (l=low)	-0.0112	(0.0004)	0.2244
	(0.0019)		(0.0692)

respectively. These probabilities confirm the persistence of each state also in the restricted setting.

We can now move to analyze the performance of the model, when the decline in the macroeconomic risk is not considered. We re-estimate the main financial figures, implied by our model with a restricted regime switching economy, fixing the utility parameters at $\gamma = 10$ and $\psi = 1.371$. The results for this calibration, obtained by discarding the first five years of data, are presented in table 6. While the equity returns are not effected by disregarding the decline in the macroeconomic risk, the implied risk free rate moves in the expected direction. In fact we obtained an estimate of 2.2% on an annual basis, that is about 90 basis point higher, than the observed risk free rate on which we calibrated the EIS parameter in the unrestricted version of the model (cf. table 3).

This result can be better interpreted if we focus on the role played by the

Table 6: Empirical series of US financial markets: restricted regime switching model

This table shows the asset returns implied by the restricted model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2004; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/ shiller/data.htm). The coefficient of risk aversion is set to 10, the EIS is set to 1.371.

Series	Mean Data	Mean Model	Std. Data	Std. Model
Equities	0.072	0.061	0.074	0.035
Risk Free	0.013	0.022	0.006	0.010
Equity premium	0.058	0.039	0.074	0.049
$\rho(r^f; r^e)$	0.047	0.064	-	-
$\rho(r^e_{t+1}; r^e_t)$	0.105	0.540	-	-
$\rho(r^f_{t+1}; r^f_t)$	0.890	0.540	-	-

volatility of the underlying state of the economy in determining the prices and thus the returns. When we shut down the regime on the volatility, we are preventing the agent from entering a persistent "high risk" regime associated with a high volatility of the economy, but we are also depriving her of the possibility of entering a persistent "low risk" regime. This creates an economy with a smoother consumption path that is appealing for the representative agent of our economy. So, an economy with such a regime gives an higher incentive to the representative agent to use the risk free asset to transfer consumption overtime, pushing its prices up and lowering the risk free return.

3.6 Time varying properties of the equity premium

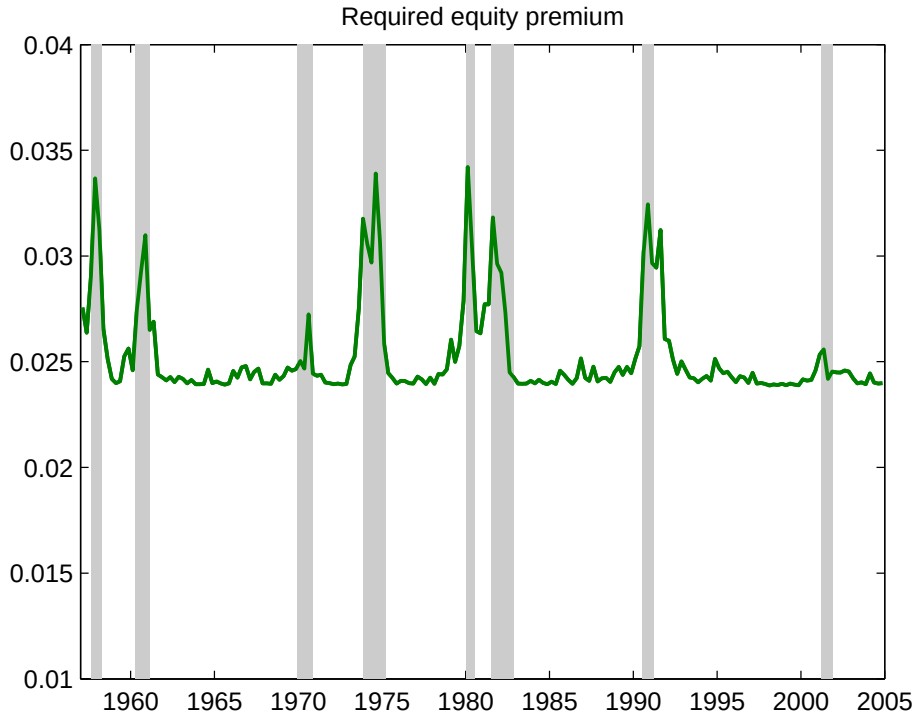
Another notable feature of the model is the ability of replicating the behavior of the equity premium over the business cycle. The model delivers a higher risk premium in recessions than in booms: the implied annualized equity premium during recession varies from 3.46% under the high volatility state to 3.32% under the low volatility state. During booms the model predicts a much lower level of equity premium on an annual basis: 2.39% in the high volatility state and 2.25% in the low volatility state.

As a thoughtful exercise, we can calculate the expected equity premium by an investor that only observes consumption realizations. In practice, at each point in time, we let investors only have access to consumption growth rates. They know the structure of the economy, but they can use only consumption data in order to infer the current state of the economy. The probabilities the investors attach to each of the two states correspond to the Hamilton's filtered state probabilities.

Figure 4 shows the expected equity premium for the US postwar period. The plotted line clearly picks up in recessions, confirming the expected coutercyclical behavior of the required premium an investor would ask in order to hold a risky asset.

Figure 4: The time varying properties of the equity premium

This figure shows the time-series of the expected equity premium by an agent that only observes consumption realizations. The state probabilities are those inferred using the Hamilton's 1989 filter. The equity premium is calculated fixing the utility parameters to the benchmark values (i.e. $\gamma = 15$, $\psi = 1.75$, and $\beta = 0.9925$).



4 Sensitivity analysis

In order to get a better grasp on the forces driving our results, we provide a sensitivity analysis of the main financial figures implied by the model to the three most relevant parameters: risk aversion (γ), elasticity of intertemporal substitution (ψ), and leverage (ν).

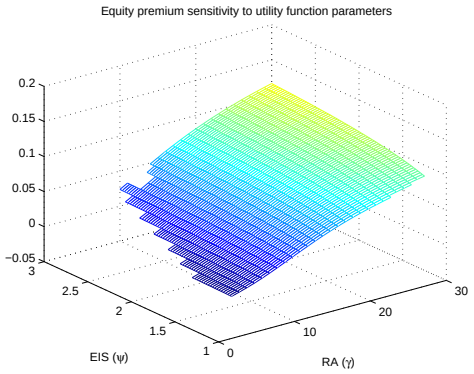
Figure 5 provides a general overview of how the equity premium is affected by different values of the parameters. Panel A shows the mesh of the equity premium by changing RAs and EISs. As expected the equity premium is monotonically increasing in the risk aversion. The same increasing relation is displayed for the intertemporal substitution and the equity premium itself, but somewhat less pronounced. Given the sizable influence of risk aversion on the equity premium, we also compare its role to the role of leverage in determining the equity premium. This is reported in Panel B of Figure 5. Here the effect by the two parameters is comparable, actually showing a large positive effect on the equity premium as the dividends become smoother (i.e. with a lower volatility).

After having discussed the general behavior of the implied equity premium with respect to risk aversion, intertemporal elasticity of substitution and leverage, it could be interesting to focus on the relevance of the utility function's parameters. Figure 6 delivers some insights on this. Panel A plots the obtained equity return by letting the risk aversion vary, keeping the remaining parameters fixed to the benchmark case. As expected the equity return is increasing in γ with a higher rate for lower values of γ. The same exercise, but on the risk free return, is reported in Panel B by letting the elasticity of intertemporal substitution vary. Again the relation is as expected: The higher the EIS the higher the willingness of an agent to transfer her wealth overtime; this leads to higher prices for a risk free asset and so to a lower risk free return.

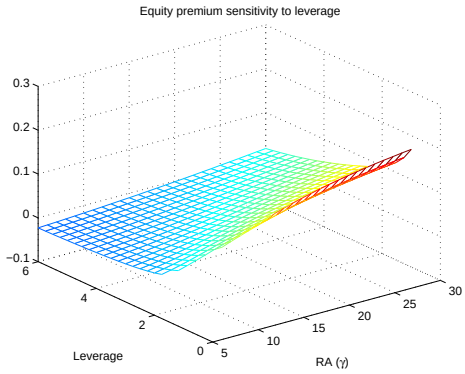
Figure 5: Equity premium's sensitivity

This figure shows the sensitivity of the implied equity premium to the relevant parameters used in the model. Panel A plots the role of both Risk Aversion and Elasticity of Intertemporal Substitution in determining the equity premium. Panel B analyzes the influence of leverage in determining the equity premium in comparison with the Risk Aversion. Leverage is plotted as $\nu = 1/\kappa$, so a lower value on the axis corresponds to a smoother dividends' path.

Panel A



Panel B

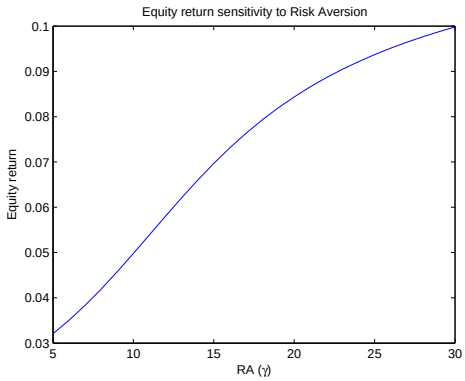


To complete the analysis, we disentangle the effect of leverage on the equity premium, by analyzing its role in determining the return on the equity asset and the return on the risk free asset respectively. Figure 7 reports this analysis, and shows the contribution of the leverage to both returns, by letting ν vary, keeping all other parameters fixed. The influence on the equity return, reported in Panel A, is in line with the finding on the influence of leverage on the equity premium: the equity return increases as dividends' smoothness increases with respect to consumption. Results for the risk free asset, shown in Panel B, are also in line with the theoretical discussion provided in subsection 3.4: the realized risk free return increases

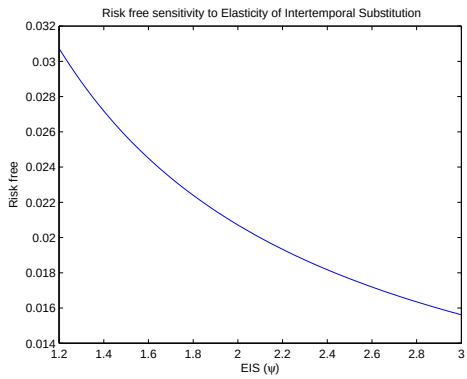
Figure 6: Returns' sensitivity to risk aversion and intertemporal substitution

This figure shows the sensitivity of assets' returns to utility function's parameters. Panel A plots the role of Risk Aversion in determining the required return on a risky asset. Panel B analyzes the influence of Elasticity of Intertemporal Substitution in determining the required risk free return.

Panel A



Panel B



as the dividends' volatility implied by the leverage parameter becomes larger.

5 Conclusion

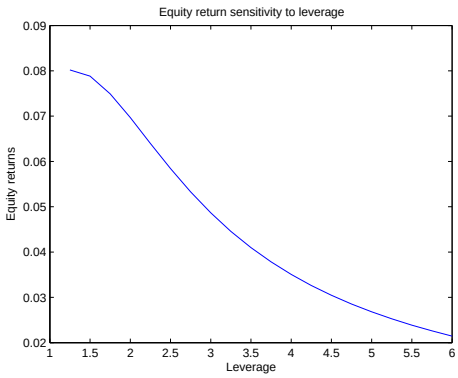
This paper deals with asset pricing implications of production economies. In particular, we propose a simple extension to a standard Real Business Cycle (RBC) model where the economy switches between booms and busts and consumers have a recursive utility.

Our first contribution is a detailed theoretical analysis of the equity premium's determinants in the proposed framework. Secondly, we show that a plausible parametrization of the model conveys financial figures that are in line with the empirical observations on the postwar U.S. data. A detailed

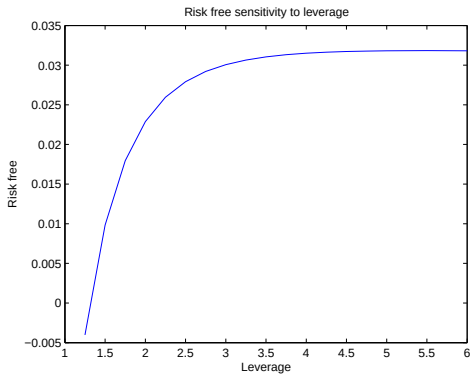
Figure 7: Returns' sensitivity to leverage

This figure shows the sensitivity of assets' returns to leverage. This is plotted as $\nu = 1/\kappa$, so a lower value on the axis corresponds to a smoother dividends' path. Panel A shows the variations of the risky asset returns obtained by letting the leverage vary. Panel B analyzes the influence of leverage on the risk free return

Panel A



Panel B



analysis on the relative contribution of prominent parameters of the model is also provided. This allows us to clarify the role of different choices on the utility function. In particular, we investigate the role of risk aversion and elasticity of intertemporal substitution in determining asset prices and thus, in determining the equity premium. Furthermore, we study the role of macroeconomic risk in the proposed economy, providing evidence in favor of modelling the "great moderation" observed in the last two decades.

The model, presented herein, is also shown to be able to replicate the empirical evidence of higher, and sizeable, required premia during a downturn of the economy, by simply letting the agent infer the state of the economy from consumption realizations.

A Deriving Asset Prices

Starting from Equation 12, we can rewrite it as

$$
\begin{aligned}
Q_{t+1} = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}} \left(\frac{P^c_{t+1} + C_{t+1}}{P_t}\right)^{\alpha-1} = \\
= \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}} \left(\frac{P^c_{t+1}}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P^c_t}{C_t}\right)^{1-\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1} = \\
= \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1} \left(\frac{P^c_{t+1}}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P^c_t}{C_t}\right)^{1-\alpha}
\end{aligned}
\tag{23}
$$

giving us an expression for the stochastic discount factor as a function of consumption and price of its claim.[16]

First we price the consumption claim $R_{c,t+1} = \left(\frac{P^c_{t+1}+C_{t+1}}{P^c_t}\right)$

$$
\mathbb{E}_t \left[\beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1} \left(\frac{P^c_{t+1}}{C_{t+1}} + 1\right)^{\alpha-1} \left(\frac{P^c_t}{C_t}\right)^{1-\alpha} \left(\frac{P^c_{t+1} + C_{t+1}}{P^c_t}\right) \right] = 1 \tag{24}
$$

Define $PC_t = \frac{P^c_t}{C_t}$

[16] It is worth noting that Q_{t+1} can be further simplified:

$$
\begin{aligned}
Q_{t+1} = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{\alpha(1-\frac{1}{\psi})-1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} \\
= \beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1-\gamma}{1-1/\psi}(1-\frac{1}{\psi})-1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} = \\
\beta^{\alpha} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha}
\end{aligned}
$$

where PC indicates the Price Consumption ratio on the consumption claim

$$PC_t^{\alpha} = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}+\alpha} (PC_{t+1}+1)^{\alpha} \right] \tag{25}$$

using $D^{\nu} = C$ we get

$$PC_t^{\alpha} = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{D_{t+1}}{D_t} \right)^{-\frac{\nu\alpha}{\psi}+\nu\alpha} (PC_{t+1}+1)^{\alpha} \right] \tag{26}$$

Then we solve for the dividend claim $R_{e,t+1} = \left(\frac{P^e_{t+1}+D_{t+1}}{P^e_t} \right)$

$$\mathbb{E}_t \left[\beta^{\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}+\alpha-1} \left(\frac{P^c_{t+1}}{C_{t+1}} + 1 \right)^{\alpha-1} \left(\frac{P^c_t}{C_t} \right)^{1-\alpha} \left(\frac{P^e_{t+1}+D_{t+1}}{P^e_t} \right) \right] = 1 \tag{27}$$

Define $PD_t = \frac{P^e_t}{D_t}$

$$PD_t = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}+\alpha-1} (PC_{t+1}+1)^{\alpha-1} (PC_t)^{1-\alpha} (PD_{t+1}+1) \left(\frac{D_{t+1}}{D_t} \right) \right] \tag{28}$$

using $D^{\nu} = C$ we get

$$PD_t = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{D_{t+1}}{D_t} \right)^{-\frac{\nu\alpha}{\psi}+\nu\alpha-\nu+1} (PC_{t+1}+1)^{\alpha-1} (PC_t)^{1-\alpha} (PD_{t+1}+1) \right] \tag{29}$$

Finally we can plug $\frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta}$ in place of $\frac{D_{t+1}}{D_t}$ in Equations 26 and 29 for getting Equations 21 and 22 respectively. Equations 14 and 15 are obtained

by simply shuting down the leverage channel (i.e imposing $\nu = 1$).

We can solve this set of four equations using the same technique as Lettau et al. (2006) and the estimation on the $\Delta \log A_{t+1}$ process (i.e. solve for the fix point and then compute expected PC and PD using posterior probabilities from the Hamilton (1989) filter.

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