Oligopolistic Competition Between Intermediaries

Alexei Alexandrov George Deltas Daniel F. Spulber*

November 2008

Abstract

Firms are intermediaries between input suppliers and consumers. This paper considers the implications of competition between strategic firms that compete in both the input and the output markets, and are differentiated both as employers and as producers. We develop a two-sided market model based on the Salop framework to characterize firm differentiation on the input and output markets. We investigate the implications competition of differentiated intermediaries for oligopoly and matching.

1 Introduction

Firms are first and foremost intermediaries, transforming the services of input providers to output desired by and sold to consumers[1]. Most of the microeconomics and industrial organization literature compartmentalizes firm decision making: the input side decisions generate a cost function, while the output side decisions maximize profits taking costs as

*Alexei Alexandrov: University of Rochester, Rochester, NY. (alexei.alexandrov@simon.rochester.edu). George Deltas: University of Illinois, Urbana-Champaign, IL. (deltas@uiuc.edu). Daniel Spulber: Northwestern University, Evanston, IL. (jems@kellogg.northwestern.edu). The authors would like to thank the participants of the Midwest Economic Theory (UIUC) and the International Industrial Organization (Arlington, VA) conferences. George Deltas would like to thank the Kellogg School of Management for its hospitality.

[1]This has been recognized long ago, see for example the seminal work of Stigler (1946).

given. Typically, this is justified on the grounds that a firm's profit function is separable in its input and its output decisions, with the input price entering as a parameter in its output pricing decision and vice versa. Even if this is not the case, the cost minimization problem often can be solved independently from the output pricing decision to generate a cost function that does not depend on the firm's pricing.

However, this is not always possible. In strategic environments, the interplay between competition in the factor and product markets implies that input and output decisions are not only interdependent within each firm, but also are jointly determined from the interaction between firms. Thus, a firm's cost function depends on the activities of other firms, which in turn depend on that firm's output decisions. As a consequence, a cost function is **not** independent of a firm's output decision.

In this paper we consider the behavior of strategic intermediaries (firms) who compete both in the input and in the output markets. We develop a base model that builds on Salop (1979) and Spulber (1999). The intermediaries are differentiated both in the eyes of customers and of suppliers as in Hotelling (1929). Such differentiation can be due to physical location in a setting with costly transportation (e.g., workers/customers preferring to work/shop in an establishment that is physically close to their home). Such differentiation can also be due to differences in the types of services that the intermediaries provide to their customers and their suppliers (some customers prefer Windows, others prefer Mac OS X; similarly, some programmers prefer to work for Microsoft and others prefer to work for Apple[2]). Differentiation of products implies that the competing intermediaries face downward-sloping demand curves, while differentiation with respect to the input providers implies that they face upward-sloping offer curves. Given strategic interaction in both the input and output markets, the demand and offer curves are interdependent across firms and jointly determined. Thus, intermediaries not only have market power in the market for both transactions, but the

[2]We do not assume that the programmers who prefer Apple are able to deliver better services to Apple customers than the programmers who prefer to work for Microsoft. As far as consumers are concerned, it does not matter what are the preferences of the workers working for the firm.

extent of this market power for each of the two markets depends on the degree of competition between firms on both of the markets.

Throughout the paper, we use the term **price** to denote what the end-consumer pays (i.e. downstream price), and **wage** to denote what supplier receives (i.e. upstream price). We use **markup** to denote intermediary's margin or the difference betwen price charged to the consumer and the price paid for the unit to the supplier (wage)[3]. Depending on the context, by **firms** we mean the intermediaries, by **suppliers/producers/workers** we mean the upstream market, and by **consumers/customers** we mean the downstream market.

Our model, which focuses on oligopolistic/oligopsonistic competition, yields the following main results. With competition between differentiated intermediaries, there is a positive markup between the upstream price and the downstream price. The markup depends on the relative sizes of the upstream and downstream markets. The model is generalized to allow for competitive entry, and positive markups survive in equilibrium. An extension of the analysis yields insights into matching markets, and gives equilibrium fees chosen by competing differentiated matchmakers.

The model that we develop applies to environments where intermediaries compete with each other both for consumers and suppliers. Applications include competition between merchants, between manufacturers, between banks and other financial intermediaries, between matchmaking firms. Consider for example, competition between supermarkets who buy produce from local farmers which they resell to local consumers. On the consumer side, the differentiation might be either the physical distance, or store specialization. On the farmer side, the differentiation might also be either physical distance to the warehouse/stores, or different trade terms such as ease of access to the managers for negotiation, logistic issues with a particular store, or a prior business relationship. The number of farmers upstream may be greater than, equal to, or less than the number of customers downstream. Suppose the bottleneck is on the producer side. Then, depending on the ratio of producers and suppli-

[3]In the financial setting, the markup becomes the bid-ask spread, the price becomes the ask price, and the wage becomes the bid price.

ers, number of intermediaries, and the differentiation parameters, one of the three equilibria occurs. The trivial case is that there are still too many suppliers, and supermarkets can act as monopolists both downstream and upstream, with comparative statics as expected from the standard Hotelling product differentiation model.

On the other hand, if there are too many supermarkets, or not enough suppliers, then firms compete for suppliers (or consumers, if the bottleneck is on that side of the market), and even though firms post prices - producers have no bargaining power - still almost all of the final consumers' reservation utility goes to suppliers via the high upstream prices. This is a kind of result which one would not get with the one-sided Salop framework - despite effectively being monopolists in the downstream markets, and having all the bargaining power upstream, strong enough competition leaves the intermediaries with just the profits due to differences in consumers' and producers' travel costs.

The third equilibrium is a kinked one, which happens at the intermediate values of the parameters, and is similar to the one in Salop's seminal paper. The comparative statics are the complete opposite of the standard intuition (like mark-ups increasing in the number of firms). Our paper is a generalization of Salop's model, accounting for differentiation on the other side of the market as well. Taking one of our differentiation parameters to zero, either in the upstream or in the downstream, brings about the familiar Salop results.

If the workers (or consumers) do not view firms as being different, then our model becomes the standard Salop model[4]. When workers do have preferences for employers, and each employer faces an upward sloping supply of labor (but does not directly compete for workers with his rivals), our model results in a generalized Salop model with increasing marginal cost. In these environments, one can describe competition in the product market without reference to competition in the input markets, as it is done in the existing literature. For example, if the suppliers are low-skilled workers providing labor, the particular set of firms

[4]Workers (consumers) seeing firms as not differentiated simply means taking the corresponding differentiation (travel cost) parameter to zero. For example, we get the familiar $\frac{t}{N}$ result if workers only care about wages.

(say banks hiring custodians) are not going to drive up wages too much.

However, if the laborers are highly skilled and specialized, for example professional sport players, or top executives, then our model applies. The upstream competition does not have to be for workers, it could be for resources. Again, if the firms competing make up a very small share of a resource's overall demand, then Salop's model with constant marginal costs is a good approximation. However if the intermediaries make up a large portion of the resource's demand, then our model provides more insight.

Despite its importance in the analysis of the firms and market equilibrium, competition between intermediaries has not been given sufficient attention[5]. Stahl (1988) analyzes strategic interaction on both the input and output sides of the market between undifferentiated firms. He shows that Bertrand competition between intermediaries generates a market equilibrium that, when it exists, features zero markup. Because in his model products are homogeneous, competition drives markups to zero and customers' demands equal suppliers' offers at the market clearing price.

In practice, however, observed markups are positive. In product markets retail prices tend to exceed wholesale prices, often by more than the marginal retail costs can explain. Wholesale prices often exceed the prices of manufactured goods by more that the marginal wholesale costs. In financial markets, bid-ask spreads for some types of financial assets can be greater than the marginal costs of brokers and dealers. In contrast to Stahl (1988), we explicitly incorporate intermediary differentiation in both the input and the output side of the market. This differentiation between intermediaries eliminates the discontinuity of the standard Bertrand model, allowing intermediaries to compete for transactions at the margin. We show that equilibrium markups are positive and depend on the relative number of customers and suppliers, on travel costs (or product differentiation), and on the entry

[5]Even the large literature on the interaction between upstream firms that provide inputs to downstream firms which ultimately supply consumers typically considers firms that set the price 'downstream' from its operations: though there is interaction between upstream and downstream pricing, these prices are set by the same decision maker. Recent extensions to Stigler (1946) that examine oligopsonistic competition in the input market abstract from competition between the firms in the product market (see Bhaskar, Manning, and To 2002 and references therein).

costs of intermediaries.[6]

The two-sided Hotelling model is introduced and briefly discussed in a reduced form in Spulber (1999). Armstrong (2006) extends the model by adding network effects and focuses on whether agents from either side of the market are forced to choose one of the platforms (single-home). Two-sided markets have been considered in a Bertrand competition setting, see Caillaud and Julien (2003) and the references therein. See Spulber (2006) for an overview of two-sided markets. For the purposes of intermediaries that we consider, supermarkets for example, we do not believe that the network effects play a significant role. Our model is about intermediaries, and is complementary to the vertical relations literature, rather than the platforms with network effects literature.

Loertscher (2007) looks at duopoly competition between market makers in a spatial model when buyers and sellers have the option of search[7]. We treat the possibility of search (and/or direct contracting) as a part of the outside option. For simplicity, we assume that this is never optimal for consumers and suppliers, however we could build this option in similar to Loertscher (2007). Our qualitative implications should not change.

Our paper is complementary to the literatures on bilateral oligopolies and bargaining with scarce suppliers or scarce resources. For examples of bilateral oligopoly papers see Hendricks and McAfee (2007), and Reisinger and Schnitzer (2007). The latter paper uses two Salop circles as well, however they concentrate on industries where there are an upstream and a downstream oligopolies. The authors show that, among other things, downstream competitive conditions dominate the overall outcome. A paper on bargaining by Marx and Shaffer (2008) examines several downstream firms contracting with one supplier, with the supplier having no bargaining power, and arriving at a similar conclusion of supplier extracting the downstream surplus despite having no bargaining power. Esö et.al. (2007) examine non-differentiated competition for a scarce resource, and find an asymmetric equilibrium

[6] Positive markups can also be due to other factors including asymmetric information and imperfect incentives (see Spulber (1999)).

[7] Bertrand competition between intermediaries is examined also by Gehrig (1993), Fingleton (1997), Spulber (1996, 2002), and Rust and Hall (2003).

with one of the downstream firms dominating the market ex-post.

2 The Base Model

2.1 Modeling Framework

Let there be two Salop circles: a circle of customers with a circumference of 1 and a circle of suppliers with a circumference of S. Customers and suppliers are uniformly distributed on their respective circles. There are also N symmetrically located intermediaries (see Figure 1). Customer and supplier locations are indexed by the clockwise distance x from the top of their respective circles. Intermediary locations with respect to customers and suppliers are defined conformably on the respective circles.[8] Each supplier is endowed with one unit of a good (or input) and each customer demands one unit of a good (or output). Intermediaries cannot produce any goods directly: they enable the exchange between the suppliers and the customers (or transform the supplier inputs into outputs). Therefore, their transaction volume is limited by quantity of goods (or inputs) they obtain from the suppliers. The intermediaries do not incur any transaction (or transformation) costs.

Let the customers have reservation utility of R for their ideal good, with the ideal good defined to be a good obtained from an intermediary in the same location as the consumer. If customers cannot get their ideal good, they incur linear transportation costs, with a per unit distance transportation cost of t. Any good delivered directly by supplier is not valued by the customers. Each supplier can sell either one good or none, and each customer can buy either one good or none. Let the suppliers' outside option be W. This is the cost to a supplier of transacting with an intermediary with the same location as him. A supplier incurrs incremental linear transportation costs of τ per unit distance to transact with intermediaries in different locations than him. Denote by w_j the price that intermediaries offer to the

[8]Equivalently, locations could be indexed with respect to the arc from the top of the circle to the location of the agents. This would be notationally more convenient in our model because the two circles have different length, but would depart from standard nomeclature.

suppliers and p_j the price they would charge to the customers. The intermediaries cannot price discriminate on either market. Assume that each intermediary (or firm) competes only with its neighbors. Finally, assume $R > W$ so that exchange between supplier and customer located on the same intermediary is efficient (and occurs in equilibrium) and that $\min\{t, \tau\} \geq R$, so that a monopolist does not serve all of the customers or purchase from (or hire) all of the suppliers.

2.2 Market Equilibrium Under Input (Resource) Abundance

We distinguish two possibilities: either there are more suppliers than customers $S > 1$, or there are more customers than suppliers $S < 1$ (the case of $S = 1$ is covered in Spulber 1999). Under the first possibility, relative input abundance is guaranteed, regardless of the number of intermediaries and other parameters: since there are more inputs than needed to satisfy all potential customers, there cannot be input scarcity. Under the second possibility, input abandance is possible if there is a sufficiently small number of firms.

We first consider the equilibrium when there are more suppliers than customers. In this case, either the firms are local monopolists/monopsonists, or they can compete in the customer (output) market and are monopsonists on the supplier (input) market. When the gains from trade between suppliers and customers normalized by their transport costs are too low relative to the number of intermediaries, each intermediary operates as a monopolist with respect to the customers and as a monopsonist with respect to the suppliers. In particular we have:

PROPOSITION 1 *Let $S > 1$ and $N < \frac{t+\tau}{R-W}$ (or, equivalently, that $\frac{R-W}{t+\tau} - \frac{1}{N} < 0$). Then, in equilibrium, the N intermediaries are (local) monopsonists in the supplier market and monopolists in the customer market. They each buy and sell $\frac{R-W}{t+\tau}$ of the good. They pay wage $w = W + \frac{(R-W)\tau}{2(t+\tau)}$ for the good, and sell it at price $p = R - \frac{(R-W)t}{2(t+\tau)}$. The markup is*

$$p - w = \frac{R - W}{2}. \tag{1}$$

Proof. We will find the optimal output for a monopolist/monopsonist intermediary, i.e., the intermediary's optimal purchases of inputs and sales of output, utilizing the fact that for a monoplist/monopsonist choice of prices is equivalent to choice of output level. We will then prove that with $N < \frac{t+\tau}{R-W}$, there is a sufficient mass of customers and suppliers to ensure that the N intermediary firms behave as if they were local monopolists/monopsonists.

Suppose an intermediary chooses to produce an output Q. Recalling that suppliers are located on either side of the intermediary's location, the upstream price, w, must be such that suppliers at a distance $\frac{Q}{2}$ away are indifferent between supplying their inputs to the firm and staying out of the market. Thus, the upstream price must be $w = W + \frac{Q\tau}{2}$, the total input cost given by $TC(Q) = QW + \frac{Q^2\tau}{2}$, and the marginal cost given by $MC(Q) = W + \tau Q$.

The inverse demand function for the intermediary is $P = R - \frac{t}{2}Q$ and the associated marginal revenue by $MR(Q) = R - tQ$. Thus, the optimal output of the intermediary is given by solving the equation $R - tQ = W + \tau Q$ for Q which yields $Q = \frac{R-W}{t+\tau}$. Note that given our assumption on the upper bound of N and given that there are more suppliers than customers, this quantity can be produced by each intermediary without exhausting the set of suppliers or customers, i.e., without any competition between the intermediaries. Substituting the optimal Q in the expressions for input and output prices, we obtain their optimal values. □

The equilibrium is shown graphically in Figure 2 which plots the opportunity cost of the marginal supplier as a function of output transacted (upward sloping solid line), the marginal cost of the intermediary (upward sloping dashed line), the intermediary's demand curve (downward sloping solid line), and the intermediary's marginal revenue (downward sloping dashed line). The equilibrium quantity transacted by an intermediary, Q^*, is less than $1/N$, the size of the customer segment that corresponds to an intermediary (in all figures, a star indicates equilibrium values). Notice that the demand curve has a kink when the price of the intermediary falls to the point that he starts poaching customers from the neighboring intermediaries. Similarly, the opportunity cost of the marginal supplier has a kink when the offer price rises to the point that the intermediary attracts suppliers who would otherwise have sold to a neighboring intermediary. For these suppliers, the opportunity cost also includes the forgone surplus from selling to a competing intermediary. Notice that the kink in the supply function is to the right of the kink of the demand function since $S > 1$.

An increase in transport costs affects the slopes of the demand and supply functions (and hence, of marginal cost and marginal revenue). It can be readily seen that an increase in either transport cost decreases transaction quantity, an increase in t reduces both prices, and an increase in τ increases both prices. Moreover, the effect of t and τ on prices are of the same magnitude. Thus, the markup is independent of the the degree of (spatial) differention of the intermediaries with respect to the customers and the with respect to the suppliers. All of these findings are standard to the model of monopsonistic monopolists.[9]

When, in contrast to Proposition (1), the gains from trade between suppliers and customers normalized by their transport costs are high relative to the number of intermediaries, the intermediaries compete for the marginal customer. There is no competition for the marginal supplier because the measure of suppliers exceeds the measure of customers. Thus, when all customers purchase one unit each and the downstream market is covered, there are still suppliers with units for sale. Competition between the intermediaries takes the form of competition between monopsonistic oligopolists. In particular we have:

PROPOSITION 2 *Let $S > 1$ and $N \geq \frac{\tau+\frac{3}{2}t}{R-W}$ (i.e., sufficiently high that all customers obtain positive surplus). Then, in equilibrium, each intermediary buys and sells $\frac{1}{N}$ units. They pay wage $w = W + \frac{\tau}{2N}$, and sell the output at price $p = W + \frac{\tau+t}{N}$. The markup is*

$$p - w = \frac{2t+\tau}{2N}. \tag{2}$$

PROOF. Since $S > 1$, intemediary interaction cannot be characterized by competition in the

[9] When the monopolist does not have monopsony power, the markup is a function of the supply and demand curve slopes. Also, the independence of the markup to changes in the demand slope is the counterpart of the constancy of the price-cost margin to demand slope changes when marginal cost is constant (in that case, monopsony power is not material).

supplier market, but only by competition in the customer market. Following the steps in the proof of Proposition 1, we obtain the intermediaries total cost to be $TC(Q) = QW + \frac{Q^2\tau}{2}$. Since the market is covered by assumption and the firms are symmetric, we will limit ourselves to symmetric equilibria in which each firm charges the same prices and produces an output level equal to $\frac{1}{N}$. In this case, the price in the supplier market will be $w = W + \frac{\tau}{2N}$. What is left to be determined is the downstream price P. Let us consider a candidate symmetric Nash equilibrium in which each each firm charges a price P_n to the customers, and then derive the value of P_n that is consistent with equilibrium. Given that N is sufficiently high so that consumers earn positive surplus, the market share of an intermediary j is determined by the distance (from his location) of the critical consumers who are indifferent from purchasing from him or from one of his two nearest competitors. This distance, d_c, is determined by the equation $P_j + td_c = P_n + t(\frac{1}{N} - d_c)$ which yields

$$d_c = \frac{1}{2}\frac{P_n - P_j + \frac{t}{N}}{t} \tag{3}$$

Since the firm obtains customers on both sides its location, the total quantity sold, as a function of P_j, is equal to

$$Q(P_j) = \frac{P_n - P_j + \frac{t}{N}}{t} \tag{4}$$

The profit function of the firm is

$$\Pi(P_j) = P_jQ(P_j) - \left(WQ(P_j) + \frac{Q(P_j)^2\tau}{2}\right) \tag{5}$$

Taking the derivative with respect to P_j, setting equal to zero, and recalling that at equalibrium $P_j = P_n = p$, we obtain the equilibrium price to be equal to

$$p = W + \frac{\tau + t}{N} \tag{6}$$

The markup is readily obtained from the difference between p and w. Given that the customer with the lowest surplus is located at a distance $\frac{1}{2N}$away from the intermediary from whom he purchases, for all customers to have positive surplus, it must be that

$$R - t\frac{1}{2N} - W - \frac{\tau + t}{N} \geq 0 \rightarrow \quad (7)$$

$$R - W - \frac{3t + 2\tau}{2N} \geq 0 \rightarrow \quad (8)$$

$$N \geq \frac{\tau + \frac{3}{2}t}{R - W} \quad (9)$$

□

When suppliers are abundant, relative to consumers, the model is equivalent to the Hotelling-Salop model with upward sloping marginal costs. The equilibrium is depicted in Figure 3, in which the depicted functions are as described above with regards to Figure 2. The difference between the two figures is that the equilibrium now results in a complete coverage of the customer Salop circle. The marginal consumer is indifferent between purchasing from two intermediaries and obtains positive surplus (in Figure 2, the marginal consumer was indifferent between purchasing from an intermediary and not purchasing at all and obtained zero surplus). Note that the kink in the demand curve is to the left of the equilibrium quantity, and corresponds to the distance from the intermediary at which the customer obtains zero surplus from purchasing from the competing (neighboring) intermediary.

With regards to the comparative statics, the main difference between the equilibria described in Figure 2 and Figure 3 is that in Figure 3 the demand is twice as responsive to consumer transport costs (or product differentiation). As a consequence, the equilibrium price rises with the consumer transport costs. An observation that persists in the supplier-scarce part of the paper as well, the bottleneck side, in this case the consumers ($S > 1$), get almost all the gain from intermediation - the intermediaries do not get the consumers' reservation utility despite the fact that consumers have no bargaining power. The whole markup comes from heterogeneity between suppliers and heterogeneity between consumers (the standard Hotelling transportation costs).

As we have mentioned before, our model is a generalization of Salop's model. In the stanard Salop (1979) model, we would have $\tau = 0$ - firms compete for consumers, and have supply at constant marginal cost. Substituting $\tau = 0$ into the markup equation from the proposition above, we get the familiar $Markup = \frac{t}{N}$. Now, we turn to our model's equivalent of the kinked equilibrium in Salop's model.

The range of the number of intermediaries, N, considered in the above two propositions leaves a gap. For $N \in [\frac{t+\tau}{R-W}, \frac{\tau+\frac{3}{2}t}{R-W}]$ the customer market is fully covered, but the marginal customer earns a surplus of zero. The equilibrium for these range of N, which is also analyzed in Salop (1979) for the case of constant marginal cost, is given by the proposition below.

PROPOSITION 3 *Let $S > 1$ and $N \in [\frac{t+\tau}{R-W}, \frac{\tau+\frac{3}{2}t}{R-W}]$ each intermediary buys and sells $\frac{1}{N}$ units. They pay wage $w = W + \frac{\tau}{2N}$, and sell it at price $p = R - \frac{t}{2N}$. The markup is*

$$p - w = R - W - \frac{t + \tau}{2N} \tag{10}$$

PROOF. For this range of N, the equilibrium downstream price is obtained by the condition that we leave the marginal customer, located at a distance $\frac{1}{2N}$ from the closest intermediary, with a surplus of zero. This price is given by the equation $R - \frac{t}{2N} - p = 0$ which solving for p yields $p = R - \frac{t}{2N}$. Given that the firms are monopsonistic and the same number of suppliers are employed, the equilibrium wage is the same as that obtained in proposition 2.

□

Note that as in Salop (1979) the effect of increasing customer transport costs differs qualitatively between the environments in Propositions 2 and 3. In the latter proposition, an increase in t leads to a decrease in price, since firms are pricing at the demand kink and an increase in t steepens the demand faced by the intermediaries. In the former proposition, an increase in t increases the price, since firm pricing power is limited by the switching of consumers from one firm to the other and a higher t makes this effect smaller.

For the rest of the paper, we will consider (unless otherwise noted) the more interesting situation in which there is a shortage of suppliers (i.e. $S \leq 1$). The following result provides sufficient conditions (S is large enough and N is low enough), such that the two cities can support N firms acting as if they are each a monopoly intermediary and gives the equilibrium prices. The proof of the proposition follows the same steps as that of Proposition 1 and is omitted.

PROPOSITION 4 *Let $S < 1$. If $\frac{R-W}{t+\tau} - \frac{S}{N} < 0$, then the equilibrium is N local monopsonists/monopolists who buy and sell $\frac{R-W}{t+\tau}$ of the good. They pay wage $w = W + \frac{(R-W)\tau}{2(t+\tau)}$, and sell it at price $p = R - \frac{(R-W)t}{2(t+\tau)}$. The markup is*

$$p - w = \frac{R-W}{2}. \tag{11}$$

Given $S < 1$ the condition for the market to accommodate N firms is that each firm would exchange at most $\frac{S}{N}$ goods in a monopoly setting instead of at most $\frac{1}{N}$. But otherwise, the prices are the same as in 1 (see Figure 4 for an illustration of the equilibrium). If the interemediaries are monopolist/monopsonists, it simply does not matter whether there are more suppliers than customers or vice-versa.

2.3 Competition Between Intermediaries Under Input (Resource) Scarcity

2.3.1 Preliminaries

In this section we consider the case when there is a scarcity of suppliers. There is a critical expression, denoted by $\overline{Q}$ and defined by,

$$\overline{Q} = \frac{R-W}{t+\tau} - \frac{S}{N}. \tag{12}$$

that determines whether the intermediaries are effective monopsonists in the upstream market or whether they are oligopsonists. Notice that the first term in the expression that defines $\overline{Q}$ is the expression for the equilibrium quantity traded by each intermediary when intermediaries are monopsonists (see Proposition 4). The second term is the quantity that each intermediary would trade if the set of suppliers were divided equally among them, as would be the case if intermediaries competed in the upstream market. Thus, the critical quantity, $\overline{Q}$, equals the difference between the monopsonist and oligopsonist equilibrium quantities, and thus a measure of how competitive the market is. It is increasing in the customer reservation utility R and in the number of firms N. It is decreasing in both the transportation costs and the number of suppliers S, since firms can compete less when there are more suppliers. It follows that if $\overline{Q}$ is negative, then firms are local monopsonists, otherwise they are oligopsonists. In this section, and for the remainder of the paper, we assume that $\overline{Q} \geq 0$, so that firms compete in the upstream market.

To reiterate, a big enough $\overline{Q}$ implies more competition - small enough number of firms (N), small enough consumer reservation utility (R), big enough differentiation both upstream and downstream (t and τ), and a big enough supplier to consumer ratio (S). A small enough $\overline{Q}$ implies less competition - big enough number of firms (N), big enough consumer reservation utility (R), small enough differentiation both upstream and downstream (t and τ), and a small enough supplier to consumer ratio (S). $\overline{Q} < 0$ implies a really not competitive market, and takes us back to the monopoly/monopsony case.

We assume that this is a simultaneous move game in which the intermediaries set both the input and output prices at the same time. Given that strategic interaction only takes place in the upstream market, this is equivalent to a simultaneous move game with respect to the upstream price only, in which the intermediaries anticipate the profit they can obtain by trading downstream as a function of the upstream quantity they secure. In the symmetric equilibrium, each intermediary obtains $\frac{S}{N}$ units of inputs (suppliers) and thus sells $\frac{S}{N}$ units in the downstream market. The downstream price that will clear the market is given by the demand curve and is equal to $p = R - \frac{St}{2N}$. One can easily verify that this price exceeds the monopsony/monopoly price given in Proposition 4 if and only if $\overline{Q} \geq 0$. If that were not the case and the downstream price were lower than the monopsony/monopoly downstream price, then the firm's will choose to sell fewer units (since the input price will be strictly higher under oligopsonistic competition than under monopsony). Therefore, the firms would not have been competing in the upstream market, a contradiction. Since the downstream price is determined through the above reasoning, one need only derive the Nash equilibrium in the upstream price to fully characterize the game.

We distinguish two possibilities. In the first possibility, competition for suppliers is sufficiently strong, so that the supplier outside option, W, is not relevant in determining the equilibrium upstream price, i.e., the supplier participation constraint does not bind in the margin. In the second possibility, competition for suppliers is weaker and the equilibrium upstream price depends on the supplier outside option, i.e., the supplier participation constraint does bind. We consider each of these cases in turn.

2.3.2 Strong Upstream Competition

There are two scenarios depending on whether or not the suppliers' outside option, W, is binding. First consider the scenarion where W does not bind. For a symmetric equilibrium, it is clear that the intermediaries must buy up the whole interval of S, so each will buy $\frac{S}{N}$. If they do not, then one of them will buy the leftovers and increase her capacity. On the customer side, since each one of the intermediaries is constrained, and together they can at most barely cover the whole market, each intermediary will act as a local monopolist with capacity K_M ($\frac{S}{N}$ if they are symmetric) and charge the price from the monopoly section $p = R - \frac{K_M t}{2}$. If the price is less than that, then the firm is giving some utility to the marginal customers, and therefore it can increase the price without affecting the quantity sold. If the price increases that means that the firm can decrease the quantity bought, but still price at the Hotelling monopoly price, just for that lower quantity. But then the intermediaries would buy less than K_M. Thus the only thing left to determine is the wage, with the possible profitable deviation being increasing your share at the expense of your neighbors.

Proposition 5 *If* $\overline{Q} \geq \frac{S\tau}{2N(t+\tau)}$*, each intermediary buys and sells* $\frac{S}{N}$ *of the good, sets the wage at* $w = R - \frac{S(\tau+t)}{N}$*, sets the price at* $p = R - \frac{St}{2N}$*, and collects a markup per unit of* $p - w = \frac{S(2\tau+t)}{2N}$.

Proof. In the upstream market there are N big sellers competing for suppliers in Bertrand fashion. Let's imagine two firms, one at 0, and the other at $\frac{S}{N}$, with the second one playing the equilibrium price $\overline{w}$. Then if the first firm is playing w, then for a seller located at $x \in [0, \frac{S}{N}]$ the utility of selling the good to the first firm is $U_1(x, \overline{w}) = w - \tau x$, i.e. as x goes down or as w goes up the utility goes up. The utility for the same supplier of selling to the second firm is $U_2(x, p) = \overline{w} - \tau(\frac{S}{N} - x)$, with the differences that now there is $\overline{w}$ instead of w, and x is with the positive constant, since as x goes up the seller gets closer to the second firm. The outside options are the same so we do not take them into account for now. Solving for the intersection point of $U_1(x)$ and $U_2(x)$:

$$x = \frac{w - \overline{w}}{2\tau} + \frac{S}{2N}. \tag{13}$$

Therefore, since the second firm has two neighbors, the supply that it can secure with a price w is $S(w) = 2x = \frac{S}{N} + \frac{w-\overline{w}}{\tau}$. The total cost will be $TC(w) = S(w) \times w = \frac{Sw}{N} + \frac{w^2 - w\overline{w}}{\tau}$. With the capacity of $S(p)$, the firm will have the total revenue of $TR(S(w)) = S(w) \times R - \frac{S(w)^2 \times t}{2}$, which is equivalent to $\left[\frac{SR}{N} + \frac{(w-\overline{w})R}{\tau}\right] - \left[\frac{tS^2}{2N^2} + \frac{tS(w-\overline{w})}{\tau N} + \frac{t(w-\overline{w})^2}{2\tau^2}\right]$. Hence, $\Pi(p) = TR(S(w)) - TC(w) = \left[\frac{SR}{N} + \frac{(w-\overline{w})R}{\tau}\right] - \left[\frac{tS^2}{2N^2} + \frac{tS(w-\overline{w})}{\tau N} + \frac{t(w-\overline{w})^2}{2\tau^2}\right] - \left[\frac{Sw}{N} + \frac{w^2 - w\overline{w}}{\tau}\right]$. This is concave in w, so the FOC is $\frac{\partial \Pi}{\partial w} = \frac{R}{\tau} - \left[\frac{tS}{\tau N} + \frac{t(w-\overline{w})}{\tau^2}\right] - \left[\frac{S}{N} + \frac{2w-\overline{w}}{\tau}\right]$. To ensure the equilibrium, we need to make $\frac{\partial \Pi}{\partial w} = 0$ and $w = \overline{w}$: $R - \frac{tS}{N} = \frac{S\tau}{N} + w \Longrightarrow$

$$w = \overline{w} = R - \frac{(t+\tau)S}{N}. \tag{14}$$

If $\overline{w} - \frac{\tau S}{2N} \geq W$, then this is the equilibrium price (i.e. the supplier with the least utility is still willing to sell to the intermediary). This happens when $R - \frac{(t+\tau)S}{N} - \frac{\tau S}{2N} - W \geq 0$, or:

$$\{R - W\} \geq \left\{\frac{(t+\tau)S}{N}\right\} + \frac{\tau S}{2N}. \tag{15}$$

We do not use the expression above outside the proof, however it is useful to explain some of the intuition behind the results. If we would simply have the expressions in the curly brackets, then we would get exactly the condition for Proposition 5. However, the inequality above does not necessarily hold with the extra term added. Therefore when this does hold, then we have (14).

The last thing to do is to insure that with this level of the upstream prices the optimal quantity bought/sold remains at $\frac{S}{N}$. We know that $\Pi(p) = (KR - \frac{K^2 t}{2} - \overline{w})K$. Since $\overline{w}$ is constant with respect to K, taking the FOC we get that $K^* = \frac{R-\overline{w}}{t} = \frac{R(t-\tau)}{t^2} + \frac{S(\tau+t)}{tN} = \frac{R(t-\tau)}{t^2} + \frac{S\tau}{tN} + \frac{S}{N}$ which, by the same logic as above, is strictly bigger than $\frac{S}{N}$. Since the profit function is concave in K, and the intermediary cannot get more supply than $\frac{S}{N}$, the optimal quantity stays at $\frac{S}{N}$ □

Two results come right out of the proposition. The equilibrium wage, w, does not depend on the suppliers' reservation utility W, and markup does not depend on the consumer reservation utility R.

Why does the wage w not depend on the reservation utility W of the suppliers? The mathematical intuition is that as the w goes below the point where all the suppliers would accept it, the constraint becomes weaker than the constraint for Proposition 4. The economic intuition is similar to that of the Hotelling model, where if the market is covered, then the prices depend on the travel cost of the consumers, and not on their reservation values.

A hasty explanation for the markup result would have been the standard Hotelling explanation - firms compete, and in a covered market the reservation utility does not matter since all the consumers buy the product anyway. This is not the case here - the market is not covered, and the firms only compete for supplies, while charging monopoly Hotelling prices to the consumers, so surely R must matter. However, since the firms are competing for the scarce suppliers, the suppliers capture all of the consumer surplus not related to the transportation costs, and firms are left with capturing some of the surplus which was left due to consumer differentiation.

Corollary 1 *If* $\overline{Q} \geq \frac{S\tau}{2N(t+\tau)}$, *the markup increases in the number of suppliers* S, *and the customer and supplier transportation costs* t *and* τ. *The markup decreases in the number of firms* N.

The equilibrium in this case is depicted graphically in Figure 5. Workers located a distance of $\hat{x}_w$ from a firm would earn zero surplus from switching to a competing firm; those located further away would earn positive surplus from working for a competing firm. Thus, the supply function is steeper to the right of $\hat{x}_w$. The equilibrium wage is determined by the interection of the supply function and the vertical line at S/N (since all workers are employed in equilibrium). Notice that W^* and $\hat{x}_w$ are determined jointly: increasing W^* would push $\hat{x}_w$ to the left. Similarly, since the equilibrium wage does not depend on the reservation utility of suppliers, an increase in W must push $\hat{x}_w$ to the right. An increase in R increases the equlibrium wage, and thus pushes $\hat{x}_w$ to the left.

2.3.3 Weak Upstream Competition

Now consider what happens if $\overline{Q}$ is greater than zero, but less than the cutoff of $\frac{S\tau}{2N(t+\tau)}$. While the intermediaries are monopolists in the customers' market, they compete for scarce suppliers. The competitive wage (bid) is derived without accounting for the outside option W, which raises two possibilities – either the outside option binds (proposition 6) or it does not (proposition 5).

Proposition 6 *If* $\overline{Q} < \frac{S\tau}{2N(t+\tau)}$, *each intermediary buys and sells* $\frac{S}{N}$ *of the good, pays wage of* $w = W + \frac{\tau S}{2N}$, *and sets the price at* $p = R - \frac{St}{2N}$.

Proof. When condition (15) is not satisfied, then the firms' wages are $w = \overline{w} = W + \frac{\tau S}{2N}$ – firms cannot pay less because some of the suppliers will not be used, and by the condition for competition ($R - W \geq \frac{(t+\tau)S}{N}$), we know that the firms would want to hire at least up to $\frac{S}{N}$. On the other hand the firms do not want to pay more, because in that case they will just be losing money for nothing. □

Corollary 2 *If $\overline{Q} < \frac{S\tau}{2N(t+\tau)}$, the markup decreases in the number of suppliers S, the customer and the supplier transportation costs t and τ, and the outside option of the suppliers' W. The markup increases in the number of firms N and the reservation utility of the customers, R.*

Why does the markup decrease in the supplier/consumer ratio S and increase in the number of firms N? The S ratio is the bottleneck for the firms, so intuitively the more suppliers there are, the less rents the firms have to give up, and similarily we are all used to the markup decreasing in the number of firms. The reason is the same for both of these effects. The wage stays the same, yet now the marginal customer is closer to any given firm, and therefore the price goes up. Just because the markup is going up, does not mean that the profits of an individual intermediary are, since the effect on the volume is in the opposite direction.

Why does the markup (and the profits) decrease in the customer differentiation (transportation cost t)? Because each firm is a constrained monopolist in the customer city, and therefore t only affects the slope of the demand curve, and bears no effect on differentiation. Essentially same reasoning applies to the supplier transportation costs since W binds. The profit function is markup times volume:

$$\Pi = S\frac{R - W}{N} - \frac{S^2(t+\tau)}{2N^2}. \tag{16}$$

Proposition 7 *If $\overline{Q} < \frac{S\tau}{2N(t+\tau)}$, the profits of each of the intermediaries increase in S and decrease in N.*

PROOF. It can be shown that $\frac{\partial \Pi}{\partial S} > 0$ and $\frac{\partial \Pi}{\partial N} < 0$. □

A graphical depiction of this equilibrium is shown in Figure 6. Notice that the marginal supplier earns zero surplus. At equilibrium, the marginal revenue function of the firm crosses its marginal cost at a point of discontinuity. A small increase in the suppliers' reservation utility must lead to an increase in the equilibrium wage; else, the firm would not be able to attract S/N workers. A small increase in the wage is consistent with the equilibrium as the MR function still crosses the MC function at the point of discontinuity. Conversely, an increase in R does not affect the equilibrium wage as the firm does not effectively compete with its neighbors for suppliers (the marginal supplier earns zero surplus from working in the neighboring firm), and thus the increased surplus is appropriate by the firm through higher prices.

2.4 Entry and Fixed Costs

In this section, rather than take the number of active firms in the market as exogenous, we consider it endogenously determined. In particular, we assume that each firm needs to pay fixed costs of $F > 0$ to enter the two markets. This cost is sunk, and therefore will not effect the equilibrium calculations above. The only thing that F affects is the decision of the firm on whether to enter, and the equilibrium number of firms in the industry. This number is achieved when the firms are getting (weakly) positive net profits, yet with the entrance of the next firm the profits will not be enough to cover the fixed costs. We continue to assume that $\overline{Q} \geq 0$ (so there is competition) and that $S < 1$ (there are more customers than suppliers). By the results of the previous section, we have to look at two different scenarios, one where W binds, and the other where it does not.

In the first scenario we have the markup that each firm charges $(R - W - \frac{S(t+\tau)}{2N})$ from proposition 6. We also know that each firm buys and sells $\dfrac{S}{N}$ of the good, since the market is covered. Therefore we have the profits of each firm.

PROPOSITION 8 *If* $\overline{Q} < \frac{S\tau}{2N(t+\tau)}$, *the maximum number of firms that the two cities support* *is*

$$N^* = \frac{S}{F} \times \left[R - W + \sqrt{(R-W)^2 - 2F(t+\tau)} \right] \tag{17}$$

Proof. $\Pi_i = S\frac{R-W}{N} - \frac{S^2(t+\tau)}{2N^2}$. To support the firms this needs to be bigger than the fixed cost F. In the limit, the maximum number of firms, N^*, there can be in the industry is characterized by the following equation:

$$S\frac{R-W}{N^*} - \frac{S^2(t+\tau)}{2N^{*2}} = F. \tag{18}$$

Re-arranging the terms we get: $2F\left[\frac{N^*}{S}\right]^2 - 2(R-W)\left[\frac{N^*}{S}\right] + (t+\tau) = 0$. This is quadratic in $\frac{N^*}{S}$, so we have two roots. Since we are looking for the maximum number of firms supported by the industry, the solution that interests us is $\frac{N^*}{S} = \frac{1}{F} \times \left[(R-W) + \sqrt{(R-W)^2 - 2F(t+\tau)} \right]$.
□

Once we know the profits of each firm, we need to check the zero profit condition and make the fixed costs equal to the profits, deriving the maximum number of firms. Once we have the closed form for the optimal number of firms, we can examine comparative statics.

Corollary 3 *If $\overline{Q} < \frac{S\tau}{2N(t+\tau)}$, the maximum number of firms supported in the equilibrium, N^*, increases with the customer reservation utility R and the number of suppliers S. N^* decreases with the increase in either the fixed costs of entry F, the increase in differentiation (the customer and supplier transportation costs t and τ), or the increase in the supplies' outside option W.*

Now we need to examine what happens if W does not bind. The proof is similar to the previous one, with the difference that now we take the markup from the proposition 5 instead.

Proposition 9 *If $\overline{Q} \geq \frac{S\tau}{2N(t+\tau)}$, the maximum number of firms that the two cities support is*

$$N^* = S \times \sqrt{\frac{2\tau + t}{2F}}. \tag{19}$$

PROOF. The markup that each firm will charge is $\frac{S(2\tau+t)}{2N}$. Therefore the profit is $\frac{S^2(2\tau+t)}{2N^2}$. Similarly to the previous proof, make this equal to F, and find N^*. □

COROLLARY 4 *If* $\overline{Q} \geq \frac{S\tau}{2N(t+\tau)}$*, the maximum number of firms supported in the equilibrium,* N^**, increases with the number of suppliers* S*, and the customer and supplier transportation costs* t *and* τ*.* N^* *will decrease with the increase in the fixed costs of entry* F*.*

Consider the benefits that the customers and the suppliers will receive from living in the two markets. These can be calculated from the results of Propositions 5 and 6, which describe the outcome of the competitive game.

COROLLARY 5 *Total of customer benefits is* $B^C = \frac{S^2 t}{4N}$ *and the benefit for a supplier is* $B^W = \frac{S\tau}{4N}$*, if* $\overline{Q} < \frac{S\tau}{2N(t+\tau)}$*, and* $B^W = R - \frac{S(\tau+t)}{N} - \frac{S\tau}{4N}$*, if* $\overline{Q} \geq \frac{S\tau}{2N(t+\tau)}$.

Note that the customers strictly benefit if there are more suppliers. Now consider a possibility that the suppliers' can move to/away from their city. To move in they need to buy a house for $H > 0$, (like F for firms). Also, let's assume that not enough suppliers enter to affect the assumptions in the previous paragraph.

First, let the suppliers decide whether they want to live in the city, and then let the intermediaries pick locations later. Then the suppliers do not know where will the intermediaries be located, they would just know the expected benefits. Then just enough suppliers live in the city so that the expected benefits just cover the costs of buying a house (assuming suppliers are risk-neutral).

COROLLARY 6 *With free entry of suppliers with a fixed cost of* H*, the maximum amount of suppliers that the cities can support is* $S^* = \frac{4HN}{\tau}$*, if* $\overline{Q} < \frac{S\tau}{2N(t+\tau)}$*, and* $S^* = 4N\frac{R-H}{S(5\tau+4t)}$ *,if* $\overline{Q} \geq \frac{S\tau}{2N(t+\tau)}$.

3 Elastic Supply

So far we have assumed that all the suppliers have unit supply - either they supply their one unit or not, and the most the firms can get out of the suppliers is S. In this part of the paper we examine the case where the supply is elastic. First, we have to decide on an elastic supply function. In the previous sections, the supply function for a given supplier x units away from the firm was

$$s(w, W, x) = \begin{cases} 1 & (w \geq W + \tau x) \\ 0 & (w < W + \tau x) \end{cases} \tag{20}$$

The most straight forward way to extend the definition to make the supply function of each individual supplier elastic is

$$s(w) = \sigma w + 1, \tag{21}$$

where σ is the elasticity, and w the wage paid as before - the higher the wage is, the more each supplier is going to supply. $\sigma = 0$ implies we are in the inelastic world of previous sections. With a small enough σ, the supply is going to still be sufficiently inelastic, so that the firms do not cover the whole consumer circle in equilibrium, as in the section above. Then the reasoning above holds, although the equations might look less appealing. We are going to concentrate on the case where firms hike up the wages enough so that overall supply is equal to 1, or in other words equal to demand.

From before (equation 13), we know that with unit supply, the supply of each firm can be described by $S = \frac{w - \overline{w}}{\tau} + \frac{S}{N}$, where w is the wage of the firm, and $\overline{w}$ is the wage of the competitors. Building in the new elastic supply function, we get

$$S(w) = \left(\frac{w - \overline{w}}{\tau} + \frac{S}{N} \right) (\sigma w + 1). \tag{22}$$

The supply function is not the only thing that changes. The whole equilibrium dynamic becomes different. Before, we knew that demand is bigger than supply, and therefore the downstream market is uncovered. That meant that there is monopoly pricing in the downstream market, and there is competition for the suppliers in the upstream market. Now that the downstream market is covered as well, the competition is on both sides of the market. Also, with elastic supply, raising (or lowering) wages means that not only you are stealing (losing) workers from (to) the competitors, but also your already employed workers are working more (less). In general, finding the equilibrium with competition on both sides would not be an easy task, but since the demand is fixed at 1, it becomes much easier. Note, that in a symmetric equilibrium $Supply(w) = \frac{1}{N}$ for each firm. If they get less, then we are back to the uncovered downstream case. If they get more, then some of it is wasted, and that's definitely not an equilibrium. However, this also means that since in the symmetric equilibrium $w = \overline{w}$, $w^* = \frac{1-S}{\sigma S}$. The result is intuitive - the more elastic the individual supply is, the less firms need to pay, and similarily, the more producers there are relative to consumers, the less the firms need to pay. Also, once we have pinned down the wage, it becomes easy to find the price charged to the consumers - we just have to make sure that the firms do not have any incentive to deviate.

Proposition 10 *If the individual supply is $\sigma w + 1$, $S < 1$, and the downstream market is covered, then in a symmetric equilibrium the firms pay the wage of $w^* = \frac{1-S}{\sigma S}$, charge consumers price of $p^* = \frac{1-S}{\sigma S} + \frac{t}{N} + \frac{\tau S}{N+\tau\sigma S^2}$, resulting in a markup of*

$$MU_{elastic} = \frac{t}{N} + \frac{\tau S}{N + \tau\sigma S^2}. \tag{23}$$

Proof. The outline of the proof is as in the paragraph above. First, let's find the optimal wage for a given price, assuming every other firm pays wage of $\overline{w}$, and charges consumers $\overline{p}$. Then for given everything, the demand is going to be as before (as in 3), $D = \frac{\overline{p}-p}{t} + \frac{1}{N}$, and supply is $Supply(w) = \left(\frac{w-\overline{w}}{\tau} + \frac{S}{N}\right)(\sigma w + 1)$. By the arguments from before, we know that for a firm which optimizes demand must be equal to supply, which gives us

$$\frac{\overline{p}-p}{t}+\frac{1}{N}=\left(\frac{w-\overline{w}}{\tau}+\frac{S}{N}\right)(\sigma w+1), \tag{24}$$

implicitly differentiating with respect to p we get

$$\left[\frac{1}{\tau}(\sigma w+1)+\sigma(\frac{w-\overline{w}}{\tau}+\frac{S}{N})\right]\frac{\partial w}{\partial p}=-\frac{1}{t} \tag{25a}$$

$$\frac{\partial w}{\partial p}=-\frac{1}{t\left[\frac{1}{\tau}+\sigma(\frac{2w-\overline{w}}{\tau}+\frac{\sigma S}{N})\right]}. \tag{25b}$$

Now we can plug in $\frac{\partial w}{\partial p}$ from the 25b into the profit maximization

$$\Pi(w)=(p-w(p))Demand(p). \tag{26}$$

Differentiating with respect to p, we get

$$\frac{\partial \Pi}{\partial p}=(1-\frac{\partial w}{\partial p})Demand(p)+(p-w)\frac{\partial Demand(p)}{\partial p} \tag{27}$$

Since this is a symmetric equilibrium, we know that $Supply(w)=\frac{1}{N}$, $p=\overline{p}$, and $w=\overline{w}=\frac{1-S}{S\sigma}$, and since this is the optimal price and wage, $\frac{\partial \Pi}{\partial p}=0$, since clearly $\frac{\partial^2 \Pi}{\partial p^2}<0$. This gets us[10]

$$p^*=\frac{1-S}{S\sigma}+\frac{t}{N}+\frac{\tau S}{N+\tau\sigma S^2}. \tag{29}$$

□

[10] Skipped steps from the derivation:

$$(1-\frac{\partial w}{\partial p})\frac{1}{N}-\frac{1}{t}(p-w)=0, \tag{28a}$$

$$p=w+\frac{t}{N}(1-\frac{\partial w}{\partial p}), \tag{28b}$$

$$p=\frac{1-S}{S\sigma}+\frac{t}{N}+\frac{1}{N\times(\frac{1+\frac{1}{S}-1}{\tau}+\frac{S\sigma}{N})}. \tag{28c}$$

Corollary 7 *Markup (and profit) increases in consumer and supplier differentiation (t and τ), and decreases in the number of firms and the elasticity of supply (N and σ).*

The result of this section is that if the bottleneck part of the market (suppliers if $S < 1$) is elastic enough, then the comparative statics are as one would expect - markup increases in both sides' differentiation, and decreases with the number of firm and elasticity of the bottleneck's side supply. If the bottleneck supply is not elastic enough, then it does not make any difference to the comparative statics from the sections above. The inelastic supply and demand per consumer/supplier is a convenient assumption that we make in all the other sections, and this section shows that it does not make a difference as far as the comparative statics are concerned.

4 Extension: Matching in a Two-Sided Market

To compare with the two-sided market literature we look at a closely related two-sided matching model. Examples of matchmaking markets, according to Spulber (2006), range from postal systems to yellow pages to brokers to internet search engines. We examine a model where there are matchmakers, and two sides to be matched – call them producers and consumers. The matchmakers are horizontally differentiated along one dimension with respect to producers, and in another with respect to consumers. An application, which we will use to illustrate the concepts in this section, is that of headhunters looking for employees for their corporate clients. Headhunters differ in one set of attributes to their clients, and in another set of attributes to the potential employees that the headhunters want to match with the clients.

There is a mass of $S < 1$ potential employees with their preferences/skills distributed along a Salop circle. There is a mass of 1 of clients and their problems (locations) are distributed along another Salop circle. There are N headhunters symmetrically located on each of the circles, with the location representing the type of skills they specialize in. Once matches are made, the joint benefit to the two sides is R. A potential employee incurs a cost of $\tau \times d_e$ to go to a headhunting firm d_e units away from him. This cost can be viewed as the costs of transacting with a headhunter who is not a specialist in evaluating that particular worker's skills, leading to more difficult negotiatins, placement, and possibly an inappropriate match requiring the worker to learn new skills. The client incurs a cost of $t \times d_c$ to hire a worker from a headhunting firm d_c units away – the matchmaker might not provide ideal conditions for the client. Potential employees can find a job outside of the headhunting firm for a wage of W.

The headhunters' objectives are to maximize their own profit. They charge each client a fee of F_c, and each employee a fee of F_e. Once the clients and the employees pay the registration fees, they obtain perfect information about all others who had paid the fees, and match efficiently (i.e. the first pair is the client and employee who gain the most overall from matching, the last is the pair who gains the least). Once matched, the pair uses Nash bargaining to split the value created, which is R, since the travel costs are sunk by that point. Notice that the outside option of the employee is W and the outside option of the client is 0 and therefore the payoffs after the match (suppressing the fees and travel costs) for the employee and the client are, respectively, $\frac{R+W}{2}$ and $\frac{R-W}{2}$.

Also note that with the headhunters setting prices charged to the clients and wages paid for the workers, their problem becomes similar to the problem of the monopolist intermediaries in Proposition 4. Denote d_e and d_c the distances such that, respectively, the employee and the client at these distances are indifferent between using the matchmaker or not. For a monopolist matchmaker it is clear that $d_e = d_c$. From definition, $d_c = \frac{\frac{R-W}{2} - F_c}{t}$ and $d_e = \frac{\frac{R-W}{2} - F_e}{\tau}$, therefore by invoking equality and substituting into the profit function, we can derive the optimal fees.

PROPOSITION 11 *If $\overline{Q} < 0$, the headhunters behave as if they are local monopolists. Headhunters charge clients and employees fees of, respectively, $F_c = \frac{R-W}{2}\frac{\tau}{\tau+t}$, and $F_e = \frac{R-W}{2}\frac{t}{\tau+t}$, while facilitating $\frac{R-W}{t+\tau}$ matches.*

Proof. Invoking $d_e = d_c$, we get

$$F_c = \frac{\frac{R-W}{2}(\tau - t) + tF_e}{\tau}. \tag{30}$$

Given this expression, and since the profit of the monopolist matchmaker is

$$\Pi(F_c, F_e) = 2d_e(F_c + F_e), \tag{31}$$

we use (30) to take the FOC of the equation above with respect to F_e, resulting in

$$F_e = \frac{R-W}{2}\frac{t}{\tau+t}, \tag{32}$$

and as a result of (30)

$$F_c = \frac{R-W}{2}\frac{\tau}{\tau+t}. \tag{33}$$

Then $d_c = \frac{\frac{R-W}{2} - \frac{R-W}{2}\frac{\tau}{\tau+t}}{t} = \frac{R-W}{2(\tau+t)}$, and therefore the monopolist facilitates $\frac{R-W}{\tau+t}$ matches. □

The monopolist sets the price such that both the marginal client and the marginal employee are indifferent between being matched or not. Moreover, the monopolist needs to make sure that the number of clients who want to get matched is equal to the number of employees who want to get matched. Note that the total fee and the number of matches is exactly the same as in Proposition 4. This is because we have the same underlying model, and it does not matter how do the employee and the client split the surplus, as long as the marginal pair would end up receiving their outside option. Also, as in proposition 4, if the number of headhunters multiplied by the optimal number of monopoly matches exceeds the supply of employees, we get the result that there is competition. However, it is not obvious whether the results hold if the marginal pair receives more than their outside options, as is the case with competition. Again as in Section 2.3 we have two results, depending on whether the outside option is binding. The intuition is the same as in the proofs of propositions 6 and 5, except now there is Nash bargaining between the employee and the client, and they split the travel costs equally. Consider what happens if W does not bind.

Proposition 12 *If* $\overline{Q} \geq \frac{S}{4N}$*, the headhunters charge clients a fee of* $F_c = \frac{R-W}{2} - \frac{St}{2N}$*, charge employees a fee of* $F_e = \frac{S(t+\tau)}{N} - \frac{R-W}{2}$*, and collect a fee per match of* $F = F_c + F_e = \frac{S(t+2\tau)}{2N}$*.*

Proof. As in proposition 5, the employees are constrained, and therefore the headhunters end up being monopolists in the client markets. Therefore we know that given a headhunter has secured $2d$ employees, the headhunter has clients with the distance of up to d away. These marginal clients will get zero surplus, and (since they split the gross surplus from the match with the employers), the fee will be $F_c = \frac{R-W}{2} - td$. Now that we know the clients fee, we fix the employee fee that the neighbor headhunters charge at F_e^*, and let the active headhunter charge some F_e. Then an employee d units away from the active headhunter will be indifferent to coming to the neighbor headhunter for services if and only if

$$\frac{R+W}{2} - \tau d - F_e = \frac{R+W}{2} - \tau(\frac{S}{N} - d) - F_e^*, \tag{34}$$

this simplifies to:

$$d = \frac{F_e^* - F_e}{2\tau} + \frac{S}{2N}. \tag{35}$$

The active headhunter will make $2d$ matches, and therefore collect a profit of

$$\Pi(F_e) = 2d(F_e) \times [F_c(F_e) + F_e]. \tag{36}$$

Notice that $\frac{\partial d}{\partial F_e} = -\frac{1}{2\tau}$, and $F_c = \frac{R-W}{2} - td = \frac{R-W}{2} - t\frac{F_e^* - F_e}{2\tau} - \frac{tS}{2N}$. Differentiating profit with respect to F_e:

$$\frac{\partial \Pi}{\partial F_e} = -\frac{1}{\tau}\left[\frac{R-W}{2} + t\frac{F_e - F_e^*}{2\tau} - \frac{tS}{2N} + F_e\right] + (\frac{F_e^* - F_e}{\tau} + \frac{S}{N})(\frac{t}{2\tau} + 1). \tag{37}$$

Notice that the second derivative is always negative, and therefore the second order conditions are satisfied. Invoking the symmetry condition ($F_e^* = F_e$) and setting the derivative of

$\Pi(F_e)$ equal to zero, it follows that:

$$F_e^* = \frac{(t+\tau)S}{N} - \frac{R-W}{2}. \tag{38}$$

Therefore $F_c^* = \frac{R-W}{2} - \frac{St}{2N}$, and the total fees are $\frac{S(t+2\tau)}{2N}$ per match.[11] □

Now consider what happens if W does bind, as in Proposition 6.

PROPOSITION 13 *If $\overline{Q} < \frac{S}{4N}$, the headhunters charge clients and employees fees of, respectively $F_c = \frac{R-W}{2} - \frac{St}{2N}$, and $F_e = \frac{R-W}{2} - \frac{S\tau}{2N}$, and collect a fee per match of $F = F_c + F_e = R - W - \frac{S(t+\tau)}{2N}$.*

PROOF. As in the proposition 6, if condition 39 is not satisfied, then the headhunters have to charge $F_e = \frac{R-W}{2} - \frac{S\tau}{2N}$, so that the marginal employee gets exactly W. □

COROLLARY 8 *In the competitive equilibria, the total fee collected by headhunters with Nash bargaining between a client and an employee is the same as the markup collected by an intermediary in the first part.*

5 Conclusion

The nature of firms as intermediaries, widely known but not often incorporated in formal models, is crucial to understand market outcomes in strategic environments. We show that the nature of competition depends on whether an environment is relatively supplier-rich or consumer-rich. The strategic interaction of firms in environments in which suppliers predominate is akin to those of oligopolistic monopsonists; when consumers are more plentiful than suppliers, firms are effectively oligopsonistic monopolists.

[11] As in the proof of propositions 5 and 6, we need to check for the outside option not to be binding. Consider the employee who is the worst off, i.e. the one located $\frac{S}{2N}$ from the nearest headhunter. His utility after the match will be $\frac{R-W}{2} - \frac{S(t+\tau)}{4N} + W - F_e^* = R - \frac{S(t+\tau)}{4N} - \frac{S(t+\tau)}{4N} - \frac{3S(t+\tau)}{4N} = R - \frac{5S(t+\tau)}{4N}$. This needs to be bigger than W. Solving, we have

$$\overline{Q} \geq \frac{S}{4N}. \tag{39}$$

The effect of changes in the market environment on market outcomes depends on whether consumers or suppliers predominate, and are not always the same as those of standard models. Changes in the economic environment that are large enough to lead to a switch between the two regimes, also lead to qualitative changes in firm behavior, for example the side which is scarce enough, derives most of the gains from intermediation, despite having no bargaining power - taking prices/wages as given. Though our results are illustrative, they suggest that a more complete characterization of strategic interaction in input and output markets can yield important insights.

References

[1] Armstrong, M. (2006). Competition in Two-Sided Markets. *Rand Journal of Economics* **37**, 668-691.

[2] Bhaskar, V., A. Manning and T. To (2002). Oligopsony and Monopolistic Competition in Labor Markets *Journal of Economic Perspectives* **16(2)**, 155-174.

[3] Caillaud, B. and B. Jullien (2003). Chicken & Egg: Competition among Intermediation Service Providers. *Rand Journal of Economics* **34(2)**, 309-328.

[4] Esö, P., Nocke, V. and L. White (2007). Competition for Scarce Resources. *Working paper.* Northwestern University.

[5] Fingleton, J. (1997). Competition among Middlemen when Buyers and Sellers can Trade Directly. *Journal of Industrial Economics* **45 (4)**, 405-427.

[6] Gehrig, T. (1993). Intermediation in Search Markets. *Journal of Economics & Management Strategy* **2**, 97-120.

[7] Hendricks, K. and R. McAfee (2007). A Theory of Bilateral Oligopoly. *Working paper.* California Institute of Technology.

[8] Hotelling, H. (1929). Stability in Competition. *Economic Journal* **39**, 41-57.

[9] Loertscher, S. (2007). Horizontally Differentiated Market Makers. *Journal of Economics & Management Strategy* **16**, 793-825.

[10] Marx, L. and G. Shaffer (2008). Buyer Power, Exclusion, and Inefficient Trade. *Working paper.* Duke University.

[11] Reisinger, M. and M. Schnitzer (2007). A Model of Vertical Oligopolistic Competition. *Working paper.* University of Munich.

[12] Rust, J. and G. Hall (2003). Middlemen versus Market Makers: A Theory of Competitive Exchange. *Journal of Political Economy* **111(2)**, 353-403.

[13] Salop, S.C. (1979). Monopolistic Competition with Outside Goods. *Bell Journal of Economics* **10(1)**, 141-156.

[14] Spulber, D. F. (1996). Market Making by Price-Setting Firms. *Review of Economic Studies* **63**, 559-580.

[15] Spulber, D. F. (1999). *Market Microstructure: Intermediaries and the Theory of the Firm*. Cambridge University Press: Cambridge, MA.

[16] Spulber, D. F. (2002). Market Microstructure and Incentives to Invest. *Journal of Political Economy* **110**, 352-381

[17] Spulber, D. F. (2006). Networks and Two-Sided Markets. To appear in: *Economics and Information Systems, Handbook in Information Systems*. Elsevier: Amsterdam, Netherlands.

[18] Stahl, D. O. (1988). Bertrand Competition for Inputs and Walrasian Outcomes. *American Economic Review* **78(1)**, 189-201.

[19] Stigler, G.J. (1946). The Economics of Minimum Wage Legislation. *American Economic Review* **36(3)**, 358-365.