Efficient Delay in Decision Making*

Evangelos Rouskas†

This Draft: April 2009

Abstract

This research places standard price dispersion models in a dynamic perspective and comes up with an explanation about the frequent delay in durables' consumption evidenced by surveys of purchase intent. First, I introduce a dynamic price information clearinghouse setup with endogenous acquisition of information and demonstrate that depending on parameter values there exist equilibria where buyers purchase early on or delay purchases and they are better-, worse-off or indifferent compared to the static benchmark. Second, in a dynamic game with a small number of capacity constrained sellers and a small number of buyers with growing demand I identify unexplored equilibria which, contrary to known results, confirm that in some occasions deliberately putting off consumption at a later time is more beneficial for buyers than the static outcome. Third, I argue that dynamics clearly confer advantages to the supply-side only in underground markets like the retail one for illicit drugs where there exists significant variation in the price/quality ratio.

JEL Classification Codes: L13, D83, D84

Keywords: price dispersion, intertemporal choice, durables, efficient vs. inefficient delay

*Preliminary and incomplete, comments welcome.

†Ph.D. Candidate, Department of Economics, Athens University of Economics and Business, 76 Patission Street, Athens 10434, Greece. Email: `rouskas@aueb.gr`

1 Introduction

Holding back from consumption although in immediate need occurs often and, more importantly, the elapsed time from the moment consumers recognize the need for a product up to the point they actually purchase it may be substantial. For example, survey evidence based on purchase intent reveals that consumers defer the decision to buy a personal computer or an automobile for months or years to come (Morwitz and Schmittlein 1992).[1] High valuation durables, however, offer utility from the time of purchase on and any delay in consumption that is since need recognition long continued seems counterintuitive, if not puzzling.[2]

This paper introduces an intertemporal oligopolistic framework and suggests that putting off expenditures on durables arises as a rational expectations equilibrium. In a class of homogeneous-good models that generate deviations from the one-price pattern, I identify parameter configurations leading to efficient intertemporal substitution of consumption that favors periods ahead. To the best of my knowledge, this is a novel result not properly accounted for theoretically.

To fix ideas, consider a continuum of unit-demanders in a frictionless environment with price-setting, constant returns-to-scale firms. The unique symmetric (degenerate) equilibrium requires the price be equal to the average cost and whenever the area of relevance touches a multi-period durables setup, it pays buyers to incur the expediture in the very first period, since for constant prices throughout the higher the valuation for the product, the higher the net utility captured. At first view, this conlusion is not challenged once one allows for informational considerations across periods that engender price dispersion.[3] Standard and more modern treatments of oligoly with costly access to price information on the demand-side (Varian 1980, Stahl 1989, Janssen and Moraga-González 2004) bear also the implication that net consumer surplus is increasing in product valuation.[4]

[1]Affordability misperception does not necessarily have a penetrating effect due to easy access to forms of credit. In point of fact, even assuming away bank lending, intra-family transfers do play a significant role in alleviating liquidity concerns and can nowadays be formally acknowledged in the sense that go-between firms emerge. (For instance, refer to the following quote from *The Wall Street Journal's* interview with Richard Branson on January 8, 2008: "In the U.S. Virgin Money is charting a new path in micro-finance, acting as a middleman to formalize lending relationships between friends or family. Such personal loans can have tax benefits and, because the parties are related, have low rates of default".)

[2]In no way can frequent delay be connected to macroeconomic uncertainty.

[3]Dispersed price equilibria may come along with strict capacity constraints. In what's next, these supply-side concerns are accommodated too.

[4]If anything deserves a bit of attention it is the low search equilibrium in Janssen and Moraga-González (2004). In this equilibrium comparative statics dictate that the net positive surplus per active consumer is the same for any valuation; however, the lower the valuation the lower the percentage of the population that is

I use a price information clearinghouse setup that admits positive access costs in the cross section of buyers. That is, buyers can pay the clearinghouse cost and purchase at the lowest price in the market or they can visit a firm randomly at no cost and pay the corresponding price. This notwithstanding, the Diamond Paradox does not obtain should one work through a solution whereby buyers randomize with regard to accessing the clearinghouse or purchasing at random. As is customary in clearinghouse models I assume an oligopolistic supply structure and a continuum of agents on the demand-side each with unit demand. Interesting results emerge by the distinction between static and dynamic analysis and I briefly point them out. These results do not depend on the number of firms and therefore sticking to the duopoly case comes without loss of generality. In fact the duopoly case has an advantage that will be highlighted shortly and the same holds for the constant returns-to-scale technology that applies hereafter.

Well, comparative statics demonstrate that someone contemplating to purchase under a low valuation faces two unfavorable effects compared to someone with a high valuation. The first effect is that the valuation in itself is lower and the second effect is that the expected price decreases with the valuation. The lower the valuation, the lower the probability that buyers with a non-zero clearinghouse fee will do pay the fee.[5] In the dynamic analysis consumers entertain the possibility of buying their unit in the course of another period in the future. Realistically, a structure that fits perfectly a durable-good market requires the valuation be lower in the second period and adding the assumption of a lower than unity discount factor amounts to a third adverse effect in regard to putting off consumption at a later time.

Nonetheless, I show clearly that in the dynamic two-period model, depending on parameters, there do exist equilibria with delay in consumption that give a net payoff to buyers higher than the payoff that corresponds to commitment to buying in the first period (i.e. higher than the payoff that corresponds to the static game). Moreover, introducing a fraction of shoppers in the total population (i.e. introducing buyers with zero access costs to the clearinghouse) makes a difference in the dynamic model, unlike comparative statics. In the one-shot game when the mass of shoppers increases the expected price remains unaltered due to the fact that buyers with a non-zero clearinghouse fee will decrease the probability that they pay the fee in equilibrium. In the dynamic game, the mass of shoppers, if sufficiently high, gives rise to some equilibria which guarantee the maximum efficiency for buyers.

Literature My work adds to several strands of literature. First and foremost, it is about price dispersion and its relation to information and search; a field that has lately comprehensively

active. Hence, any buyer would prefer to purchase the durable straightway.

[5]That says the dispersion in prices for expensive products is higher than for cheap ones.

been surveyed by Baye *et al.* (2006). By focusing to the duopoly case, I drive a wedge between clearinghouse and search models. What I use is a duopolistic constant returns-to-scale version of the Varian (1980) model and this comes in handy since in terms of qualitative results the analysis does pertain to the duopolistic version of Burdett and Judd's (1983) fixed-sample search model recently advanced by Janssen and Moraga-González (2004); with more than two firms, for example, there is no exact matching. That equivalence seems not that much of a novelty in one-shot environments; however it does make a good case for comparisons in respect to intertemporal choice. If the first price quote is free then dynamics matter. Someone in the first period may visit one firm at random and, roughly, if the price encountered is that high that the net surplus today falls short of the outside option to purchase the product tomorrow then delay occurs. This behavior in practice alters the effective valuation for consuming the product in the first period. Since the lower the valuation the higher the mass of endogenously uninformed buyers, a lower effective valuation in the first period raises the expected price in the first period and it turns out that this expected price is higher than the expected price in the static game. In the absence of shoppers in the economy dispersed price dynamics hurt buyers yet with shoppers dispersed price dynamics benefit buyers. The reason is that with only shoppers being active in the second period one gets the Bertrand Paradox and this implies that for shoppers the effective valuation of consuming the product in the first period is even lower than the effective valuation for non-shoppers. Simply put, sellers would be forced to lower prices too much in the first period in order to avoid delay in consumption by shoppers. In the dynamic equilibrium, when agents are patient and the forgone utility by not consuming in the first period is low, shoppers and non-shoppers are non-synchronized across periods with non-shoppers purchasing early on at a price higher than the one corresponding to the static game and with shoppers delaying consumption and purchasing at the marginal cost. Given the proper conditions, a higher mass of shoppers squeezes profits to a higher degree.

Second, this paper is part of an emerging literature that examines oligopolistic pricing with intertemporal capacity constraints and costless search for price information. I am not aware of any other structure fitting these standards than the one developed by Biglaiser and Vettas (2007). They employ a two-period finite buyer economy where firms sell a durable and have limited capacity to satisfy demand; however they downplay delay in durables' consumption since in their dynamic dispersed price equilibrium buyers are worse off compared to the one-shot game. That result comes to favor the comparative statics implications of oligopolistic models with costly price information. Nevertheless, I work through Biglaiser and Vettas again with their assumptions being held intact and demonstrate that their result is not robust. I identify a new equilibrium whereby buyers delay consumption and in spite of that their net payoff is equal to the static payoff while for arbitrarily small parameter perturbations buyers' net payoff in the new equilibrium is superior to the static one.[6]

Third, answers about the observed delay in consumer decision making are much sought after in the marketing literature (see Greenleaf and Lehmann 1995 and the references therein). That literature has proposed a delay typology that applies particularly to high-priced products and addresses delay which is substantial ("at least one month elapses between need recognition and purchase") but does not continue indefinitely and eventually leads to a purchase. For example, individuals may delay consumption because they have "too many other things to do", because they "feel shopping for the product is an unpleasant task that they wish to avoid" or because they want to "gather more information on alternative product offerings". However, the model I present is an unexplored alternative free of any product differentiation concerns or *ad hoc* behavioral rules that seems relevant for a wide class of high-priced items and is based solely on imperfect information about prices. By allowing buyers to get one price quote at a point in time and possibly not purchasing the product at that point, I try to capture the micro-foundations of delay in consumption. Shlightly paraphrasing Greenleaf and Lehmann (1995), "a consumer might spend [sometime] searching for [price] information on fax machines and then purchase in the same day, or spend [sometime] in March, [sometime] in July, and purchase in August".

More to be added.

The paper is organized as follows. In Section 2, I present the static benchmark setup. Section 3 solves for the equilibrium in the dynamic clearinghouse model, and in view of the main findings, Section 4 discusses similarities with dynamic markets where price dispersion arises through capacity constraints. In Section 4, I also sketch a counterargument to recent research about price dispersion in underground markets with addictive consumption. Section 5 concludes. Unless stated in the main body of the paper, proofs are relegated to the Appendix.

[6] Somehow related to this contribution of mine is a recent paper by Deneckere and Peck (2005) that studies price dispersion due to demand uncertainty. Deneckere and Peck use a perfectly competitive structure on the supply-side and in effect introduce dynamics in Prescott's (1975) "hotels" model allowing consumers to "delay their purchases in the hopes of clinching a better deal in the future". The point is that dynamics restore efficiency whenever consumers are heterogeneous.

2 Detailed Preliminary Results

2.1 Assumptions

I examine an oligopolistic market for a homogeneous product and as a starting point I focus to the static duopoly case. Risk neutral firms compete in prices and each one can supply any quantity demanded subject to a constant returns-to-scale technology with marginal cost normalized to zero. The demand is made up by a continuum of risk neutral consumers in [0,1], each willing to buy at most one unit of the product as long as the price does not exceed the common reservation value $v > 0$. However, there is not full transparency with regard to price quotes. In order to identify the lowest price in the market, a buyer must first visit a price information clearinghouse by paying the corresponding fee $c_s \in (0, v)$. (Implicitly, it is assumed that firms can list their prices in the clearinghouse costlessly and both do so with probability equal to one.) Otherwise, a buyer may get only one price quote for free.

2.2 No Price Dispersion Equilibrium

Firms charging the monopolistic price with all buyers being uninformed is an equilibrium. This equilibrium best exemplifies the Diamond Paradox. Indeed, when buyers expect that all firms charge v, then no one enters the clearinghouse since the expected benefits from doing so are zero while the cost is strictly positive. This works the other way around too. If firms hold the belief that none has full information, then it is in their best interest to charge v with probability equal to one.

The non-existence of an equilibrium where all buyers access the clearinghouse and firms price aggressively at the marginal cost is another manifestation of the impossibility of informationally efficient markets. More precisely, a buyer may expect that all others in the economy have full information. This translates to prices not varying across sellers and consequently inasmuch as this expectation is shared among all buyers anyone would be better off by purchasing at random. Then again the equilibrium in prices is that both firms charge v, while no one engages in price comparison.

Conclusion 1 *In the duopolistic clearinghouse model with the assumptions put forward in Subsection 2.1, there exists only one equilibrium with no price dispersion which corresponds to none of the buyers incurring the clearinghouse cost and to firms charging the monopolistic price v.*

2.3 Price Dispersion Equilibrium: The Static Benchmark Case

I consider buyers who randomize between accessing the clearinghouse or not. Suppose that a buyer does believe that all others in the economy pay the clearinghouse fee with $prob = \theta \in (0, 1)$. Then this corresponds to the expectation that θ is the population with full information, while $1 - \theta$ is the population that purchases at the average price and the following Lemma obtains.

Lemma 1 *When θ denotes the fully informed population and $1 - \theta$ are the buyers that purchase randomly, firms use mixed strategies in prices where the common minmax profit equals $\frac{1}{2}(1-\theta)V$ and the atomless c.d.f. is $F(p) = \frac{1+\theta}{2\theta} - \frac{(1-\theta)v}{2p\theta}$ with support $[\frac{(1-\theta)v}{1+\theta}, v]$.*

However, in order for the c.d.f. in Lemma 1 to be indeed a rational expectations equilibrium, the following condition must be satisfied.

Condition 1 $E[p] = E[min(p_1, p_2)] + c_s$

This condition says that the said buyer must be indifferent between purchasing randomly with $prob = \theta$ and thereby paying the average price or getting full information with $prob = 1-\theta$ and purchasing at a generalized price equal to $E[min(p_1, p_2)] + c_s$. Computing the expected price and the expected lowest price (see A.1) and rearranging terms, one can express the expected benefit from paying the clearinghouse fee in the LHS and the cost in the RHS as follows:

$$\frac{(1-\theta)v}{2\theta} \ln\left(\frac{1+\theta}{1-\theta}\right) - \frac{(1-\theta)v}{\theta} + \frac{v}{2}\left(\frac{1-\theta}{\theta}\right)^2 \ln\left(\frac{1+\theta}{1-\theta}\right) = c_s \tag{1}$$

Proposition 1 *There exists at least one and at most two values of θ that satisfy Equation (1). That is, when all consumers have to pay a positive fee in order to access the clearinghouse, prices are dispersed according to the c.d.f. F(p) defined in Lemma 1.*

Indeed, the LHS in Equation (1) is a stricty concave function with respect to θ, which achieves a positive maximum value. When c_s is sufficiently low, a line parallel to the horizontal axis (which represents values of θ) at the level of c_s intersects the concave function two times (one intersection point is in the increasing and the other in the decreasing part of the function). However, it can be argued that only the intersection point in the decreasing part of the function constitutes a stable equilibrium. This is so because when $\theta < \theta_{stable}$, the expected benefit from getting access to full information is greater than the clearinghouse fee and this leads buyers to increase the probability θ. Whenever $\theta > \theta_{stable}$, the expected benefit is lower than the cost and subsequently θ tends to decrease. Unlike that, at the intersection point in the increasing part of the function, when $\theta < \theta_{non-stable}$ the expected benefit from getting full information is lower than the cost and this decreases θ. Also, for $\theta > \theta_{non-stable}$, θ tends to increase.

Collorary 1 *For a given value of c_s, θ_{stable} is increasing in v.*

When v increases, the function in the LHS of Equation (1) moves upwards and the stable equilibrium point moves to the right. In practice, the lower the valuation for the good, the lower the net consumer surplus.

Collorary 2 *No matter what the total of individuals on the demand-side, the equilibrium c.d.f. remains unchanged and follows directly from Proposition 1. Put differently, the net surplus per buyer $v - E(p)$ is the same for any population mass.*

2.3.1 The Role of Shoppers in the Static Equilibrium Outcome

Let's set *ex ante* that a fraction of the unitary population mass equal to $\lambda > 0$ has access to full information for free. As a consequence, it is now only the mass $1 - \lambda$ that gets access to the clearinghouse with $prob = \theta$ or remains uninformed with $prob = 1 - \theta$. The counterpart of Lemma 1 in the economy with shoppers is the Lemma that follows:

Lemma 2 *With the fully informed population equal to $\lambda + (1-\lambda)\theta$ and the uninformed population equal to $(1-\lambda)(1-\theta)$, firms use mixed strategies in prices where the common minmax profit equals $\frac{1}{2}(1-\lambda)(1-\theta)v$ and the atomless c.d.f. is $F(p) = \frac{1+\lambda+(1-\lambda)\theta}{2[\lambda+(1-\lambda)\theta]} - \frac{(1-\lambda)(1-\theta)v}{2p[\lambda+(1-\lambda)\theta]}$ with support $[\frac{(1-\lambda)(1-\theta)v}{1+\lambda+(1-\lambda)\theta}, v]$.*

The proof resembles the one for Lemma 1, and using Condition 1, equilibrium would require:

$$\frac{(1-\lambda)(1-\theta)v}{2[\lambda+(1-\lambda)\theta]} \ln\left(\frac{1+\lambda+(1-\lambda)\theta}{(1-\lambda)(1-\theta)}\right) - \frac{(1-\lambda)(1-\theta)v}{2[\lambda+(1-\lambda)\theta]}$$

$$+\frac{v}{2}\left(\frac{(1-\lambda)(1-\theta)}{\lambda+(1-\lambda)\theta}\right)^2 \ln\left(\frac{1+\lambda+(1-\lambda)\theta}{(1-\lambda)(1-\theta)}\right) = c_s \qquad (2)$$

$E(p \mid \theta = 0)$	$E(p)$	θ_{stable}	v	λ	v	θ_{stable}	$E(p)$	$E(p \mid \theta = 0)$
No Dispersion	9.3725009536543	0.7391864700931	28	0	40	0.9018950964193	6.4495024132023	No Dispersion
26.622199976157	9.3725009536543	0.7254594422033	28	.05	40	0.8967316804414	6.4495024132023	38.031714251653
22.706046054057	9.3725009536543	0.6739830876164	28	0.2	40	0.8773688705241	6.4495024132023	32.437208648653
17.793255068131	9.3725009536543	0.5653107834885	28	0.4	40	0.8364918273655	6.4495024132023	*non-stable*
15.380572041354	9.3725009536543	0.4783729401862	28	0.5	40	0.8037901928386	6.4495024132023	*non-stable*
non-stable	9.3725009536543	0.3479661752327	28	0.6	40	0.7547377410482	6.4495024132023	*non-stable*
7.6902860206767	N/A	N/A	28	0.8	40	0.5094754820964	6.4495024132023	*non-stable*
2.6994664760955	N/A	N/A	28	.95	40	N/A	N/A	3.8563806801364
N/A	No Dispersion	N/A	28	1	40	N/A	No Dispersion	N/A

Table 1: Numerical illustration of the stable equilibrium value of θ and $E(p)$ for different values of λ with v given and $c_s = 2.8$. None non-shopper paying the clearinghouse fee constitutes a stable equilibrium for sufficiently low and sufficiently high values of λ; the expected price then equals $\frac{(1-\lambda)v}{2\lambda}\ln(\frac{1+\lambda}{1-\lambda})$. In fact, for low positive values of λ there exist multiple dispersed price equilibria.

The numerical analysis in Table 1, however, shows that, when the mass of shoppers is sufficiently low, λ itself has no impact in the determination of the expected price. For example, when λ increases (decreases), Equation (2) ensures that the probability (θ_{stable}) a non-shopper will enter the clearinghouse decreases (increases) that much so as the expected price remains the same. For $\lambda = 0$, θ_{stable} achieves its maximum value. Next Proposition states the full description of the equilibrium prices under a positive mass of shoppers and suggests that Equation (2), if at all, captures only one out of the two stable dispersed price equilibria.

Proposition 2 *In the duopolistic clearinghouse model with the assumptions put forward in Subsection 2.1, there does not exist any equilibrium with no price dispersion when the mass of shoppers λ is positive. However, with $\lambda > 0$, depending on parameters there is a unique dispersed price equilibrium or multiple dispersed price equilibria. For any relative value $\frac{v}{c_s}$ be it low or high, if none of the non-shoppers pays the clearinghouse fee, the expected benefit from paying the fee is lower or higher than c_s depending on λ:*

(i) as long as λ is sufficiently high there exists one equilibrium whereby none of the non-shoppers pays the fee and firms price according to the atomless c.d.f. $F(p) = \frac{1+\lambda}{2\lambda} - \frac{(1-\lambda)v}{2p\lambda}$ with support $[\frac{(1-\lambda)v}{1+\lambda}, v]$,

(ii) as long as λ takes intermediate values there exists only one equilibrium whereby non-shoppers mix with respect to paying the fee or not and firms price according to the c.d.f. in Lemma 2; this equilibrium gives a higher net payoff to consumers while the expected price equals the one derived by Proposition 1,

(iii) for sufficiently low values of λ there exist two equilibria; one whereby none of the non-shoppers pays the fee and firms price according to the atomless c.d.f. $F(p) = \frac{1+\lambda}{2\lambda} - \frac{(1-\lambda)v}{2p\lambda}$ with support $[\frac{(1-\lambda)v}{1+\lambda}, v]$, and one whereby non-shoppers mix with respect to paying the fee or not and firms price according to the c.d.f. in Lemma 2.

There is no equilibrium without price dispersion due to the existence of shoppers that generates incentives for undercutting the rival firm. Like in Proposition 1, here it goes that Equation (2) delivers one stable (θ_{stable}) and one unstable ($\theta_{non-stable}$) equilibrium, when λ takes sufficiently low values. Nevertheless, for any $\theta \in [0, 1]$ that satisfies $\theta < \theta_{non-stable}$ and $\theta > \theta_{stable}$, it holds that the expected benefit from paying the fee is lower that the fee itself. To the degree that the population is heterogeneous in regard to the fee, an endogenous equilibrium is well defined as long as

$$\underbrace{0}_{shoppers'\ fee} < \underbrace{E(p) - E[min(p_1, p_2)]}_{expected\ benefit\ from\ paying\ the\ fee} < \underbrace{c_s}_{non-shoppers'\ fee} \tag{3}$$

Since any $\theta \in [0,1]$ that satisfies $\theta < \theta_{non-stable}$ tends to decrease and any $\theta > \theta_{stable}$ tends to increase, the stable equilibrium values of θ properly defined are $\theta_{STABLE-1} = 0$ and $\theta_{STABLE-2} = \theta_{stable}$. When λ is sufficiently high, $\forall \theta \in [0,1]$ it holds that $E(p) - E[min(p_1,p_2)] < c_s$, neither θ_{stable} nor $\theta_{non-stable}$ exist, and $\theta_{STABLE-1}$ is the unique stable equilibrium. When λ takes intermediate values, $\forall \theta \in [0, \theta_{stable})$ it holds that $E(p) - E[min(p_1,p_2)] > c_s$, and $\theta_{STABLE-2}$ is the unique stable equilibrium.

To be rigorous, λ being sufficiently high means that $\lambda > \widetilde{\lambda}$, where $\widetilde{\lambda}$ is the unique solution to the equation $\frac{(1-\lambda)v}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda}) = E(p \mid \theta_{stable} > 0)$ with v being exogenously given and $E(p \mid \theta_{stable} > 0)$ being a constant.[7] The LHS of Equation (2), when one fixes θ equal to zero, is a stictly concave function of λ; in that case, for appropriate (sufficiently high) values of $\frac{v}{c_s}$, it has two distinct roots, $\lambda_{low} > 0$ and $\lambda_{high} = \widetilde{\lambda}$. Explicitly, for the purposes of Proposition 2, intermediate and sufficiently low values of λ correspond to $(\lambda_{low}, \widetilde{\lambda})$ and $(0, \lambda_{low})$ respectively.

Corollary 3 *Under (i) and (iii) in Proposition 2, λ is such that there exists a stable equilibrium with no agent paying the clearinghouse fee; in this equilibrium comparative statics bring forth that the net surplus per consumer $v - E(p \mid \theta_{stable} = 0)$ is increasing in product valuation. Under (ii) and (iii) in Proposition 2, λ is such that there exists a stable equilibrium whereby each non-shopper pays the clearinghouse fee with some positive probability; in this equilibrium again comparative statics yield that the net consumer surplus $v - E(p \mid \theta_{stable} > 0)$ increases with the valuation v.*

Let's focus to the equilibrium where $\theta_{stable} = 0$: For $v_1 > v_2$ and holding λ fixed, one gets that $v_1 \left[1 - \frac{(1-\lambda)}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda})\right] > v_2 \left[1 - \frac{(1-\lambda)}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda})\right]$ because $1 - \frac{(1-\lambda)}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda})$ is a positive constant (see Footnote 7). Since $v_i \left[1 - \frac{(1-\lambda)}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda})\right]$ is the net consumer surplus under valuation v_i, it holds that $\frac{\partial [v - E(p \mid \theta_{stable}=0)]}{\partial v} > 0$. Also, it is the case that $\frac{\partial E(p \mid \theta_{stable}=0)}{\partial \lambda} < 0$ which means that *ceteris paribus* a larger fraction of shoppers in the total population corresponds to a greater cost of delay. Now, let's focus to the equilibrium where $\theta_{stable} > 0$: For given λ, the function at the LHS of Equation (2) is a function of θ only and when v gets higher this function moves upwards thereby increasing the equilibrium value of θ_{stable}; in point of fact, θ_{stable} increases so much that the lower bound of the support of the c.d.f. in Lemma 2 grows less in number which in turn means that prices probabilistically get lower. Strictly comparable

[7] The function $\frac{(1-\lambda)v}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda})$ is strictly decreasing in λ since $\frac{\partial \left[\frac{(1-\lambda)}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda})\right]}{\partial \lambda} = \frac{1}{\lambda(1+\lambda)} - \frac{1}{2\lambda^2} \ln(\frac{1+\lambda}{1-\lambda}) < 0$ for all $\lambda > 0$. Indeed, $\frac{\ln(\frac{1+\lambda}{1-\lambda})}{2\lambda}$ achieves its minimum value which equals 1 when $\lambda \downarrow 0$, while $\frac{1}{1+\lambda}$ achieves its maximum value which equals 1 again when $\lambda \downarrow 0$. Also, when $\lambda \downarrow 0$, $\frac{(1-\lambda)v}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda}) \uparrow v$ (for $\lambda \to 0^+$, $\frac{\ln(\frac{1+\lambda}{1-\lambda})}{2\lambda} \overset{L'H\hat{o}pital}{\to} \frac{1}{(1+\lambda)(1-\lambda)} \to 1^-$), and when $\lambda \uparrow 1$, $\frac{(1-\lambda)v}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda}) \downarrow 0$.

to Corollary 1, when the product valuation intrinsically gets higher, the expected price gets lower.

2.3.2 Comparison to the Search-Theoretic Approach

Janssen and Moraga-González (2004) use an oligopolistic version of the fixed-sample search model of Burdett and Judd (1983). They impose exogenously that a fraction λ of a measure-one population has zero costs in searching for the best price in the market. The rest of the population has to incur a cost for each price quote (define it the same as above), i.e. even visiting a firm at random yields a generalized price equal to $E(p) + c_s$, while getting two price quotes yields a generalized price $E[min(p_1, p_2)] + 2c_s$. In that model, consumers are indifferent between searching for one price or searching for two, for the reason that the expected benefit from search is decreasing in the number of searches. Letting μ_1 denote the probability according to which uninformed consumers search for one price quote, there are $1 - \mu_1$ consumers who search for two quotes.

Remark 1 *In the endogenous high search equilibrium with two firms in Janssen and Moraga-González (2004), letting* $\lambda = 0$*, gives a stable equilibrium value of* μ_1 *that is identical to the stable equilibrium value of* $1 - \theta$ *given by Equation (1) in Subsection 2.3 above. The only difference in the two models then is that the net consumer surplus in equilibrium is always greater in Subsection 2.3 by* c_s*.*

Remark 2 *In the endogenous high search equilibrium with two firms and* $\lambda > 0$ *in Janssen and Moraga-González (2004) the stable equilibrium value (or multiple stable equilibrium values) of* μ_1 *is (are) identical to the stable equilibrium value (values) of* $1 - \theta$ *given by Proposition 2 in Subsubsection 2.3.1 above. Again, the only difference in the two models is that the net consumer surplus in equilibrium is always greater in Subsubsection 2.3.1 by* c_s*.*

3 A Dynamic Clearinghouse Model

3.1 Structure

I introduce a two-period model with the assumptions employed in Subsection 2.1. In the second period each buyer's valuation (v_2) is lower than the valuation in the first period (v_1). Besides, both buyers and sellers discount the future with the factor $\delta \in (0, 1]$, e.g. consuming a unit tomorrow gives a gross consumer surplus δv_2 as of today, and the prices quoted by firms are valid only for a single period. Actually, it turns out that the equilibrium determination in the dynamic game is sensitive to λ. For this reason, I examine the environments with shoppers and no shoppers separately.

3.2 The Case with No Shoppers: $\lambda = 0$

I set out to find a perfect equilibrium.

Definition 1 *In every period potential demand is the population mass that is present in the market and has not purchased the product in the previous period. Active demand refers to the potential buyers who purchase the product.*

Obviously, in the final period, there is no distinction between potential and active demand. Whoever did not consume the product in the first period will do so in the next period. Either way, one need not be wary of losses in terms of allocative efficiency. In the second period the upper bound of the support coincides with v_2 and this follows suit from the static analysis.

Claim 1 *In the first period potential demand equals the total population mass (unity), but active demand may be lower.*

The intuition underlying this Claim is that the second period serves as an outside option for buyers being present in the first period and by the next Lemma this outside option gives a non-negative payoff. Therefore all buyers will be present in the first period, but depending on parameter values some or all buyers may opt for their outside option as Proposition 3 demonstrates.

Lemma 3 *In the second period there are two equilibria; one with no price dispersion by Conclusion 1 and one with endogenous information acquisition by Proposition 1.*

This Lemma says that since the second period is the last one and since by Corollary 2 the population mass that consumed the product in the first period does not affect prices, everything in the second period is the same compared to the static game where the valuation equals v_2. Define $E_2(p)$ the expected price in the static dispersed equilibrium with valuation v_2. Then, those buyers that decide to be active in the second period of the dynamic game anticipate at most a net surplus equal to $v_2 - E_2(p) > 0$.

Proposition 3 *Solving the game backwards, whenever price dispersion prevails in the second period, the effective monopolistic price in the first period is no longer equal to v_1. More precisely, should none of the buyers incur the information cost in the first period:*

(i) for any $\frac{v_2}{c_s}$ that satisfies $\frac{1}{\theta_{v_2}^{stable}} \ln\left(\frac{1+\theta_{v_2}^{stable}}{1-\theta_{v_2}^{stable}}\right) \geq 1$, both sellers set a price equal to $p^* = v_1 - \delta\left[v_2 - E_2(p)\right]$ with $prob = 1$ in the first period no matter what the value of the discount factor δ. In this case, the equilibrium is that all buyers purchase in the first period.

(ii) for any $\frac{v_2}{c_s}$ that satisfies $\frac{1}{\theta_{v_2}^{stable}} \ln\left(\frac{1+\theta_{v_2}^{stable}}{1-\theta_{v_2}^{stable}}\right) < 1$, both sellers set a price equal to $p^* = v_1 - \delta\left[v_2 - E_2(p)\right]$ with $prob = 1$ in the first period as long as the discount factor $\delta < \frac{v_1}{max\left(v_1, [v_2 - E_2(p) + \frac{(1-\theta_{v_2}^{stable})v_2}{2}]\right)}$. Again, the equilibrium is that all buyers purchase in the first period.

(iii) as long as $\frac{v_2}{c_s}$ is such that $\frac{1}{\theta_{v_2}^{stable}} \ln\left(\frac{1+\theta_{v_2}^{stable}}{1-\theta_{v_2}^{stable}}\right) < 1$ and furthermore if the discount factor satisfies $\frac{v_1}{max\left(v_1, [v_2 - E_2(p) + \frac{(1-\theta_{v_2}^{stable})v_2}{2}]\right)} \leq \delta \leq 1$, then both sellers playing $p^* = v_1 - \delta\left[v_2 - E_2(p)\right]$ with $prob = 1$ and all agents buying in the second period is an equilibrium. In fact, for these relative parameter values there are other infinite many equilibria with buyers deferring purchases until next period and sellers charging at least p^*. For example, each firm i quoting a price $\widehat{p_i} \in (p^*, +\infty)$ with any positive probability or with probability equal to one is an equilibrium. Sellers playing an atom at p^* and allocating the remaining probability mass in any arbitrary way in the interval $(p^*, +\infty)$ is also an equilibrium with no trade in the first period.

In case price dispersion obtains in the second period, a representative buyer out of the total mass that is present in the market in the first period cannot accept any price in the first period which is greater than the one that guarantees a net surplus in the first period equal to $\delta\left[v_2 - E_2(p)\right]$. Let p^* denote the threshold for prices above which no sales are possible by firms. This threshold satisfies the equation $v_1 - p^* = \delta\left[v_2 - E_2(p)\right]$ or $p^* = v_1 - \delta\left[v_2 - E_2(p)\right]$. Thus, any seller who sets a price higher than p^* in the first period forces half the uninformed population to defer consumption until next period. Consequently, when the total population is uninformed in the first period, seller i with price $p_i > p^*$ gets zero profits today and expects a minmax profit equal to $\frac{(1-\theta_{v_2}^{stable})v_2}{4}$ in the next period as long as in the first period the rival firm set a price equal to or lower than p^*, else seller i expects a minmax profit equal to $\frac{(1-\theta_{v_2}^{stable})v_2}{2}$ in the second period.

By Table 2, when none of the buyers incurs the information cost in the first period, setting a price equal to p^* with $prob = 1$ is a strictly dominant strategy for sellers in the first period provided that

$$\frac{1}{2}\left[v_1 - \delta[v_2 - E_2(p)]\right] > \delta\frac{(1-\theta_{v_2}^{stable})v_2}{4} \tag{4}$$

since it also goes that, without any restrictions on parameters, for any firm i setting a price $p_i < p^*$ is strictly donimated by $p_i = p^*$. For the purposes of the proof, the inequality above is equivalent to

$$\ln\left(\frac{1+\theta_{v_2}^{stable}}{1-\theta_{v_2}^{stable}}\right) - \theta_{v_2}^{stable} < 0 \tag{5}$$

There exists a unique critical value $\widehat{\theta}_{v_2}^{stable}$ below which the range of the function in the LHS of Inequality (5) takes only strictly positive values. In turn, $\widehat{\theta}_{v_2}^{stable}$ depends on the ratio $\frac{v_2}{c_2}$. The lower the c_s, the higher the $\widehat{\theta}_{v_2}^{stable}$.

The economic rationale here is the following. To the extent that $\ln\left(\frac{1+\theta_{v_2}^{stable}}{1-\theta_{v_2}^{stable}}\right) - \theta_{v_2}^{stable} \geq 0$, Inequality (4) that takes the form $v_1 > \delta\left(v_2 - E_2(p) + \frac{(1-\theta_{v_2}^{stable})v_2}{2}\right)$ holds regardless of how small or large the difference $v_1 - v_2 > 0$ is, and both sellers always prefer to price at p^* in the first period irrespective of how patient they are. (One can think this equilibrium as if both sellers set a price equal to $p^* - \epsilon$, where ϵ is positive and arbitrarily close to zero.) To the extent that $\ln\left(\frac{1+\theta_{v_2}^{stable}}{1-\theta_{v_2}^{stable}}\right) - \theta_{v_2}^{stable} < 0$, if v_1 is much higher compared to v_2, both sellers always prefer to price at $p^* - \epsilon$ in the first period even if $\delta \longrightarrow 1$; however, if v_1 is only a bit higher than v_2, both sellers set a price equal to $p^* - \epsilon$ in the first period provided that they are sufficiently impatient, i.e. $\delta \leq \frac{v_1}{v_2 - E_2(p) + \frac{(1-\theta_{v_2}^{stable})v_2}{2}}$.

Corollary 4 *Compared to the static benchmark where the valuation is v_1 and no one pays the clearinghouse fee, buyers are better off in the dynamic equilibrium where none of them pays the clearinghouse fee in the first period as long as trade takes place only in the first period and there is an anticipation of a dispersed price equilibrium in the second period. Also, buyers in the dynamic equilibrium where none of them pays the clearinghouse fee in the first period are better off, compared to the static benchmark where the valuation is v_1 and no one pays the clearinghouse fee, whenever the parameter values dictate that trade takes place only in the*

	$p_2 > p^*$	$p_2 = p^*$
$p_1 > p^*$	$\delta\frac{(1-\theta_{v_2}^{stable})v_2}{2}, \delta\frac{(1-\theta_{v_2}^{stable})v_2}{2}$	$\delta\frac{(1-\theta_{v_2}^{stable})v_2}{4}$, $\frac{1}{2}p^* + \delta\frac{(1-\theta_{v_2}^{stable})v_2}{4}$
$p_1 = p^*$	$\frac{1}{2}p^* + \delta\frac{(1-\theta_{v_2}^{stable})v_2}{4}$, $\delta\frac{(1-\theta_{v_2}^{stable})v_2}{4}$	$\frac{1}{2}p^*$, $\frac{1}{2}p^*$

Table 2: Determination of the first period's effective monopolistic price with a dispersed price equilibrium in the second period.

second period with dispersed prices.

These results follow directly from Proposition 3 and Conclusion 1.

Proposition 4 *Solving the game backwards, whenever price dispersion is established in the second period, should buyers mix with respect to accessing the clearinghouse or not in the first period, in the dynamic equilibrium firms price in the first period according to the atomless c.d.f.* $F_{p^*}(p) = \frac{1+\theta}{2\theta} - \frac{(1-\theta)p^*}{2p\theta}$ *with support* $[\frac{(1-\theta)p^*}{1+\theta}, p^*]$ *while all consumers purchase the product in the first period.*

When $v_1 - v_2 < v_2 - E_2(p)$ and $\delta > \frac{v_1-v_2}{v_2-E_2(p)}$, $p^* < v_2$ holds. That is, if the difference $v_1 - v_2$ is relatively small and if agents in the economy are relatively patient, sellers realize that the effective monopolistic price in the first period p^* is lower than the monopolistic price in the second period. By Corollary 1, when the reservation value decreases consumers search less intensively for the best price quote (θ_{stable} decreases), prices rise probabilistically, and sellers are better off (the minmax profit increases). To the degree that $\frac{(1-\theta_{v_2}^{stable})v_2}{2} < \frac{(1-\theta_{p^*}^{stable})p^*}{2}$ which as explained is always true with $p^* < v_2$, both sellers prefer to trade in the market only in the first period. In particular, the high price seller in the first period attracts only the uninformed population ($\frac{1-\theta_{p^*}^{stable}}{2}$) and will never choose to quote a price above p^* because this way the seller forgoes profits equal to $\frac{(1-\theta_{p^*}^{stable})p^*}{2}$ today (the uninformed population today optimally decides to defer consumption) and expects tomorrow minmax profits $\frac{(1-\theta_{v_2}^{stable})(1-\theta_{p^*}^{stable})v_2}{4}$ since the low price seller in the first period would set p^* and corner all the informed and half the uninformed mass. Subsequently, the minmax profit for sellers in the first period is $\frac{(1-\theta_{p^*}^{stable})p^*}{2}$ and the argumentation for the determination of equilibrium reduces to the one in Proposition 1 where now v is represented by p^*.

	$p_2 > p^*$	$p_2 = p^*$
$p_1 > p^*$	$\frac{(1-\theta_{v_2}^{stable})v_2}{2}, \frac{(1-\theta_{v_2}^{stable})v_2}{2}$	$\frac{(1-\theta_{v_2}^{stable})(1-\theta_{p^*}^{stable})v_2}{4}, \frac{(1+\theta_{p^*}^{stable})p^*}{2} + \frac{(1-\theta_{v_2}^{stable})(1-\theta_{p^*}^{stable})v_2}{4}$
$p_1 = p^*$	$\frac{(1+\theta_{p^*}^{stable})p^*}{2} + \frac{(1-\theta_{v_2}^{stable})(1-\theta_{p^*}^{stable})v_2}{4}, \frac{(1-\theta_{v_2}^{stable})(1-\theta_{p^*}^{stable})v_2}{4}$	$\frac{1}{2}p^*, \frac{1}{2}p^*$

Table 3: Determination of the upper bound of the c.d.f. in the first period when (i) buyers randomize in regard to paying the fee or not in the first period, (ii) $p^* > v_2$, (iii) $\delta \to 1$, and (iv) there is a dispersed price equilibrium in the second period.

The case $p^* = v_2$ is obvious. In case $p^* > v_2$ (see Table 3), which is of interest when the difference $v_1 - v_2 > 0$ is relatively large no matter what the discount factor δ, sellers would prefer to coordinate so as both price above p^* in the first period and none of the buyers gets full information in the first period which results in that trade takes place only in the second period. For $p^* > v_2$, it holds that $\frac{(1-\theta_{v_2}^{stable})v_2}{2} > \frac{(1-\theta_{p^*}^{stable})p^*}{2}$ and therefore $\delta\frac{(1-\theta_{v_2}^{stable})v_2}{2} > \frac{(1-\theta_{p^*}^{stable})p^*}{2}$ is true for all $\delta > \frac{(1-\theta_{p^*}^{stable})p^*}{(1-\theta_{v_2}^{stable})v_2}$. (For $\delta < \frac{(1-\theta_{p^*}^{stable})p^*}{(1-\theta_{v_2}^{stable})v_2}$ the analysis in the previous paragraph applies.) However, $\delta\frac{(1-\theta_{v_2}^{stable})v_2}{2} < \theta_{p^*}^{stable}p^* + \frac{(1-\theta_{p^*}^{stable})p^*}{2} + \delta\frac{(1-\theta_{v_2}^{stable})(1-\theta_{p^*}^{stable})v_2}{4}$ is valid for any δ (and for $\delta \to 1$) and if in the first period consumers randomize with regard to paying the fee or not, the high price seller had better not price above p^* because the optimal response to such a behavior by the rival firm would be to charge p^*. Any seller would prefer $\frac{(1-\theta_{p^*}^{stable})p^*}{2}$ to $\frac{(1-\theta_{v_2}^{stable})(1-\theta_{p^*}^{stable})v_2}{4}$ for all values of the discount factor.

Corollary 5 *Compared to the static benchmark where the valuation is v_1 and prices are dispersed, buyers are worse off in the dynamic equilibrium where they are all active in the first period and randomize between paying the clearinghouse fee or not while firms use the atomless c.d.f. $F_{p^*}(p) = \frac{1+\theta}{2\theta} - \frac{(1-\theta)p^*}{2p\theta}$ with support $[\frac{(1-\theta)p^*}{1+\theta}, p^*]$.*

It suffices that $p^* < v_1$ and then Corollary 1 guarantees the result. Strictly speaking, one must rule out the inequality $E_1(p) > E_{p^*}(p)$. This amounts to checking on

$$v_1 \left(\frac{1-\theta_{v_1}^{stable}}{2\theta_{v_1}^{stable}} \ln\left(\frac{1+\theta_{v_1}^{stable}}{1-\theta_{v_1}^{stable}}\right) - \frac{1-\theta_{p^*}^{stable}}{2\theta_{p^*}^{stable}} \ln\left(\frac{1+\theta_{p^*}^{stable}}{1-\theta_{p^*}^{stable}}\right)\right)$$
$$> -\delta[v_2 - E_2(p)]\frac{1-\theta_{p^*}^{stable}}{2\theta_{p^*}^{stable}} \ln\left(\frac{1+\theta_{p^*}^{stable}}{1-\theta_{p^*}^{stable}}\right) \qquad (6)$$

It can be shown that Inequality (6) does not hold for any parameters.[8] Some explanation is due: by Corollary 1, it is known that $\theta_{v_1}^{stable} > \theta_{p^*}^{stable}$, and from this it can be inferred that the LHS of Inequality (6) is negative; then for high values of v_1 (6) cannot hold. On the other hand, v_1 and v_2 being close together serves so as to put a downward pressure to the highest price consumers are willing to accept in order to trade in the first period; for example, when $\theta_{p^*}^{stable} \to 0$, (6) gives $E_1(p) - v_1 > -\delta[v_2 - E_2(p)]$ which is false.

Proposition 5 *Solving the game backwards, whenever no price dispersion is established in the second period and none of the buyers accesses the clearinghouse in the first period, the*

[8]Numerical results are available by the author upon request.

effective monopolistic price in the first period is equal to v_1 *and the dynamic equilibrium is that both sellers set a price equal to* v_1 *in the first period while buyers are indifferent between periods.*

The outside option of deferring consumption until tomorrow gives a zero payoff and the first period of the dynamic game is identical to the static game.

Proposition 6 *Solving the game backwards, whenever no price dispersion is established in the second period, and when buyers in the first period mix with respect to getting access to full information or not, in equilibrium sellers use the c.d.f. by Proposition 1 where the valuation* v *now equals* v_1 *and trade takes place only in the first period.*

Both sellers would like to coordinate by setting in the first period prices above v_1 so that trade takes place only in the second period. However, this is not possible because if the high price seller in the first period does price above v_1, the best response of the rival firm is to charge in the first period v_1 and this is due to the existence of the informed population $\theta_{v_1}^{stable}$.

3.3 The Case with Shoppers: $\lambda > 0$

By Proposition 2, one could argue in the spirit of Lemma 3 that in the second period there is no equilibrium with no price dispersion, but depending on λ there exists at least one and at most two stable dispersed price equilibria. This conclusion is not necessarily true.

Claim 2 *Each segment on the demand-side is synchronized in equilibrium. That is, all non-shoppers purchase the product in the same period. This is the case for shoppers too. However, synchronization across segments need not occur.*

In the second period, say, should only shoppers purchase the product, pricing is aggressive. Moreover, non-shoppers are aware of the fact that if all of them purchase in a given period then their net payoff (by Table 1) is the same irrespective of the mass of shoppers, if any, in that period.

Lemma 4 *The effective monopolistic price in the first period for non-shoppers is* $p^* = v_1 - \delta[v_2 - E_2(p)]$*, while shoppers' reservation value for purchasing in the first period is* $\underline{p} = v_1 - \delta v_2$*.*

If prices in the first period are such that all non-shoppers purchase in the first period, then it is obvious that shoppers by purchasing tomorrow can capture a surplus δv_2 as of today. If prices in the first period are such that all non-shoppers purchase in the second period, then shoppers will purchase for sure in the second period. So it is right to say that in any case shoppers are willing to pay up to $\underline{p} = v_1 - \delta v_2$ in the first period.

Claim 3 *In the first period no firm will ever set a price within $(\underline{p}, p^*)$, should none non-shopper incur the clearinghouse cost throughout the game.*

Any price in this interval is strictly dominated, in terms of profits accrued to the seller, by p^*.

Proposition 7 *Suppose $\lambda \in (0, \lambda_{low}) \bigcup (\widetilde{\lambda}, 1)$, where the notation follows from Proposition 2, so that if both shoppers and non-shoppers are about to purchase the product in the second period, there exists a stable equilibrium at that time whereby none non-shopper pays the clearinghouse fee. Let's stick to this equilibrium in the second period, if there is a positive active mass of both shoppers and non-shoppers then. Whenever in the first period also none non-shopper pays the fee, and with $\widehat{\lambda}$ denoting the unique solution to the equation $p^* = \frac{\underline{p}(1+\lambda)}{1-\lambda}$, the dynamic equilibrium is as follows:*

(i) if $p^* > \frac{\delta v_2}{2}$ and $\lambda < \widehat{\lambda}$, then in the first period both firms charge p^* with prob $= 1$ and all non-shoppers purchase the product in the first period. All shoppers delay consumption and purchase the product in the second period at the marginal cost.

(ii) if $p^* > \frac{\delta v_2}{2}$, and $\lambda > \widehat{\lambda}$, and $\lambda > \frac{p^* - \underline{p}}{p^*}$, then trade takes place only in the first period with both firms charging $\underline{p}$ with prob $= 1$.

(iii) if $p^* > \frac{\delta v_2}{2}$, and $\lambda > \widehat{\lambda}$, and $\lambda < \frac{p^* - \underline{p}}{p^*}$, and $\frac{v_2}{c_s}$ is sufficiently low, then in the first period either $p_1 = \underline{p}$, $p_2 = p^*$ or $p_1 = p^*$, $p_2 = \underline{p}$, or each firm i randomizes by charging $p_i = p^*$ with probability α equal to $\frac{(1-\lambda)p^* - \underline{p}}{\lambda \underline{p}}$ and $p_i = \underline{p}$ with probability equal to $1 - \alpha$. In all three equilibria all non-shoppers purchase only in the first period. Particularly, the two pure strategy equilibria guarantee that all shoppers and half non-shoppers purchase at price $\underline{p}$ in the first period while half non-shoppers purchase at price p^*; in the mixed strategy equilibrium, there is positive probability equal to α^2 that shoppers purchase in the second period at the marginal cost.

(iv) if $p^* < \frac{\delta v_2}{2}$ and $\lambda < \frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1}$, then for each seller i setting any price $p_i > p^*$ with prob $= 1$ or with any positive probability constitutes an equilibrium with all consumers purchasing the product in the second period.

(v) if $\delta v_2 - \lambda v_1 + \delta E_2(p) < p^* < \frac{\delta v_2}{2}$, and $3 - \frac{2v_1}{\delta v_2} > \frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1}$, and $\frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1} < \lambda < 3 - \frac{2v_1}{\delta v_2}$, then the equilibrium is the same as the one under (iv).

(vi) if $p^* < \frac{\delta v_2}{2}$*, and* $3 - \frac{2v_1}{\delta v_2} > \frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1}$*, and* $\lambda > 3 - \frac{2v_1}{\delta v_2}$*, then* $\underline{p}$ *is a strictly dominant strategy for both sellers in the first period and no trade takes place in the second period.*

(vii) if $p^* < \frac{\delta v_2}{2}$*, and* $3 - \frac{2v_1}{\delta v_2} < \frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1}$*, and* $\lambda > \frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1}$*, then the equilibrium is the same as the one under (vi).*

(viii) if $p^* < \delta v_2 - \lambda v_1 + \delta E_2(p) < \frac{\delta v_2}{2}$*, and* $3 - \frac{2v_1}{\delta v_2} > \frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1}$*, and* $\frac{\delta\left[\frac{v_2}{2} - E_2(p)\right]}{v_1} < \lambda < 3 - \frac{2v_1}{\delta v_2}$*, and* $\frac{v_2}{c_s}$ *is sufficiently low, then there exist two pure strategy equilibria,* $p_1 > p^*$*,* $p_2 = \underline{p}$*, and* $p_1 = \underline{p}$*,* $p_2 > p^*$*, and one equilibrium in mixed strategies whereby each firm sets prices higher than* p^* *with probability* β *equal to* $\frac{2\underline{p} - \delta(1-\lambda)v_2}{\delta(1-\lambda)v_2 - 2\underline{p}\lambda}$ *while* $\underline{p}$ *is set with probability equal to* $1-\beta$*. Alternatively,* $p_i > p^*$*,* $p_j = \underline{p}$*,* $(i \neq j)$*, may be meant to represent a mixed strategy equilibrium with firm* j *setting* $\underline{p}$ *with prob* $= 1$ *and firm* i *randomizing among prices in the interval* $(p^*, +\infty)$*. In all, to the extent that a single firm charges* $\underline{p}$ *for sure, all shoppers and half non-shoppers purchase in the first period; otherwise with probability equal to* β^2 *trade takes place only in the second period, and with probability* $(1-\beta)^2$ *trade takes place only in the first period.*

It is straightforward that for any $\lambda < \widehat{\lambda}$, $p^* > \frac{\underline{p}(1+\lambda)}{1-\lambda}$, and for any $\lambda > \widehat{\lambda}$, $p^* < \frac{\underline{p}(1+\lambda)}{1-\lambda}$. (Note that $\underline{p}$ is independent of λ, p^* is strictly decreasing in λ since here $E_2(p)$ is strictly decreasing in λ, and $\frac{1+\lambda}{1-\lambda}$ is strictly increasing in λ. For $\lambda \downarrow 0$, $\frac{1+\lambda}{1-\lambda} \downarrow 1$ while $p^* \uparrow v_1$ $(> \underline{p})$. For $\lambda \uparrow 1$, $\frac{1+\lambda}{1-\lambda} \rightarrow +\infty$ while p^* achieves its minimum value.) Using the fact that $\frac{(1-\lambda)(1+\lambda)}{2\lambda} \ln(\frac{1+\lambda}{1-\lambda}) < 1$ $\forall \lambda > 0$, in Table 4 the strategy $p_i < \underline{p}$ is strictly dominated by $p_i = \underline{p}$. Then, by symmetry, the equilibria are easily identifiable.

An important caveat in regard to this Proposition is the following: one has to make sure that non-shoppers who do not pay the fee in the first period do so optimally (see also 2.3.1). Under (i), (ii), (iv), (v), and (vi) no dispersion is established in the first period and the requirement that non-shoppers endogenously end up not paying the fee is trivially satisfied. Under (iii) there are three cases to be examined: in the two pure strategy equilibria, a non-shopper that goes shopping without full information about prices chooses a seller at random and purchases at the price encountered at that seller; so the expected price is equal to $E(p) = \frac{1}{2}p^* + \frac{1}{2}\underline{p}$ while the minimum price in the market is for sure $\underline{p}$. Analogously to Equation (3), here equilibrium requires

$$\frac{1}{2}(p^* - \underline{p}) < c_s \tag{7}$$

Inequality (7) is true whenever $\frac{(1-\lambda)v_2}{4\lambda} \ln(\frac{1+\lambda}{1-\lambda}) < c_s$ or otherwise put whenever the ratio $\frac{v_2}{c_s}$ is sufficiently low. For high relative values of $\frac{v_2}{c_s}$, it holds that $\frac{(1-\lambda)v_2}{4\lambda} \ln(\frac{1+\lambda}{1-\lambda}) < c_s$ if and only if $\lambda \rightarrow 1^-$.

	$p_2 > p^*$	$p_2 = p^*$	$p_2 = \underline{p}$	$p_2 < \underline{p}$
$p_1 > p^*$	$\frac{\delta(1-\lambda)v_2}{2}, \frac{\delta(1-\lambda)v_2}{2}$	$\frac{\delta(1-\lambda)v_2}{4}, \frac{\delta(1-\lambda)v_2}{4} + \frac{(1-\lambda)p^*}{2}$	$\frac{\delta(1-\lambda)v_2}{4}, \frac{\delta(1-\lambda)v_2}{4} + \underline{p}\left[\frac{1+\lambda}{2}\right]$	$\frac{\delta(1-\lambda)v_2}{4}, \frac{\delta(1-\lambda)v_2}{4} + E(p_2)\left[\frac{1+\lambda}{2}\right]$
$p_1 = p^*$	$\frac{\delta(1-\lambda)v_2}{4} + \frac{(1-\lambda)p^*}{2}, \frac{\delta(1-\lambda)v_2}{4}$	$\frac{(1-\lambda)p^*}{2}, \frac{(1-\lambda)p^*}{2}$	$\frac{(1-\lambda)p^*}{2}, \underline{p}\left[\frac{1+\lambda}{2}\right]$	$\frac{(1-\lambda)p^*}{2}, E(p_2)\left[\frac{1+\lambda}{2}\right]$
$p_1 = \underline{p}$	$\frac{\delta(1-\lambda)v_2}{4} + \underline{p}\left[\frac{1+\lambda}{2}\right], \frac{\delta(1-\lambda)v_2}{4}$	$\underline{p}\left[\frac{1+\lambda}{2}\right], \frac{(1-\lambda)p^*}{2}$	$\frac{\underline{p}}{2}, \frac{\underline{p}}{2}$	$\underline{p}\left[\frac{1-\lambda}{2}\right], E(p_2)\left[\frac{1+\lambda}{2}\right]$
$p_1 < \underline{p}$	$\frac{\delta(1-\lambda)v_2}{4} + E(p_1)\left[\frac{1+\lambda}{2}\right], \frac{\delta(1-\lambda)v_2}{4}$	$E(p_1)\left[\frac{1+\lambda}{2}\right], \frac{(1-\lambda)p^*}{2}$	$E(p_1)\left[\frac{1+\lambda}{2}\right], \underline{p}\left[\frac{1-\lambda}{2}\right]$	$\underline{p}\left[\frac{1-\lambda}{2}\right], \underline{p}\left[\frac{1-\lambda}{2}\right]$

Table 4: Determination of the equilibrium in the first period when (i) there exists a positive mass of shoppers in the economy, (ii) in the second period none non-shopper, if any is active then, pays the clearinghouse fee, and (iii) in the first period also none non-shopper pays the fee. Notation: $p_i < \underline{p}$ means that firm i uses the atomless c.d.f. $F(p) = \frac{1+\lambda}{2\lambda} - \frac{(1-\lambda)\underline{p}}{2p\lambda}$ with support $[\frac{(1-\lambda)\underline{p}}{1+\lambda}, \underline{p}]$ that gives $E(p_i) = \frac{(1-\lambda)\underline{p}}{2\lambda}\ln(\frac{1+\lambda}{1-\lambda})$.

Under (iii) again, when one focuses on the mixed strategy equilibrium, the endogenous non-acquisition of information requires

$$(1-\alpha)\alpha(p^* - \underline{p}) < c_s \tag{8}$$

Inequality (8) is always true when Inequality (7) is satisfied because $2\alpha(1-\alpha) < 1$. Under (viii), the counterpart of Equation (7) is

$$(v_1 - \underline{p}) - \left(\frac{1}{2}(v_1 - \underline{p}) + \frac{\delta}{2}\left(v_2 - E_2(p)\right)\right) < c_s \tag{9}$$

Inequality (9) holds whenever $\delta\frac{(1-\lambda)v_2}{4\lambda}\ln(\frac{1+\lambda}{1-\lambda}) < c_s$; this constraint is identical to or less stringent than the one referring to Inequality (7) since $\delta \leq 1$. Should one multiply only the LHS of Inequality (9) with $2\beta(1-\beta)$ (< 1), one gets an inequality which corresponds to the requirement for an endogenous equilibrium under the mixed strategy in (viii).

Corollary 6 *Compared to the static benchmark where the valuation is v_1, $\lambda > 0$, and none non-shopper incurs the clearinghouse cost which results in that the expected price equals $\frac{(1-\lambda)v_1}{2\lambda}\ln(\frac{1+\lambda}{1-\lambda})$, under Proposition 7:*

(i) if trade takes place only in the second period, all shoppers and all non-shoppers are worse off,

(ii) non-shoppers who purchase in the first period at price p^* are worse off,

(iii) shoppers who purchase in the second period at the marginal cost are worse off,

(iv) when both non-shoppers and shoppers purchase in the first period at price $\underline{p}$, ...

More to be added.

3.3.1 Comparison to the Search-Theoretic Approach

To be added.

3.3.2 A Contribution to the Microfoundations of Business Cycles

Fershtman and Fishman (1992)

To be added.

4 Discussion

4.1 Price Dispersion in Games with Capacity Constraints

Here, I argue that efficient delay has quite general applicability in games with price dispersion. Although buyers may have full information about price quotes at no cost, for example, price dispersion arises through supply-side capacity constraints. Recently, dynamic pricing for durables with intertemporal capacity constraints and full information about prices has been examined by Biglaiser and Vettas (2007). In what follows, I rework Biglaiser and Vettas. Fine testing lends their model a novel intellectual appeal.

Assumptions In a two period duopoly game with a homogeneous durable good there exist two buyers each one with demand for three units. For the first two units the gross per unit surplus equals v in each period, while the third unit yields a positive gross surplus equal to v_3 only in the second period. That said, if consumption of all three units takes place in the first period then a buyer's gross utility equals $2v(1+\delta)+\delta v_3$, where δ is the discount factor and $0 < v \leq v_3$. Intertemporally, each seller is able to supply up to five units. The marginal cost of production is normalized to zero up to capacity and infinite thereafter. Finally, economy-wide agents are risk neutral.

In a few words, Biglaiser and Vettas argue that in a dynamic setting like the one just described, buyers get hurt by thinking strategically. Strategic buyers buy two units in the first period and would prefer to split purchases between sellers in the first period (if the price difference is not significant) in order to support competition by sellers subsequently. Due to capacity constraints, if a firm sells out today, in the course of the next period buyers will engage in trade with a monopolist. Buyers buying two units (with probability equal to one) in the first period, however, are worse off compared to buyers who commit (before observing any price quotes) to buying all three units in period-1. Namely, the equilibrium Biglaiser and Vettas derive says that multi-period interactions benefit sellers.

I claim that this reasoning is not thoroughly carried out, since it fails to take into account another possibility whereby buyers randomize between purchasing two units in period-1 and one unit in period-2 or purchasing one unit in period-1 and two units in period-2. Proposition 8 comes after the following Remark and shows clearly that buyers, by employing the aforementioned strategy which entails delay in consumption, can achieve an equivalent payoff to the one that corresponds to buying all three units in the first period. More strikingly, sellers then get zero profits. Should one allow for arbitrarily small parameter perturbations, the stratery

I propose gives in equilibrium a payoff to buyers higher than the static one, while sellers do price competitively.

Remark 3 *In the Biglaiser and Vettas (2007) model buyers committing to buy all three units in the first period get a net surplus equal to* $2v(1+\delta)$*, while each seller's expected profit amounts to* δv_3*.*

When buyers commit to buying all of their units in the first period, then the structure of the game is as follows. Trade takes place only in the first period, demand is equal to six units (i.e. both buyers are active and each purchases three units) and sellers engage in one-shot price competition with capacities $k_1 = 5, k_2 = 5$. In this case, sellers have a minmax payoff that equals $\delta v_3 > 0$, i.e. if a seller sets the highest price in the market, then buyers will take care to purchase their high valuation units from the seller with the low price, thereby leaving demand of one unit (corresponding to valuation δv_3) to be satisfied by the high price seller. So, the upper bound of the support is δv_3. Following the Proof of Lemma 1 (see A.1), in equilibrium prices are dispersed according to the atomless c.d.f. $F(p) = \frac{5}{4} - \frac{\delta v_3}{4p}$ with lower bound of the support $\frac{\delta v_3}{5}$.

Summing up, when buyers commit to purchase all of their units in the first period, then each one captures a gross payoff equal to $2v(1+\delta)+\delta v_3$ and pays (assuming an equal probability for buyers being rationed by the low-price firm and taking account of the equality $\frac{1}{2}E[min(p_i, p_j)] + \frac{1}{2}E[max(p_i, p_j)] = E(p)$)

$$2E[min(p_i, p_j)] + E(p).$$

As a result, each buyer expects a net surplus equal to:

$$2v(1+\delta) + \delta v_3 - 4\int_{\frac{\delta v_3}{5}}^{\delta v_3} p\,[1 - F(p)]\,dF(p) - \int_{\frac{\delta v_3}{5}}^{\delta v_3} p dF(p)$$

$$= 2v(1+\delta) + \delta v_3 - 2\left(\frac{\delta v_3}{2} - \frac{\delta v_3}{8}\ln 5\right) - \frac{\delta v_3}{4}\ln 5$$

$$= 2v(1+\delta) + \delta v_3 - \delta v_3$$

$$= 2v(1+\delta).$$

To establish delay in consumption, consider that each buyer randomizes between purchasing the first and the third unit in the first period and the second unit in the second period or purchasing the first unit in the first period and the remaining units in the second period. Let ϑ the probability that each buyer purchases the first and the third unit in the first period and the second unit in the second period.

Proposition 8 *In the Biglaiser and Vettas (2007) model buyers capture the greatest surplus possible, i.e.* $2v(1+\delta)$*, by randomizing between purchasing two units (the first and the third) in the first period and one unit (the second) in the second period or purchasing one unit (the first) in the first period and two units (the second and the third) in the second period. This holds for* $\delta = 1$ *and* $v = v_3$*. Sellers, in this case, get zero profit margin.*

The net payoff of a buyer purchasing the first and the third unit in the first period equals:

$$\vartheta \left[v(1+\delta) + \delta v_3 + \delta v \right] + (1-\vartheta) \left[v(1+\delta) + \delta v_3 + \delta v \right]$$

$$= v(1+\delta) + \delta v_3 + \delta v.$$

Following is some interpretation. When a buyer buys the first and the third unit in period-1, then with $prob = \vartheta$ the other buyer does the same thing. Hence, in the first period there is an aggregate demand for 4 units, while $k_1 = 5, k_2 = 5$. Accordingly, each firm serves half the first period demand and both firms price at the marginal cost. In the second period, aggregate demand equals 2 units, while $k_1 = 3, k_2 = 3$, and again the Bertrand Paradox obtains. When a buyer buys the first and the third unit in period-1, then with $prob = 1 - \vartheta$ the other buyer purchases only the first unit in the first period. More precisely, in the first period aggregate demand equals 3 units and in the second period the capacities are $k_1 = 3.5, k_2 = 3.5$. It easily checks out that in both periods prices are competitive, even with indivisibility in production, as long as in the first period each buyer chooses a different seller.

The net payoff of a buyer purchasing only the first unit in the first period is also:

$$\vartheta \left[v(1+\delta) + \delta v_3 + \delta v \right] + (1-\vartheta) \left[v(1+\delta) + \delta v_3 + \delta v \right]$$

$$= v(1+\delta) + \delta v_3 + \delta v.$$

Indeed, when a buyer purchases only the first unit in the first period, then with $prob = \vartheta$ the other buyer buys both the first and the third unit in the first period. In both periods, $p_1 = p_2 = 0$, since in the second period aggregate demand is 3 units and $k_1 = 3.5, k_2 = 3.5$. Again, indivisibility does not affect the argument. When a buyer purchases only the first unit

	1st unit	2nd unit	3rd unit
1st period	v	$v-\epsilon$	
2nd period	v	v	v_3

Table 5: Slight modification of the assumptions in Biglaiser and Vettas (2007) with regard to the utility from consumption, where $\epsilon \to 0^+$.

in the first period, then with $prob = 1 - \vartheta$ the other buyer does the same and so in the second period $k_1 = 4, k_2 = 4$.

For an extended treatment of why $2v(1+\delta)$ is the greatest consumer surplus possible (as stated in Proposition 8), see the Appendix A.3.

Corollary 7 *In the Biglaiser and Vettas (2007) model with $\delta = 1$ and $v_3 = v$ when buyers commit to buying all three units in period-1, the outcome is Pareto superior to the equilibrium derived when buyers randomize between purchasing the first and the third unit in the first period or purchasing the first unit in the first period and the remaining ones in the second period.*

By risk neutrality, in anyone of these cases buyers capture the same net surplus which equals, in value or expected value, $2v(1+\delta)$; however when buyers commit to buy all units in period-1, expected industry profits are $2\delta v_3 > 0$.

Corollary 8 *In the Biglaiser and Vettas (2007) model with $\delta = 1$ and $v_3 = v$ and with a slight modification as in Table 2, buyers prefer to delay consumption by purchasing only the first unit in period-1 with probability $1 - \vartheta$ or purchasing both the first and the third unit in period-1 with probability ϑ. This strategy yields a payoff for the buyers superior to any other.*

If buyers buy all three units in the first period then the expected net surplus per buyer (by Table 5) is $v(1+2\delta) + v - \epsilon$ (beware that ϵ being positive does not affect pricing decisions), whereas the net surplus by mixing with respect to buying only the first unit in period-1 or buying the first and the third unit in period-1 is higher (for $\delta = 1$ and $v_3 = v$) in view of the fact that it equals $v(1+\delta)+\delta v_3+\delta v$ even though the second unit gives utility $v-\epsilon$ in the first period.

Corollary 9 *In the Biglaiser and Vettas (2007) model, delay in consumption holds even for values of $\delta < 1$, when slight modifications (in the utility assumptions), as the one in Table 2, apply.*

For example, in the event that $v_3 = v$, buyers using the strategy in Proposition 8 get a higher net surplus than the one under the static outcome provided that $\delta \in (\frac{v-\epsilon}{v}, 1]$. Actually, when $v_3 = v$, ϵ can be as high as v without any distortion in pricing decisions and so delay in consumption may hold for any $\delta > 0$.

4.2 Price Dispersion in Underground Markets

Galenianos *et al.* (2008).

To be added.

5 Conclusion

This work is also part of a research program attempting to establish that rational agents' delay is prevalent in a fair number of important decision making processes. Another theoretical paper (Rouskas 2009) highlights efficient delay in stock investing.

More to be added.

References

[1] Baye M. R. Morgan J. and P. Scholten 2006. Information, Search, and Price Dispersion. *Handbook in Information Systems*

[2] Biglaiser G. and N. Vettas 2007. Dynamic Price Competition with Capacity Constraints and Strategic Buyers. *Portuguese Competition Authority Working Paper 24*

[3] Burdett K. and K. Judd 1983. Equilibrium Price Dispersion. *Econometrica*

[4] Deneckere R. and J. Peck 2005. Dynamic Competition with Random Demand and Costless Search: A Theory of Price Posting. *Working Paper*

[5] Fershtman C. and A. Fishman 1992. Price Cycles and Booms: Dynamic Search Equilibrium. *American Economic Review*

[6] Galenianos M. Pacula R. L. and N. Persico 2008. A Search-Theoretic Model of the Retail Market for Illicit Drugs. *Working Paper*

[7] Greenleaf E. A. and D. R. Lehmann 1995. Reasons for Substantial Delay in Consumer Decision Making. *Journal of Consumer Research*

[8] Janssen M. and J. Moraga-González 2004. Strategic Pricing, Consumer Search and the Number of Firms. *Review of Economic Studies*

[9] Morwitz V. G. and D. Schmittlein 1992. Using Segmentation to Improve Sales Forecasts Based on Purchase Intent: Which Intenders Actually Buy? *Journal of Marketing Research*

[10] Prescott E. C. 1975. Efficiency of the Natural Rate. *Journal of Political Economy*

[11] Rouskas E. 2009. Spurious Biases in Capital Market Investment Decisions. *Working Paper*

[12] Stahl D. 1989. Oligopolistic Pricing with Sequential Consumer Search. *American Economic Review*

[13] Varian H. 1980. A Model of Sales. *American Economic Review*

A Appendix

A.1 Proof of Lemma 1

Obviously no firm has an incentive to price above v and competitive pricing is mitigated due to the existence of uninformed agents. If a firm is about to set the highest price, it will do so by charging v, since the rival firm will set a price that is lower or equal to v for sure. That is, each firm has a minmax profit equal to $\frac{(1-\theta)v}{2}$. The equilibrium in mixed strategies requires each price give the same expected payoff and this payoff should equal the minmax profit. By symmetry, both firms do not play any atoms in equilibrium and use the self-same mixed strategy c.d.f. $F(p)$, where $F(p)$ solves the following equation:

$$F(p)p\left(\frac{1-\theta}{2}\right)+[1-F(p)]p\left(\frac{1-\theta}{2}+\theta\right)=\frac{(1-\theta)v}{2}$$

$$\Leftrightarrow F(p)=\frac{1+\theta}{2\theta}-\frac{(1-\theta)v}{2p\theta}, p\in[\frac{(1-\theta)v}{1+\theta},v]$$

For example, when a firm has the highest price in the market which occurs with $prob=F(p)$, then the demand accruing to this firm is $\frac{1-\theta}{2}$; i.e. half the uninformed population buys from the high price firm. As is standard, it is assumed that each uninformed buyer visits each firm with $prob=\frac{1}{2}$. Correspondingly, a firm has the lowest price in the market with $prob=1-F(p)$ and serves then all the informed buyers and half the uninformed, i.e. in total $\frac{1-\theta}{2}+\theta$.

With respect to the support, the lower bound is simply the price that satisfies $F(p)=0$.

Both firms using $F(p)$ in equilibrium gives an expected price in the market equal to $E(p)=\int_{\frac{(1-\theta)v}{1+\theta}}^{v}pdF(p)=\frac{(1-\theta)v}{2\theta}\ln(\frac{1+\theta}{1-\theta})$ and an expected minimun price $E[min(p_1,p_2)]=2\int_{\frac{(1-\theta)v}{1+\theta}}^{v}p[1-F(p)]dF(p)=\frac{(1-\theta)v}{\theta}-\frac{(1-\theta)^2v}{2\theta^2}\ln(\frac{1+\theta}{1-\theta})$. Recall that for two firms and common c.d.f., the density of the lowest price is $2f(p)[1-F(p)]$.

A.2 Proof of Corollary 2

Suppose that the population mass equals $\mu>0$. In this case, the uninformed and the informed population equals $(1-\theta)\mu$ and $\theta\mu$ respectively. One determines the c.d.f. $F(p)$ as the solution to the equation $F(p)p\left(\frac{1-\theta}{2}\right)\mu+[1-F(p)]p\left(\frac{1-\theta}{2}+\theta\right)\mu=\frac{\mu(1-\theta)v}{2}$ but then dividing both the LHS and the RHS of the equation with μ results in that the population mass disappears.

A.3 Proof of Proposition 4

In this subsection, I prove that there does not exist any other equilibrium strategy with demand-side timing uncertainty across periods in regard to consumption but the one stated in Proposition 4 in the main body of the paper. First, I take up the question of whether probabilistic delay can be established and then I deal with some other randomization strategies.

A.3.1 Probabilistic Delay

Consider that each buyer randomizes between purchasing all three units in the first period or purchasing all three units in the second period and let θ the probability that each buyer purchases all three units in the first period.

The net payoff of a buyer purchasing all three units in period-2 is:

$$\theta\delta\left(2v + v_3 - 2\int_{\frac{v_3}{3}}^{v_3} \frac{v_3(2v_3 - p)}{6p^2}dp - \int_{\frac{v_3}{3}}^{v_3} \frac{v_3}{3p}dp - \frac{v_3}{3}\right)$$

$$+(1-\theta)\delta\left(2v + v_3 - \frac{5}{2}\int_{\frac{v_3}{5}}^{v_3} \frac{v_3(v_3 - p)}{8p^2}dp - \frac{1}{2}\int_{\frac{v_3}{5}}^{v_3} \frac{v_3(v_3 - p)}{8p^2}dp - \frac{4}{2}\int_{\frac{v_3}{5}}^{v_3} \frac{v_3(5p - v_3)}{8p^2}dp\right)$$

$$= \theta\delta\left(2v + v_3 - 2(\frac{2v_3}{3} - \frac{v_3}{6}\ln 3) - \frac{v_3}{3}\ln 3 - \frac{v_3}{3}\right)$$

$$+(1-\theta)\delta\left(2v + v_3 - 3(\frac{v_3}{2} - \frac{v_3}{8}\ln 5) - 2(\frac{5v_3}{8}\ln 5 - \frac{v_3}{2})\right)$$

$$= 2v\delta - \theta\delta v_3(\frac{7}{6} - \frac{7}{8}\ln 5) + \frac{\delta v_3}{2}(1 - \frac{7}{4}\ln 5)$$

When θ is sufficiently large, i.e. $\theta > \frac{5}{6}$, the net payoff of the buyer purchasing all three units in period-1 equals:

$$2v(1+\delta) + \delta v_3 - (3-\theta)\int_{\frac{(6\theta-5)\delta v_3}{5}}^{\delta v_3} \left(2\left(\frac{(6\theta-5)\delta v_3}{(10-6\theta)p}\right)^2 - \frac{2\delta v_3}{p}\left(\frac{6\theta-5}{10-6\theta}\right)^2\right)dp$$

$$-\theta\int_{\frac{(6\theta-5)\delta v_3}{5}}^{\delta v_3} \frac{(6\theta-5)\delta v_3}{(10-6\theta)p}dp$$

$$= 2v(1+\delta) + \delta v_3 - (3-\theta)\left(\frac{2(6\theta-5)\delta v_3}{10-6\theta} - \frac{2\delta v_3(6\theta-5)^2 \ln\left(\frac{5}{6\theta-5}\right)}{(10-6\theta)^2}\right)$$
$$-\frac{\theta}{2}\frac{(6\theta-5)\delta v_3}{10-6\theta}\ln\left(\frac{5}{6\theta-5}\right)$$

On the basis of numerical calculations, it can be concluded that the two payoffs above cannot be made equal for any values in the predefined parameter space.

Right below, I provide explanations on how to determine the payoffs in the first place.

Whenever the buyer in question is active in period-2, with $prob = \theta$ the other buyer buys all three units in period-1. In this case, period-2 setup consists of firms with capacities $k_1 = 2, k_2 = 5$ and total demand equal to 3 units. This gives an atomless c.d.f. $F_1(p) = \frac{3}{2} - \frac{v_3}{2p}$ for the low capacity firm, while the high capacity firm prices according to $F_2(p) = 1 - \frac{v_3}{3p}$ playing an atom at v_3 with $prob = \frac{1}{3}$. The common support is $[\frac{v_3}{3}, v]$ and the density of the lowest price is $f_1(p)(1-F_2(p)) + f_2(p)(1-F_1(p)) = \frac{v_3(2v_3-p)}{6p^2}$. Beware that the third unit will be purchased for sure by the high capacity firm. In case, however, that the other byuer buys all three units in period-2 which occurs with $prob = 1-\theta$, period-2 setup consists of firms with capacities $k_1 = 5, k_2 = 5$ and total demand equal to 6 units. This gives rise to a common c.d.f. $F(p) = \frac{5}{4} - \frac{v_3}{4p}$ defined on $[\frac{v_3}{5}, v_3]$. The density of the lowest price is $2f(p)(1-F(p)) = \frac{v_3(5p-v_3)}{8p^3}$ and the density of the highest price is $2f(p)F(p) = \frac{v_3(5p-v_3)}{8p^3}$.

Whenever the buyer in question is active in period-1, with $prob = 1-\theta$ the other buyer buys all three units in period-2; thereafter, first period purchases cost $3E[min(p_i, p_j)]$. However, with $prob = \theta$ the other buyer buys all three units in period-1, and this results in per buyer total purchasing cost $2E[min(p_i, p_j)] + \frac{1}{2}E[min(p_i, p_j)] + \frac{1}{2}E[max(p_i, p_j)]$ assuming an equal probability for buyers being rationed by the low-price firm. The expected demand in the first period from the sellers' point of view equals $6\theta^2 + 6\theta(1-\theta) = 6\theta$ and as long as $\theta > \frac{5}{6}$ this leads to minmax profits $(6\theta-5)\delta v_3 > 0$. Subsequently, sellers mix over the support $[\frac{(6\theta-5)\delta v_3}{5}, \delta v_3]$ according to the common atomless c.d.f. $F(p) = \frac{5}{10-6\theta} - \frac{(6\theta-5)\delta v_3}{(10-6\theta)p}$.

More strikingly, for $\theta \leq \frac{5}{6}$, a buyer that purchases all three units in period-1 enjoys zero prices regardless of whether the other buyer is active in period-1 or not. This is so because when θ takes these values the minmax payoff for firms in the first period is zero. Hence, the net consumer surplus equals $2v(1+\delta) + \delta v_3$ which obviously is superior to any other. In this case, indifference requires $2v(1+\delta)+\delta v_3 = 2v\delta - \theta\delta v_3(\frac{7}{6}-\frac{7}{8}\ln 5)+\frac{\delta v_3}{2}(1-\frac{7}{4}\ln 5)$. It checks out that there does not exist relative arrangement of parameters that satisfy the indifference condition.

A.3.2 Other Mixed Strategies with Delay

First Case ...

Second Case ...

Third Case ...