Environmental Policy, Spatial Spillovers and the Emergence of Economic Agglomerations

Efthymia Kyriakopoulou*and Anastasios Xepapadeas†

Athens University of Economics and Business

30 January 2009

Abstract

We explain the spatial concentration of economic activity, in a model of economic geography, when the cost of environmental policy - which is increasing in the concentration of emissions - and an immobile production factor act as centrifugal forces, while positive knowledge spillovers and iceberg transportation costs act as centripetal forces. We study the agglomeration effects caused by trade-offs between centripetal and centrifugal forces. The above effects govern firms' location decisions and as a result, they define the distribution of economic activity across space. We derive the rational expectations equilibrium and the optimum, compare the outcomes and characterize the optimal spatial policies.

JEL classification: R3, Q5, H2.

Keywords: Agglomeration, Spatial Economics, Environmental Policy, Knowledge Spillovers, Transportation Cost.

*Department of International and European Economic Studies, Athens University of Economics and Business (e-mail: efik@aueb.gr).

†Department of International and European Economic Studies, Athens University of Economics and Business and Beijer Fellow (e-mail: xepapad@aueb.gr).

1 Introduction

The distribution of population and activity across the landscape is undoubtedly uneven. Metropolitan areas - large population centers - play a major role in economic activity. Most OECD metropolitan areas have a higher GDP per capita than their national average, a higher labor productivity level and many of them tend to have faster growth rates than their countries as a whole. For example, Île-de-France is the most populated region of France - it accounts for 2.2% of the area of the country and 18.1% of its population. Economically, Île-de-France is one of the richest regions in the world and produces 33% of the GDP of France.[1]

The concept "metropolitan area" is based on the concept of a business or labor market area and is typically defined as an employment core (an area with a high density of available jobs) and the surrounding areas that have strong commuting ties to the core. Tokyo, Seoul, Mexico City, New Delhi and New York City are some examples of the largest metropolitan areas in the world, which include a large number of industries.[2] This process of clustering of economic activity in space is studied by agglomeration economics.

Agglomeration, once created, is sustained as a result of circular logic. For example, a shop is more likely to locate in a shopping street than in the centre of a residential area with no shops around. The same happens with specialized economic regions, like Silicon Valley. Silicon Valley is so famous for its development as a high-tech economic center and its large number of innovators and manufacturers, that the term is now generally used as a metonym for the high-tech sector. In Europe, large corporations and service or research centers, such as IBM, General Motors Europe, Toyota Europe, Google and Microsoft are moving to Zurich, which is a leading financial centre.

Despite their particular importance and interest, the spatial decisions of firms and economic agents haven't attracted a lot of attention from mainstream economics until recently. Some exceptions are earlier works that belong to the field of urban economics

[1] Its total GDP was €533.5 billion in 2007 with per capita GDP of €45,982 the same year. http://www.insee.fr/fr/ppp/bases-de-donnees/donnees-detaillees/cnat-region/pib_reg.xls

[2] For a review of metropolitan areas, see OECD (2006).

and have some kind of spatial structure.[3] Those works are very well analysed in the monograph, "The Spatial Economy", by Fujita, Krugman and Venables (1999).

In the 1990's, space started attracting the interest of economists again. According to Krugman (1998), the reason for this renewed interest was the fact that it is now possible to model imperfect competition and concepts like unexhausted scale economies are no longer intractable. The result was the emergence of New Economic Geography (NEG), which represents a new branch of spatial economics. The purpose of NEG is to explain the formation of a large variety of economic agglomerations in geographical space.

Economic models of agglomeration are based on centripetal forces that promote the concentration of economic activity and centrifugal forces that oppose it, studying the trade-offs between various forms of increasing returns and different types of mobility costs.[4] Moreover, some recent studies have included knowledge externalities - as agglomeration forces - in a spatial context.[5] These kinds of models have three forces that define the equilibrium allocation of business and residential areas: transportation costs and production externalities, that both pull economic activities together, and immobility of factors that pushes them apart.

Another characteristic of the models we have already referred to is the assumption that the spatial area under study is homogeneous. Contrary to this fact, economic activities are spatially concentrated because of dissimilarities in natural features, such as rivers, harbors or even exhaustible resources that are available in certain points in space. This "first nature" advantage hasn't been studied in depth yet. An exception is Fujita and Mori's (1996) paper that explained the role of ports in the formation of cities, using an increasing returns model. This assumption of nonuniformity in geographical space will be introduced in our model too.

Furthermore, cities are important generators of wealth, employment and productivity growth. The growing economic importance of places with high concentration of economic activity, such as metropolitan areas, raises important policy issues. More precisely, these

[3] For example, von Thünen (1826), Alonso (1964), and Henderson (1974).

[4] Some examples are Krugman (1991, 1993a, 1993b), Fugita, Krugman and Mori (1999). For a detailed discussion on forces that affect geographical concentration see Krugman (1998).

[5] See Lucas (2001), Lucas and Rossi-Hansberg (2002), Rossi-Hansberg (2005).

economic activities are not only associated with positive externalities, but also with certain negative externalities, such as congestion, pollution or high crime rates. Considering these issues, we try to model the problem of pollution and explain the spatial patterns of economic activity under environmental policy.[6]

According to Rauscher (2008), if pollution is taken into account, two spatial patterns are formed. In the first case, consumers trying to avoid pollution leave agglomerations, but the industry follows them in order to locate where consumption is. On the other hand, if environmental damage is large, consumers and the industry agglomerate in different locations. In that case, the high concentration of pollution promotes dispersion. An example of this dispersion is nuclear power stations, which locate in regions with low concentration of economic activity and population. As far as policy issues are concerned, there is a lot of research on how environmental regulation affects the location decisions of firms. Henderson (1996) shows that air quality regulation affects industrial location. A greater regulatory effort, leading to air quality improvement, results in the spreading out or the exiting of polluting industries. Greenstone (2002), using data from the Clean Air Act in the US, finds that environmental regulation restricts industrial activity. According to another recent study (Elbers and Withagen, 2004), pollution and environmental policy tend to countervail clustering that would occur in their absence. To put it differently, environmental policy acts as a centrifugal force.

In that context, we study the spatial structure of a single city or well defined region when firms are free to choose where to locate. As far as production is concerned, we assume positive knowledge spillovers. There are a lot of empirical studies that confirm the role of knowledge spillovers in the location decisions of firms and explain why firms choose to locate near each other. More specifically, Keller (2002) finds that technological knowledge spillovers are significantly local and their benefits decline with distance. Boltazzi and Peri (2003), using data for European regions, prove that knowledge spillovers resulting from patent applications are very localised and affect positively regions within a distance of 300 km. Carlino, Chatterjee and Hunt (2006) also provide evidence on patent

[6] van Marrewijk (2005), Lange and Quaas (2007) study the effect of pollution on agglomeration using Forslid and Ottaviano's (2003) core-periphery model.

intensity for metropolitan areas in US and conclude that it is affeced by employment density. These two characteristics: the employment density around a spatial point and the distance between firms, are used in the modeling of knowledge spillovers.

There is also a “first nature” advantage in a certain point in the area under study. We further assume here that firms use emissions as an input. The government, in order to avoid a high concentration of emissions in a single area, adopts environmental regulations. These regulations impose an additional production cost on firms. We assume that the environmental regulations refer to general environmental costs such as taxes or the cost of controlling the environment and imposing zoning systems which all increase with the concentration of emissions. As a result, the production process becomes more expensive, as firms have to pay an extra amount of money that depends not only on their own emissions but also on the aggregate concentration of emissions at the point where they decide to locate. The higher the number of firms at a certain point, the higher will be the concentration of emissions at that point.

Now, firms have to take into account two things: if they locate near other firms, they will benefit from positive knowledge spillovers, but they will have to pay a higher price for each unit of emissions used in the production of output. They also have to consider different transportation costs at each spatial point and the immobility of production factors. Under these assumptions, we define a rational expectations equilibrium (REE) and an optimal concentration of economic activity, we identify deviations between the two solutions and discuss policies with spatial characteristics.

The plan of the paper is as follows. In Section 2, we will present the model and its mathematical structure and prove the existence and the uniqueness of the REE. In Section 3, we determine the optimum of the model, while in Section 4, we derive optimal spatial policies. In Section 5, we make some numerical experiments and compare the different output distributions corresponding to the REE and the optimal solutions both in cases where environmental policy is imposed and in cases where environmental policy issues are not taken into account. We also refer to the concept of “pollution haven hypothesis” and show how it could be modelled using our modelling structure. In the final Section, we make some concluding remarks and give some ideas for future research.

2 Rational Expectations Equilibrium under Centripetal and Centrifugal Forces

We consider a single city or region located on a line of length S. So, 0 and S represent the western and eastern borders of the city, which is part of a large economy. In the city, there is a large number of small, identical firms that produce a single good. There are also workers who are geographically immobile and take no location decisions. The production process is characterized by externalities in the form of positive knowledge spillovers. This means that firms benefit from locating near each other and the total advantage they take depends on the amount of labor used in nearby areas and on the distance between them.

There is a port available at the point $\bar{r} \in [0, S]$, in our city, which is used to import machinery. So, machinery is another production factor that arrives at the port at a given price.[7] But the transportation of machinery inside the city is costly. This means that the point $\bar{r}$ has a spatial advantage, because if firms decide to locate there, they will pay no transportation cost for machinery. At all other points, the transportation cost will add an additional cost to the production process.

The last assumption we make is that emissions are used as an input in the production process.[8] According to Brock (1977), the idea behind this assumption is that techniques of production are less costly in terms of capital input (machinery in our case) if more emissions are allowed - a situation which is observed in the real world. In other words, if we use polluting techniques, we can reduce the total cost of production.

The borders of the city under study are strictly defined and firms can locate nowhere else.[9] Our intention is to study the location decisions of firms. More specifically, we aim to consider the equilibrium spatial distribution of production in order to determine the

[7] We assume that machinery producers provide their products at a given price that includes the transportation cost to the port.

[8] The concept of emissions as an input in the production function was first introduced by Brock (1977) and later used by other authors, eg. Rauscher (1994), Tahvonen and Kuuluvainen (1993), Xepapadeas (2005).

[9] Land is owned by landlords who play no role in our analysis.

distribution of firms over sites $r \in [0, S]$.

All firms produce the same traded good using labor, machinery, emissions and land. The good is sold around the world at a competitive price assuming no transportation cost. Production *per unit of land* at location $r \in [0, S]$ is given by:

$$q(r) = e^{\gamma z(r)} L(r)^a K(r)^b E(r)^c \tag{2.1}$$

where q is the output, L is the labor input, K is the machinery input, E is the amount of emissions used in production and z is the production externality, which depends on how many workers are employed at all locations and represents positive knowledge spillovers.[10]

$$z(r) = \delta \int_0^S e^{-\delta(r-s)^2} \ln(L(s)) \, ds \tag{2.2}$$

The function $k(r, s) = e^{-\delta(r-s)^2}$ is called a kernel. The production externality is a positive function of labor employed in all areas and is assumed to be linear and to decay exponentially at a rate δ with the distance between r and s. A high δ indicates that only labor in nearby areas affects production positively. In other words, the higher δ is, the more profitable it is for firms to locate near each other. When the production externality plays a major role in location decisions, each firm chooses to locate where all other firms are located. In terms of agglomeration economics, the production externality is a *centripetal* force, i.e. a force that promotes the spatial concentration of economic activity.

As already stated, there is a port at the point $\bar{r} \in [0, S]$, which is used for the imports of machinery. It is clear that the point $\bar{r}$ has spatial advantages over other possible locations. If the price of machinery at $\bar{r}$ is p_K, then iceberg transportation costs imply that the price at location r can be written as: $p_K(r) = p_K \; e^{\beta(r-\bar{r})^2}$. In other words, if one unit of machinery is transported from $\bar{r}$ to r, only a fraction $e^{-\beta(r-\bar{r})^2}$ reaches

[10] This kind of external effects that is interpreted as knowledge spillovers is used by Lucas (2001) and by Lucas and Rossi-Hansberg (2002) - with a different structure - and is consistent with Fujita and Thisse's (2002) analysis. The idea is that workers at a spatial point benefit from labor in nearby areas and thus, the distance between firms determines the production of ideas and the productivity of firms in a given region.

r.[11] So, β is the transportation cost per square unit of distance, which is assumed to be positive and finite. It is obvious that the total transportation cost of machinery increases with distance.[12] Thus, firms have an incentive to locate near point $\bar{r}$ to avoid a higher transportation cost. Like knowledge spillovers, the transportation cost is a *centripetal* force.

The emissions used in the production process damage the environment. The damage (D) at each spatial point is a function of the total concentration of emissions (X) at the same point.

$$D(r) = X(r)^{\phi} \tag{2.3}$$

where $\phi \geq 1,\ \ D'(X) > 0,\ \ D''(X) \geq 0,$ and the marginal damage function is:

$$MD(r) = \phi\ X(r)^{\phi-1} \tag{2.4}$$

Each firm has to pay a "price" or a tax for each unit of emissions used as an input. This tax τ is a function of the marginal damage (MD):

$$\tau(r) = \theta\ MD(r) \tag{2.5}$$

where $0 \leq \theta \leq 1$, and $\theta = 1$ means that the full marginal damage at point r is charged as a tax. In other words, each firm pays an amount of money for the emissions it uses in the production of the output, but the per unit tax depends not only on its own emissions, but on the total concentration of emissions at the spatial point where it decides to locate.The tax function can be written as:

$$\tau(r) = \theta\ \phi\ X(r)^{\phi-1} = \psi\ X(r)^{\phi-1} \tag{2.6}$$

[11] For a detailed analysis of "iceberg costs", see Fujita, Krugman and Venables (1999) and Fujita and Thisse (2002). Conceptually, with the "iceberg" forms, we assume that a fraction of the good transported melts away or evaporates in transit.

[12] We can use another formulation of iceberg transportation cost: $p_K(r) = p_K\ e^{\beta|r-\bar{r}|}$, instead of $p_K(r) = p_K\ e^{\beta(r-\bar{r})^2}$, without changing the conclusions of the analysis. We prefer the latter so as to have the same exponential terms in the whole model.

where $\psi = \theta\ \phi$, $\tau'(X) > 0$, $\tau''(X) \geq 0$.

When solving our model, we use the logarithm of the tax function, thus:

$$\ln \tau(r) = \ln \psi + (\phi - 1)\ \ln X(r) \tag{2.7}$$

where

$$\ln X(r) = \int_0^S e^{-\zeta(r-s)^2} \ln(E(s))\ ds \tag{2.8}$$

Equation (2.8) implies that aggregate emissions (X) at a point r are a weighted average of the emissions used in nearby locations, with kernel $k(r,s) = e^{-\zeta(r-s)^2}$. This might capture the movement of emissions in nearby places. A high ζ indicates that only nearby emissions affect the total concentration of emissions at point r. In the real world, the value of ζ depends on weather conditions and on natural landscape. As we have assumed that the only dissimilarity in our land is the existence of a port, we suppose that ζ is influenced only by weather conditions. Specifically, if it is windy, ζ takes a low value and areas at a long distance from r are polluted by emissions generated at r. As ζ increases, the concentration of emissions in certain areas does not affect other areas so much.

Thus, the cost of environmental policy, $\tau(X(r))$, increases the total production cost for the firms. The extra amount of money that a firm pays, in the form of taxation, depends on the total emissions at the point where it has decided to locate. To put it differently, the higher the concentration of industry at an interval $[s_1, s_2] \in [0, S]$, the higher the cost firms will be obliged to pay. In that way, the environmental policy is a *centrifugal* force, i.e. a force that opposes spatial concentration of economic activity.

Let w be the wage rate, which is the same across sites, and let p be the competitive price of output. A firm located at r chooses labor, machinery and emissions to maximize its profits. Thus, the *profit per unit of land*, $\hat{Q}$, at location r, is given by:

$$\hat{Q}(r) = \max_{L,K,E}\ p\ e^{\gamma z(r)} L(r)^a K(r)^b E(r)^c - wL(r) - p_K\ e^{\beta(r-\bar{r})^2} K(r) - \tau(r)\ E(r) \tag{2.9}$$

A firm located at the site r treats the production externality $z(r)$ and the aggregate emissions $X(r)$ as exogenous parameters z^e and X^e respectively. Assuming $X(r)$ is exogenous to the firm implies that the tax $\tau(r)$ is treated as a parameter at each r. Then, the first order conditions (FOC) for profit maximization are:

$$pa\ e^{\gamma z(r)} L(r)^{a-1} K(r)^b E(r)^c = w \tag{2.10}$$

$$pb\ e^{\gamma z(r)} L(r)^{a} K(r)^{b-1} E(r)^c = p_K\ e^{\beta(r-\overline{r})^2} \tag{2.11}$$

$$pc\ e^{\gamma z(r)} L(r)^{a} K(r)^b E(r)^{c-1} = \tau(r) \tag{2.12}$$

Setting $z(r) = z^e$, $X(r) = X^e$, the FOC define a *rational expectations equilibrium spatial distribution* of labor, machinery and emissions at each point $r \in [0, S]$. After taking logs on both sides and doing some transformations, which are described in Appendix A, the FOC result in a system of second kind Fredholm integral equations with symmetric kernels:

$$\begin{aligned} \tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}y(s)ds + \tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_1(r) = y(r) \qquad (2.13)\\ \tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}y(s)ds + \tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_2(r) = x(r)\\ \tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}y(s)ds + \tfrac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_3(r) = \varepsilon(r) \end{aligned}$$

where $y(r) = \ln L(r)$, $x(r) = \ln K(r)$, $\varepsilon(r) = \ln E(r)$ and $g_1(r)$, $g_2(r)$, $g_3(r)$ are some known functions.

Proposition 1 : *Assume that: (i) the kernel $k(r, s)$ defined on $[0, 2\pi] \times [0, 2\pi]$, is an L_2-kernel which generates the compact operator W, (ii) $1-a-b-c$ is not an eigenvalue of W, and (iii) G is a square integrable function, then a unique solution determining the rational expectations equilibrium distribution of inputs and output exists.*

The proof of existence and uniqueness of the REE is presented in the following steps:[13]

[13] See Moiseiwitsch (2005) for more detailed definitions.

- A function $k(r,s)$ defined on $[a,b]\times[a,b]$ is an L_2-kernel if it has the property that $\int_a^b\int_a^b |k(r,s)|^2\,drds<\infty$.

  The kernels of our model have the following formulation: $e^{-\xi\ (r-s)^2}$ with $\xi=\delta,\ \zeta$ (positive numbers) and are defined on $[0,2\pi]\times[0,2\pi]$.

  We need to prove that $\int_0^{2\pi}\int_0^{2\pi}\left|e^{-\xi\ (r-s)^2}\right|^2 drds<\infty$.

  Rewriting the left part of inequality, we get: $\int_0^{2\pi}\int_0^{2\pi}\left|\frac{1}{e^{\xi\ (r-s)^2}}\right|^2 drds$.

  The term $\frac{1}{e^{\xi\ (r-s)^2}}$ takes its highest value when $e^{\xi\ (r-s)^2}$ is very small. But the lowest value of $e^{\xi\ (r-s)^2}$ is obtained when either $\xi=0$ or $r=s$ and in that case $e^0=1$. So, $0<\left|\frac{1}{e^{\xi\ (r-s)^2}}\right|<1$. When $\left|\frac{1}{e^{\xi\ (r-s)^2}}\right|=1$, then: $\int_0^{2\pi}\int_0^{2\pi}\left|\frac{1}{e^{\xi\ (r-s)^2}}\right|^2 drds=4\ \pi^2<\infty$. Thus, the kernels of our system are L_2-kernels.

- If $k(r,s)$ is an L_2- kernel, the integral operator

$$(W\phi)(r)=\int_a^b k(r,s)\,\phi(s)\,ds\ ,a\le s\le b$$

  that it generates is bounded and

$$\|W\|\le\left\{\int_a^b\int_a^b |k(r,s)|^2\,drds\right\}^{\frac{1}{2}}$$

  So, in our model the upper bound of the norm of the operator generated by the L_2-kernel is:

  $\|W\|\le\left\{\int_a^b\int_a^b |k(r,s)|^2\,drds\right\}^{\frac{1}{2}}=\left\{\int_0^{2\pi}\int_0^{2\pi}\left|\frac{1}{e^{i\ (r-s)^2}}\right|^2 drds\right\}^{\frac{1}{2}}\le 2\pi$

- If $k(r,s)$ is an L_2- kernel and W is a bounded operator generated by k, then W is a compact operator.

- If $k(r,s)$ is an L_2-kernel and generates a compact operator W, then the integral equation

$$Y-\left(\tfrac{1}{1-a-b-c}\right)W\ Y=G \tag{2.14}$$

  has a *unique* solution for all square integrable functions G, if $(1-a-b-c)$ is not an eigenvalue of W (Moiseiwitsch, 2005). If $(1-a-b-c)$ is not an eigenvalue of

W, then $\left(I - \frac{1}{1-a-b-c}W\right)$ is invertible. We show in Appendix C that the system (2.13) can be transformed into a second kind Fredholm Integral equation of the form (2.14).

To solve the system (2.13) numerically, for the REE, we use a modified Taylor-series expansion method (Maleknejad et al., 2006). More precisely, a Taylor-series expansion can be made for the solutions $y(s)$ and $\varepsilon(s)$ in the integrals of the system (2.13). We use the first two terms of the Taylor-series expansion (as an approximation for $y(s)$ and $\varepsilon(s)$) and substitute them into the integrals of (2.13). After some substitutions, we end up with a linear system of ordinary differential equations of the form:

$$\theta_{11}(r)\ y(r) + \theta_{12}(r)\ y'(r) + \theta_{13}\ y''(r) + \sigma_{11}\ \varepsilon(r) + \sigma_{12}\ \varepsilon'(r) + \sigma_{13}\ \varepsilon''(r) = g_1(r) \quad (2.15)$$

$$x(r) + \theta_{21}(r)\ y(r) + \theta_{22}(r)\ y'(r) + \theta_{23}\ y''(r) + \sigma_{21}\ \varepsilon(r) + \sigma_{22}\ \varepsilon'(r) + \sigma_{23}\ \varepsilon''(r) = g_2(r)$$

$$\theta_{31}(r)\ y(r) + \theta_{32}(r)\ y'(r) + \theta_{33}\ y''(r) + \sigma_{31}\ \varepsilon(r) + \sigma_{32}\ \varepsilon'(r) + \sigma_{33}\ \varepsilon''(r) = g_3(r)$$

In order to solve the linear system (2.15), we need an appropriate number of boundary conditions. We construct them and then we obtain a linear system of three algebraic equations that can be solved numerically.

The maximized value of the firm's profit $\hat{Q}(r)$ is also the land-rent per unit of land that a firm would be willing to pay to operate with these cost and productivity parameters at location r. Since the decision problem at each location is completely determined by the technology level z, the wage rate w, the price of machinery p_K , the output price p and the concentration of emissions X, the FOC of the maximization problem give us the REE values of labor, machinery and emissions used at each location: $L = \hat{L}(z, w, p_K, p, X)$, $K = \hat{K}(z, w, p_K, p, X)$ and $E = \hat{E}(z, w, p_K, p, X)$. Finally, the equilibrium distribution of output is given by: $q = \hat{q}(z, w, p_K, p, X)$.

3 The Optimal Solution

After having solved for the REE, we study the optimal solution, by assuming the existence of a regulator who takes all the location decisions. The regulator's objective is to maximize the total value of land in the city, which implies maximization of the profits net of damages caused by the concentration of emissions across the spatial domain. It's clear that the regulator takes into account the real damage caused in the city by the concentration of emissions. So, the optimal problem is:

$$\max_{L,K,E} \int_0^S \left[p\ e^{\gamma z(r)} L(r)^a K(r)^b E(r)^c - wL(r) - p_K\ e^{\beta(r-\overline{r})^2} K(r) - D(r) \right] dr \tag{3.1}$$

Substituting (2.3) for the damage function, inside the integral, we get:

$$\max_{L,K,E} \int_0^S \left[p\ e^{\gamma z(r)} L(r)^a K(r)^b E(r)^c - wL(r) - p_K\ e^{\beta(r-\overline{r})^2} K(r) - X(r)^\phi \right] dr \tag{3.2}$$

The FOC for the optimum are:

$$ape^{\gamma z(r)} L(r)^{a-1} K(r)^b E(r)^c + pe^{\gamma z(r)} L(r)^a K(r)^b E(r)^c \gamma \frac{\partial z(r)}{\partial L(r)} = w \tag{3.3}$$

$$p\ b\ e^{\gamma z(r)} L(r)^a K(r)^{b-1} E(r)^c = p_K\ e^{\beta(r-\overline{r})^2} \tag{3.4}$$

$$cp\ e^{\gamma z} L(r)^a K(r)^b E(r)^{c-1} - \phi X(r)^{\phi-1} \frac{\partial X(r)}{\partial E(r)} = 0 \tag{3.5}$$

Comparing the FOC for the optimum to those for the REE, we notice some differences. First, the FOC with respect to $L(r)$ (3.3) contains one extra term - the second term on the left-hand side. That is, the regulator, when choosing $L(r)$, takes into account the positive impact of $L(r)$ on the production of all other sites, through knowledge spillovers. So, increasing labor at r has two effects: it increases output in the standard way, but it increases the positive externalities at all other sites as well. In the same way, labor increases at other sites increase the externality in r. This externality is now taken into account, while the firm, maximizing its own profits, considered the externality as a fixed parameter.[14]

The second difference between the optimum and the REE concerns the FOC with respect to $E(r)$, i.e. equation (3.5). The first term in the left-hand side is the marginal product of emissions, which is the same as the FOC of the REE. The difference is in the second term, which shows how changes in the value of emissions at r affect the total concentration of emissions, not only at r but also at all other sites. This damage, which is caused by the total concentration of emissions in our spatial economy and is altered every time emissions increase or decrease, is now taken into account by the regulator.

After making some transformations, we end up with the following system of second kind Fredholm integral equations with symmetric kernels:

$$\phi \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s) ds + g_1'(r) = y(r) \tag{3.6}$$

$$\phi \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s) ds + g_2'(r) = x(r) \tag{3.7}$$

$$-\frac{\gamma\delta}{c} \int_0^S e^{-\delta(r-s)^2} y(s) ds + \frac{(1-a-b)\phi}{c} \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s) ds + g_3'(r) = \varepsilon(r) \tag{3.8}$$

where $y(r) = \ln L(r),\ x(r) = \ln K(r),\ \varepsilon(r) = \ln E(r)$ and $g_1'(r),\ g_2'(r),\ g_3'(r)$ are some known functions. The existence and the uniqueness of the solution can be proved following the same steps which were presented in Section 2. To determine a numerical solution of the problem, we follow the same method of Taylor-series expansion used in the REE case. This approach provides an accurate approximate solution of the integral system as demonstrated by some numerical examples in Section 5.

4 Optimal Policy Issues

The differences between the REE and the optimum give us some intuition about the design of optimal policies. These differences come from the fact that the production externality $z(r)$ and the aggregate emissions $X(r)$ are taken as parameters in the case of REE, while the regulator takes them into account. Specifically, comparing equations

[14] See Appendix B.

(2.10) and (2.12) with (3.3) and (3.5) respectively, we observe that the latter ones have one extra term each. Thus, the FOC with respect to $L(r)$ for the REE equates the marginal product of labor with the given wage rate: $MP_L = w$, while the same condition for the optimum which takes into account the positive knowledge spillovers of labor to nearby areas is: $MP_L + q\ z'(L) = w$. The design of optimal policy in that case is determined by the extra term: $q\ z'(L)$. So, setting $v^*(r) = q\ z'(L) = pe^{\gamma z(r)} L(r)^a K(r)^b E(r)^c \gamma \frac{\partial z(r)}{\partial L(r)}$, the regulator can impose a new wage rate that is no longer constant across space and is equal to: $w^*(r) = w - v^*(r)$.[15] Now, the FOC for the REE becomes $MP_L = w^*(r) = w - v^*(r)$. Conceptually, the new wage rate $w^*(r)$ takes into account the changes in the knowledge spillovers across space, when a firm decides to employ more or fewer workers. The function $v^*(r)$ can also be considered as a subsidy that is given to firms to employ more workers. In that way, firms pay a lower wage $w^*(r) < w$, employ more labor, benefit from the higher knowledge spillovers and produce more output.

Probably more interesting is the design of optimal environmental policy. Thus, the firm trying to maximize its own profits equates the marginal product of emissions with the tax imposed on each unit of emissions used in the production process: $MP_E = \tau(r)$, while the regulator equates the marginal product of emissions with the marginal damage of emissions: $MP_E = MD_E$. But the difference between the $MD_E = \phi X(r)^{\phi-1} \frac{\partial X(r)}{\partial E(r)}$ and the tax function $\tau(r) = \ \phi\ X(r)^{\phi-1}$ is created by the term $\frac{\partial X(r)}{\partial E(r)}$.[16] This term shows that when a firm increases (decreases) the amount of emissions used in the production process, the total concentration of emissions at all spatial points increases (decreases) too. On the other hand, in the REE case, each firm decides about the amount of the emissions used as an input, taking the total concentration of emissions across space as given. It does not account for the fact that its own emissions at r affect the aggregate concentration of emissions in other areas. Thus, the designer of optimal environmental policy has to consider the extra damage caused at all spatial points from the use of emissions at r. As a result, the optimal tax function has to satisfy: $\tau^*(r) = MD_E = \phi X(r)^{\phi-1} \frac{\partial X(r)}{\partial E(r)}$ and

[15] The term $\frac{\partial z(r)}{\partial L(r)}$ is defined in Appendix B.

[16] We assume here that taxation at the REE charges the full marginal damage caused by aggregate emissions at a specific site, so $\theta = 1$ and $\psi = \phi$. The term $\frac{\partial X(r)}{\partial E(r)}$ is defined in Appendix B.

firms now equate $MP_E = \tau^*(r)$. Thus, in the spatial model, a tax equal to full marginal damages at the REE, as defined in (2.5) with $\theta = 1$, does not mean full internalization of the social cost as it is usually understood in environmental economics without spatial considerations. This is because setting $\tau(r) = \ \phi \ X(r)^{\phi-1}$ ignores this spatial externality which is captured by the term $\frac{\partial X(r)}{\partial E(r)}$. We will refer to setting the emission tax at $\tau(r)$ as REE-internalization (RRE-I) and setting it at $\tau^*(r)$ as the optimal internalization. Imposing the optimal policy rules, $v^*(r)$ and $\tau^*(r)$, the REE can reproduce the optimum.

It's worth noticing that the enforcement of the optimal taxation, $\tau^*(r)$, implies the implementation of a different tax at each spatial point. Nevertheless, this is considered to be an unusual scheme of taxation if we compare it to the real environmental policies. For this reason, based on the optimal taxation analysed above, the regulator could enforce zoning taxation. In that case, we have areas or zones with flat environmental tax. More specifically, in the areas with high concentration of economic activity, which suffer from serious pollution problems, the environmental tax will be high, but constant. The opposite will be true for the areas of low production, where aggregate emissions and therefore, the cost of environmental policy will be low. Following the optimal policy instruments above, in the zone $[s_1, s_2] \in [0, S]$, the optimal flat environmental tax (per unit of land) will be: $ztax^* = \frac{1}{s_1 - s_2} \int_{s_1}^{s_2} \ \tau^*(s) \ ds$.

Finally, the regulator, in order to implement the efficient allocation as an equilibrium, uses the two instruments analysed above: the subsidy $v^*(r)$ and the environmental tax $\tau^*(r)$. As far as the subsidy is concerned, the regulator gives some money to the firms, so as to encourage them to employ more workers. The total amount of money he has to spend is equal to: $\int_0^S \ v^*(s) \ ds$. In a similar way, the aggregate amount of money he receives from the enforcement of the optimal environmental tax is: $\int_0^S \ \tau^*(s) \ ds$. It's easy to predict that the tax and subsidy will not equal one another in most cases. However, in the optimum, we should achieve a balanced budget. In case where the expenditures are greater than the revenues, or $\int_0^S \ v^*(s) \ ds > \int_0^S \ \tau^*(s) \ ds$, the regulator could impose a lump-sum tax on land owners, so as to cover the difference. Then, the tax per unit of land would be equal to: $\overline{tax} = \frac{1}{S} \int_0^S \ [v^*(s) - \tau^*(s)] \ ds$. In the opposite case, when the revenues exceed the expenditures, or $\int_0^S \tau^*(s)\ ds > \int_0^S v^*(s)\ ds$, the regulator could give a lump-sum subsidy to firms. In order to receive this financial support, the firms will be obliged to finance R&D in pollution control and clean production processes. The subsidy per unit of land, in that case, is: $\overline{sub} = \frac{1}{S}\int_0^S\ [\tau^*(s) - v^*(s)]\ ds$.

5 Numerical Experiments

The model of the business sector of a single city analysed above involves fourteen parameters $\alpha,\ b,\ c,\ \beta,\ \delta,\ S,\ \gamma,\ p,\ w,\ p_K,\ \bar{r},\ \zeta,\ \phi,\ \psi$. Given these parameters, we can predict both the REE and the optimal patterns of output on the given interval. The corresponding distributions of output, labor, machinery and emissions will determine the location of firms in both cases and will characterize the optimal spatial policies. The numerical approach of the Taylor-series expansion, described above, will give us the equilibrium and the optimal values of inputs and output.

The map of a city will be defined by the two opposing forces already mentioned. On the one hand, there are the production externalities and the transportation cost of machinery that pull economic activity together and, on the other hand, there are the cost of environmental policy and the "immobility" of land that push it apart. This trade-off between centripetal and centrifugal forces will determine the geographical structure of the economy. Giving different values to the parameters, we may end up in a monocentric economy which presents the clustering of economic activity around a spatial point, or in a pattern where the economic activity will be concentrated in two or more regions.

The simulations discussed in this Section will provide maps resulting from the models of Sections 2 and 3. The share of labor is set to $\alpha = 0.6$, the share of the machinery is $b = 0.25$ and the share of emissions is $c = 0.05$. Given these values, we let the implied share of land be 0.1. The length of the city is $S = 2\pi$. In the business sector analysed here, we consider wages ($w = 1$) and the price of machinery ($p_K = 1$) as given and the same is assumed for the price of output which is $p = 10$. We set a reasonable value for

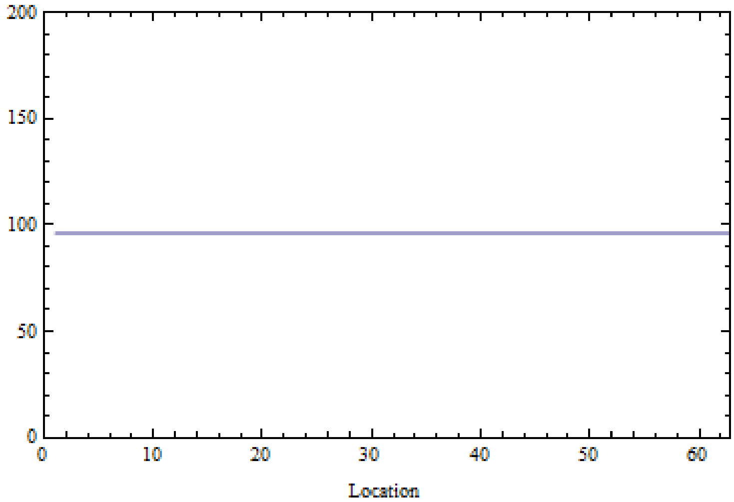


Figure 1: Benchmark case: The distribution of production under no agglomeration forces.

γ, that is 0.01.[17] We also assume that there is a port at the point $\bar{r} = \pi$. The value $\phi = 1.5$ implies an increasing and convex damage function. Finally, the ζ parameter, which shows how much the concentration of emissions at site r affects the concentration of emissions at other sites, is set to 0.5.[18] To study the economy's possible spatial structure, we hold the above parameters constant and vary just the transportation cost β, the "strength" of knowledge spillovers δ and the ψ parameter, which indicates whether the taxation internalizes fully or partly the marginal damage caused by the concentration of emissions.

As a benchmark case, we study the distribution of economic activity under no agglomeration forces, i.e. $\gamma = 0$, $\delta = 0$, $\beta = 0$, $\zeta = 0$, $\psi = 0$. This means that there is no production externality, no transportation cost for machinery and no environmental policy to increase the cost of the production process. In other words, a firm doesn't benefit at all from nearby firms, doesn't pay anything for the emissions used in production, and the per unit cost of machinery is the same at all locations. As expected, Figure 1 shows that the distribution of production is uniform over the given interval. In that case, firms have no incentives to locate at any special point of our economy.

[17] This value of γ is low enough to ensure that the "no black hole" assumption, described in Fujita, Krugman and Venables (1999), holds.

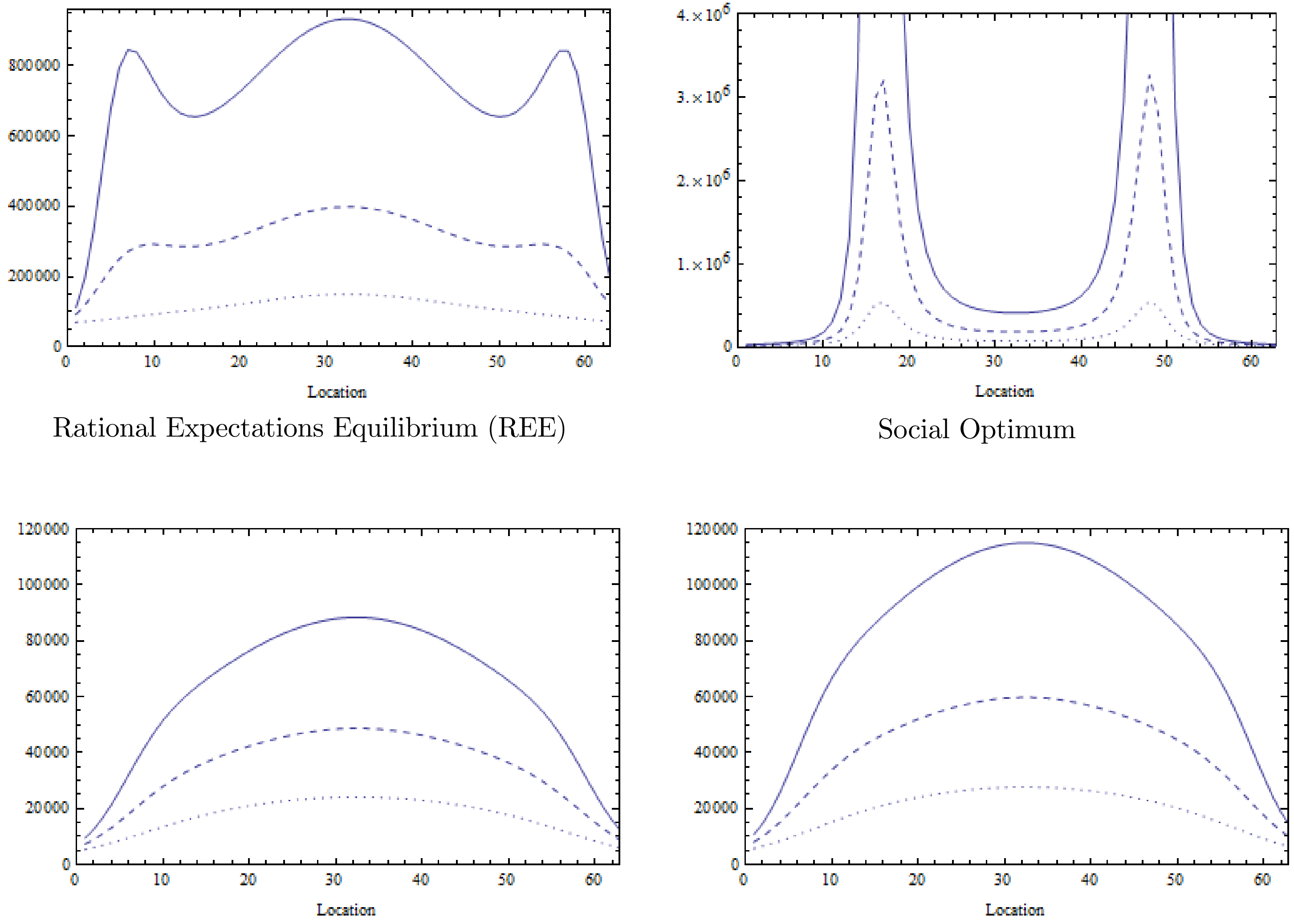
[18] When we study the effect of taxation in the spatial structure, we will give one more value to ζ in order to see how, under the assumption of "more localized" pollution, the environmental policy changes the concentration of economic activity.

Changing the parameters results in different maps. As we have a lot of parameters in our model, the results we can obtain are numerous too. We will present some interesting cases below which are worth mentioning and explain the structure of the model. Each set of parameters will provide four maps, all presenting the distribution of economic activity. We will study in detail the REE and the optimum, both in the presence of environmental regulations and in the case where there are no environmental considerations. This allows us to present the differences between the REE and the optimum and to explain how environmental policy affects the spatial structure of our city, on the interval $[0, S]$.

5.1 *Knowledge Spillovers*

Figures 2 and 3 present the distribution of production resulting from the δ values of 1, 2 and 3. The higher δ is, the more profitable it is for firms to locate near each other, so as to benefit from positive knowledge spillovers. In other words,the centripetal force of production externality is stronger when δ is high, and as a result, economic activity is more concentrated at certain sites. Figure 2 uses a low value of transportation cost ($\beta = 0.045$) and Figure 3 uses a higher one ($\beta = 0.075$). Moreover, in these examples, the marginal damage caused by the concentration of emissions is fully internalised ($\psi = 1.5$).

The first map, in Figure 2, presents the rational expectations equilibrium under environmental policy. The lowest value of δ is the closest to the benchmark case of the uniform distribution of production. In that case, the transportation cost is low. Also, the low value of δ means that knowledge spillovers do not decline fast with distance. So, the two centripetal forces do not have a strong effect and the result is the distribution of production given by the dotted line. When δ increases, there are two effects: first, spillovers affect the output more and the production increases at each site, and second, there are more incentives for agglomerations because benefits decline faster with distance. In that way, we observe a higher production at each spatial point. But, to produce more output, firms use more emissions and the total concentration of emissions increases at each point. When the concentration of emissions is very high, the price of emissions is high too. So, when firms decide where to locate, they take into account the centripetal



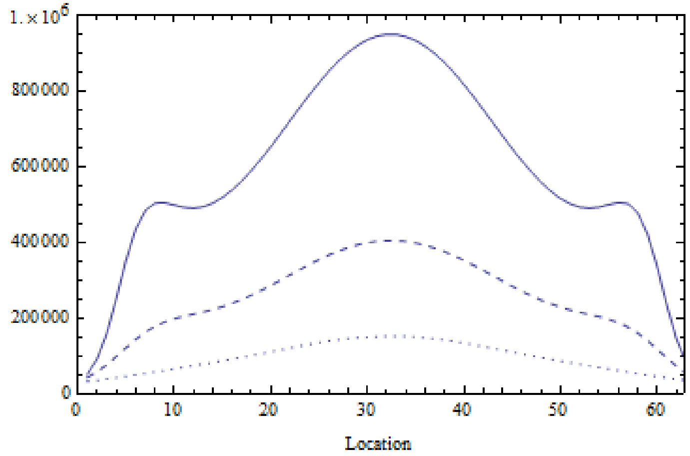
Rational Expectations Equilibrium (REE)

Social Optimum

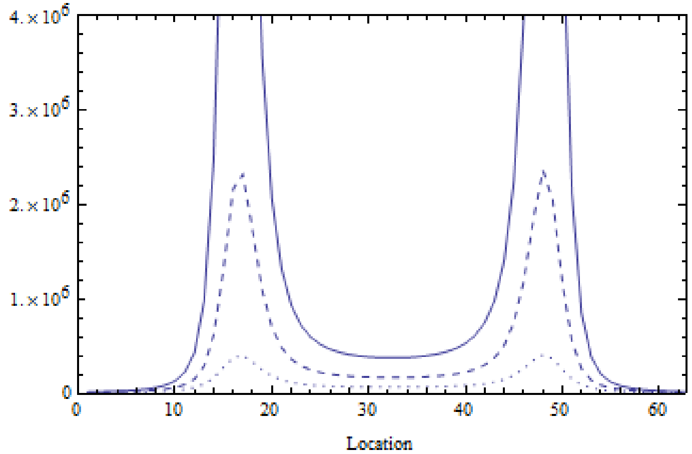
Social Optimum without Environmental Considerations

REE without Environmental Considerations

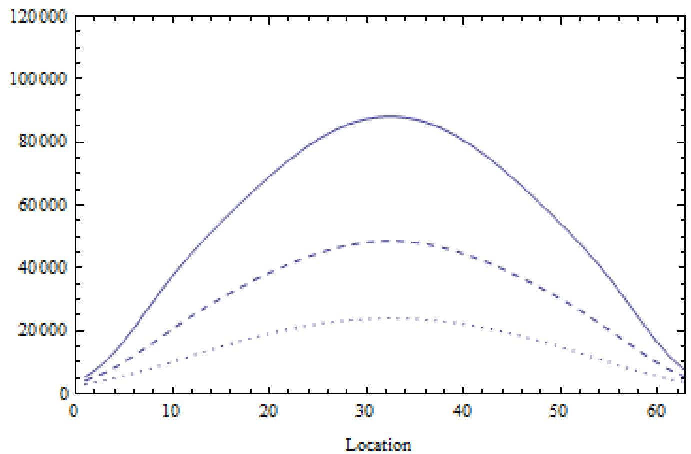
Figure 2: The Distribution of Economic Activity: Changes in the values of delta for low transportation cost (beta=0.045). Dotted Line: delta=1, Dashed Line: delta=2, Solid Line: delta=3



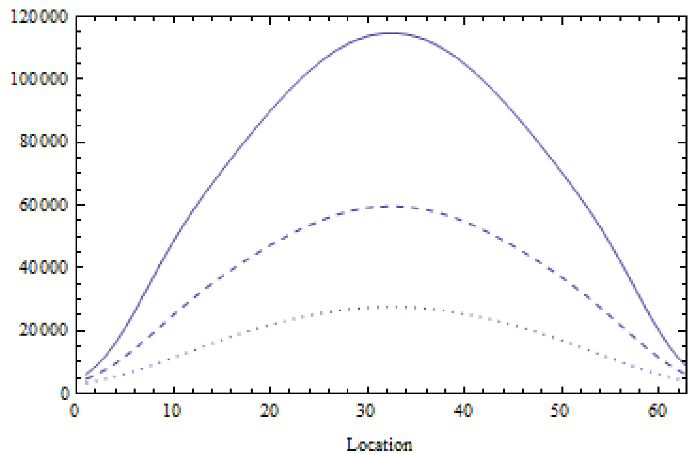
Rational Expectations Equilibrium (REE)



Social Optimum



Social Optimum without Environmental Considerations



REE without Environmental Considerations

Figure 3: The Distribution of Economic Activity: Changes in the values of delta for high transportation cost (beta=0.075). Dotted Line: delta=1, Dashed Line: delta=2, Solid Line: delta=3

force of strong knowledge spillovers and the centrifugal force of environmental policy. The trade-off between these two opposing forces forms the three peaks we observe in case $\delta = 3$ (solid line). The conclusions are, more or less, the same, if we observe Figure 3. The only difference comes from the higher value of transportation cost ($\beta = 0.075$). So, here the higher transportation cost decreases the concentration of economic activity in areas near the boundaries. In that way, the two peaks near the boundaries (in case $\delta = 2,\ 3$), are lower. However, the central peak is higher, as the transportation cost near the city centre is low in every case.

In the same Figures (2, 3), we also observe the REE without environmental considerations. The economic activity is now concentrated around the city centre, because of the two centripetal forces: the transportation cost and the knowledge spillovers. Stronger knowledge spillovers lead to a higher distribution of economic activity at the city centre. The use of land as a production factor deters economic activity from concentrating entirely at the city centre. The absence of environmental policy allows the formation of a unique peak.

Finally, Figure 2 shows the optimal distribution of economic activity under environmental considerations. As stated above, the regulator takes into account how labor in one area benefits from labor in nearby areas and how emissions in one area affect the total concentration of emissions in other areas. In that way, the regulator internalizes the production externality and the damage caused by the use of emissions in the production function. The result is the formation of two peaks near the points $r = 1.6, \quad 4.7$. The explanation is that at the optimum with environmental considerations, the regulator realizes the positive interaction of firms located at nearby areas, but he also takes into account the fact that if all firms locate around "one" spatial point, then the cost of environmental policy will be very high. So, the optimal solution is to cluster around "two points". The same is true for Figure 3, but, here the higher transportation cost leads to a lower concentration of economic activity around the two peaks.[19] The optimal

[19] In the social optimum, the solid line corresponds to $\delta = 3$. The two peaks of that line do not appear in the Figure, because we wanted to draw all three curves in one Figure, so as to point out the differences. So, at the spatial points $r = 1.6,\ 4.7,$ where we have the two peaks, the corresponding distribution value if $\beta = 0.045$ (Figure 2) is 1.5×10^7 and if $\beta = 0.075$ (Figure 3) is 1×10^7.

distribution, if there is no environmental policy, is illustrated by a unique peak. One final remark is that when comparing the REE and the optimum, if there are no environmental considerations, then the regulator, by internalizing the production externality, leads to a higher distribution of economic activity at each spatial point.

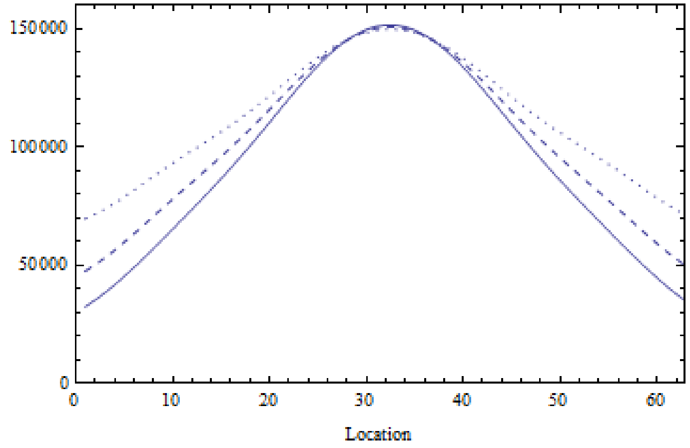
Taking everything into account, in both cases (REE and optimum), the environmental policy works to discourage the clustering of economic activity around one spatial point, which would occur in its absence.

5.2 *Transportation Cost*

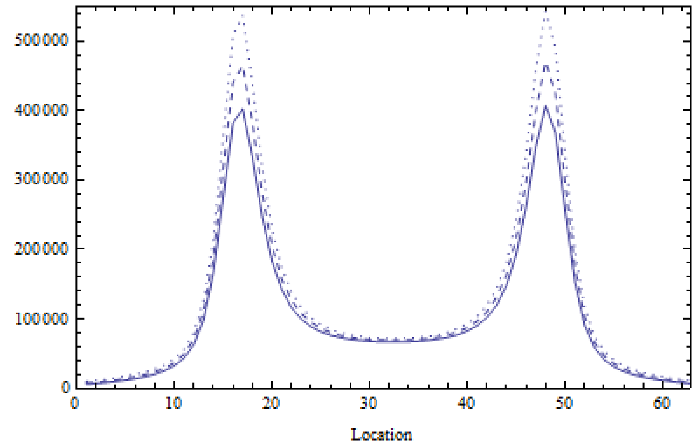
Machinery is available at the city centre ($\bar{r} = \pi$) without transportation cost. When firms use machinery at a site different from $\bar{r} = \pi$, they are obliged to pay a transportation cost. To study how changes in transportation cost affect the spatial structure of our city, we use the values $\beta = 0.045$, 0.06 and 0.075. The high value of β (0.075), was selected to double the per unit price of machinery at the boundaries ($r = 0,\ S$) and the low value of β (0.045) to increase the per unit price of machinery by 50% at the same points. These values of β are combined with δ values of 1 and 3 and the results are shown in Figures 4 and 5.

In Figure 4, we observe the clustering of economic activity around the city centre, which is the result of the low value of δ.[20] Higher transportation costs (solid line) imply lower densities at the boundaries and at all other points, except for $\bar{r} = \pi$. This is the case of REE. The centripetal and centrifugal forces are not very strong here and economic activity is concentrated around the centre, as the point $\bar{r} = \pi$ has a spatial advantage: machinery is available without transportation cost. For $\delta = 3$ (Figure 5), the low transportation cost forms three peaks. This is the result of the trade-off between knowledge spillovers and environmental policy.[21] However, higher values of β lead to a lower concentration around the two boundary peaks, as it is more expensive now to transport a lot of machinery to points far from the city centre.

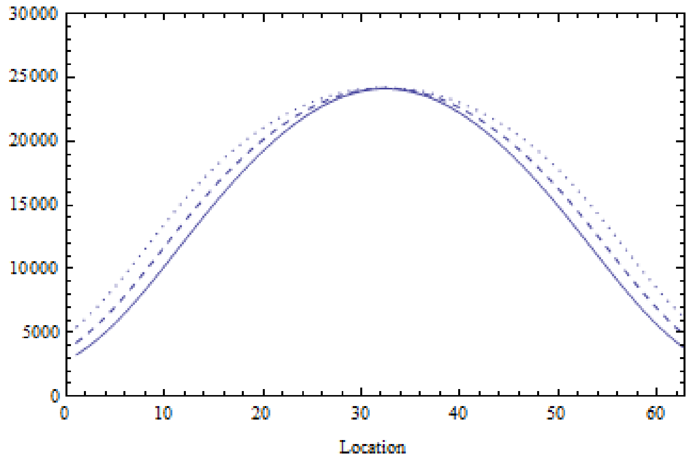
[20] See the analysis for Knowledge Spillovers above.
[21] See the analysis for Knowledge Spillovers above.



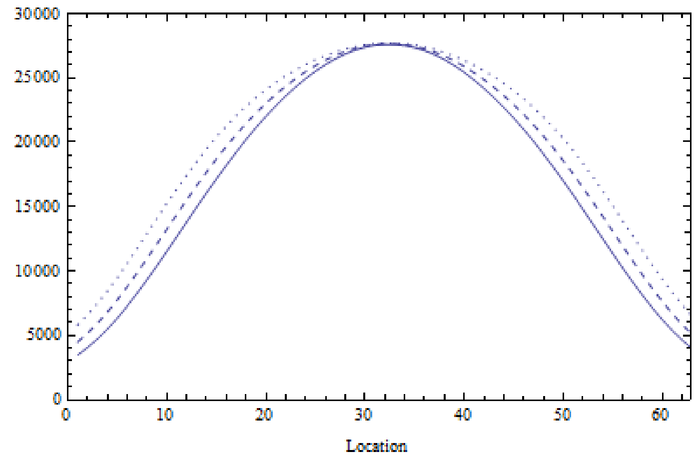
Rational Expectations Equilibrium (REE)



Social Optimum

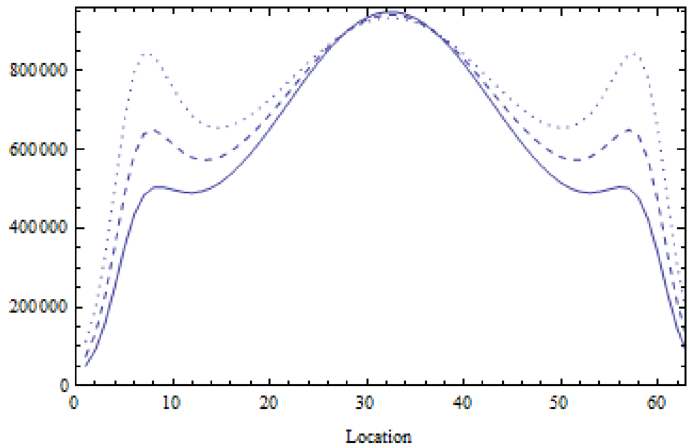


Social Optimum without Environmental Considerations

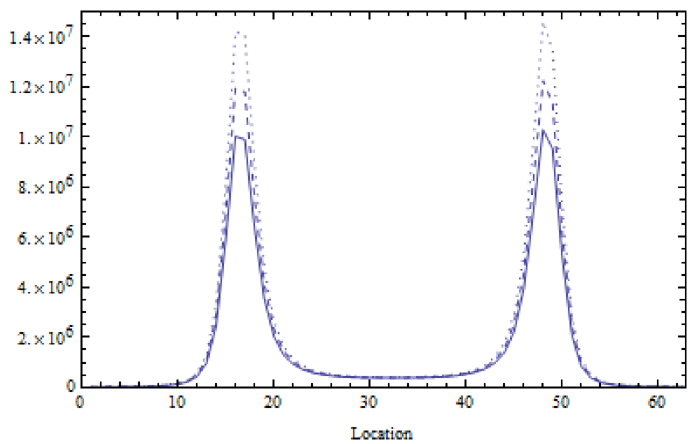


REE without Environmental Considerations

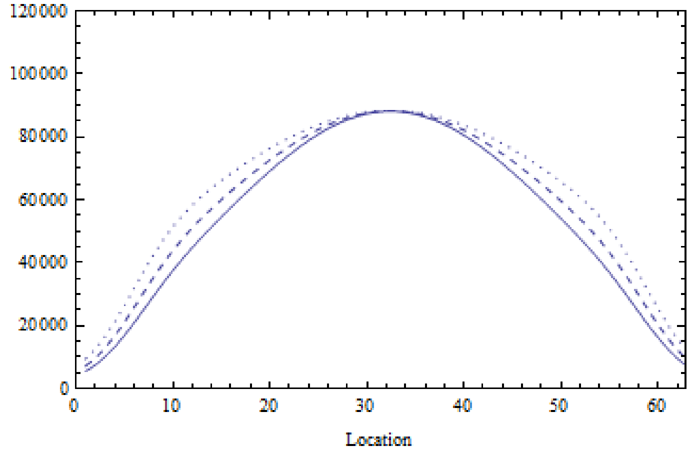
Figure 4: The Distribution of Economic Activity: Changes in transportation cost when delta=1. Dotted Line: beta=0.045, Dashed Line: beta=0.06, Solid Line: beta=0.075



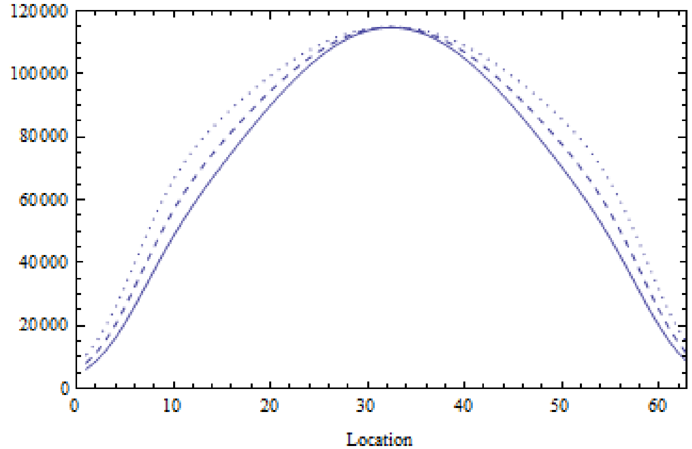
Rational Expectations Equilibrium (REE)



Social Optimum



Social Optimum without Environmental Considerations



REE without Environmental Considerations

Figure 5: The Distribution of Economic Activity: Changes in transportation cost when delta=3. Dotted Line: beta=0.045, Dashed Line: beta=0.06, Solid Line: beta=0.075

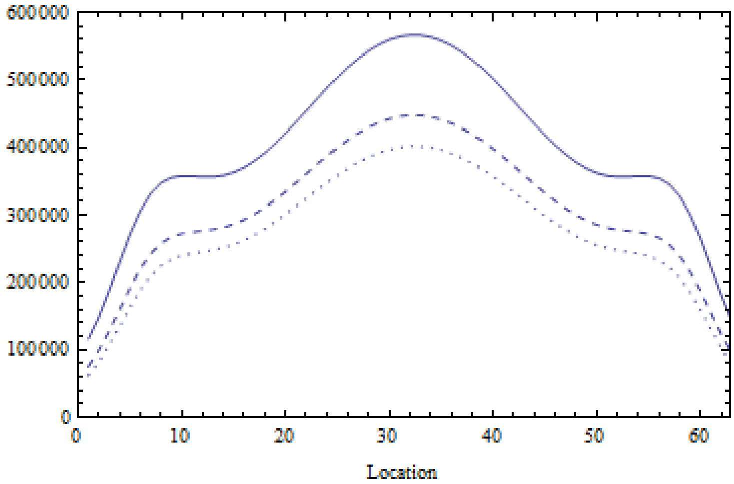


Figure 6: The Distribution of Production: Changes in the values of ψ with $\zeta = 0.5$. Dotted Line: $\psi = 1.5$, Dashed Line: $\psi = 0.9$, Solid Line: $\psi = 0.3$.

As far as the optimum is concerned, we have the formation of two peaks analysed above. Higher values of β imply lower densities of output around the two peaks in both cases. In all examples, where there is no environmental policy to act as a centrifugal force, we have a unique peak around the central point. In all other spatial points, increases in β decrease the distribution of economic activity. The only difference is that the total output produced in the optimum case is higher if compared to the REE.

5.3 *Environmental Policy*

In analysing environmental policy, we do not consider optimal emission taxes as defined in Section 4, but emission taxes at the REE as defined by (2.5).[22] Firms, at the REE, pay a "tax" or a "price" for each unit of emissions used in the production process. As already said, this tax depends on the total concentration of emissions at each spatial point and the tax rate is a function of the marginal damage caused in the economy by the concentration of emissions at a given point. Depending on the stringency of environmental policy, this form of taxation could fully (REE-I) or partly (REE-P) internalize the marginal damage. The strict (REE-I) or the lax (REE-P) environmental policy determines the amount of money firms are obliged to pay for their emissions. So, the ψ parameter shows the

[22] The imposition of optimal taxes would reproduce the social optimum (Figures 2, 3, 4, 5).

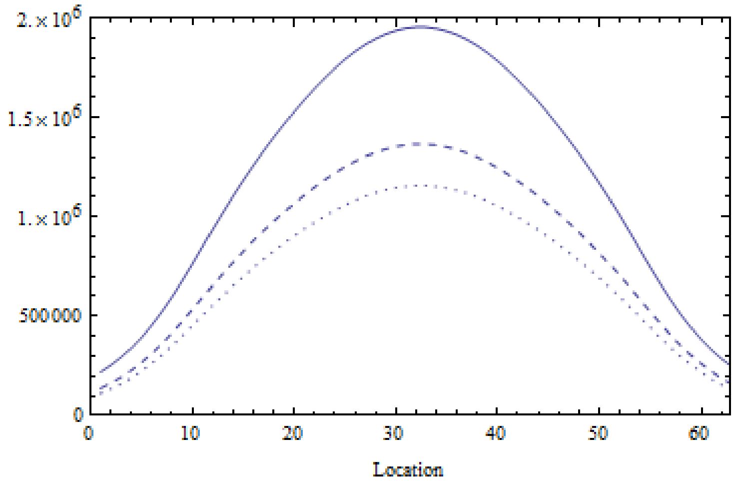


Figure 7: The Distribution of Production: Changes in the values of ψ with $\zeta = 2$. Dotted Line: $\psi = 1.5$, Dashed Line: $\psi = 0.9$, Solid Line: $\psi = 0.3$.

degree of internalization: $\psi = 1.5$ means full internalization and every value of ψ which is $0 < \psi < 1.5$ implies lower taxation and less strength of centrifugal force.

In Figures 6 and 7, we observe the distribution of economic activity using different values of ψ. Figure 6 is drawn for $\zeta = 0.5$ and Figure 7 for $\zeta = 2$. The higher value of ζ means that pollution is more localized and affects only nearby areas compared to the lower one. Let's explain first why $\zeta = 0.5$ (Figure 6) leads to the clustering of economic activity in three peaks, while $\zeta = 2$ (Figure 7) forms a unique peak. Under low values of ζ, emissions at each site pollute other sites that are far away. But, if each site is affected by emissions concentrated at a lot of sites, farther or closer, the total concentration of emissions would be higher at each spatial point. In that case, firms avoid locating at the same spatial point as others, so as not to increase further the "price" of emissions. For this reason, we have the clustering of production in three peaks. When pollution is more localized ($\zeta = 2$) the concentration of emissions at one site does not affect other sites a lot and so the "price" of emissions is lower. Then, firms have a stronger incentive to locate near each other in order to benefit from knowledge spillovers. This is the case presented in Figure 7.

As far as the stringency of environmental policy is concerned, the results could be easily predicted. Strict environmental policy and full internalization of marginal damage lead to a lower distribution of production in every case. On the other hand, more lenient

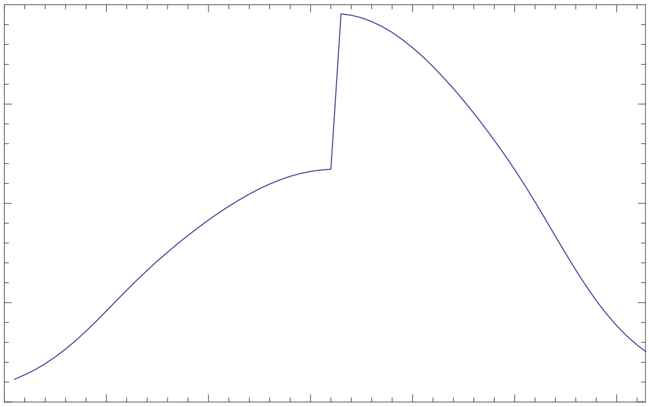


Figure 8: The Distribution of Economic Activity: Region A (left): $\psi = 1.5$ and Region B (right): $\psi = 0.3$ (with $\beta = 0.06$, $\delta = 2$ and $\zeta = 2$)

environmental regulations not only lead to a higher distribution at each site but also promote the agglomeration of economic activity around the city centre. So, the intuition is simple: environmental policy deters the clustering of production and makes the distribution of economic activity flatter.[23] In other words, strict environmental policy makes the distribution of economic activity less uneven. This result is consistent with the empirical literature, according to which environmental regulations restrict economic activity and result in a spreading out or an exiting of polluting firms.[24]

5.4 *Two Regions: Pollution Haven Hypothesis*

In previous sections, we have assumed the existence of a single city with borders 0 and 2π. Now, we can divide this space into two regions, where the first region (A) is located between 0 and π and the second one (B) between π and 2π. Let's suppose that each region adopts different environmental regulations: in region A, environmental policy is very strict, while in region B, it is more lenient. The degree of stringency, in our model, is determined by the parameter ψ. If $\psi = 1.5$, the government charges the full marginal damage caused by the concentration of emissions (RRE-I) and if $0 < \psi < 1.5$, the environmental policy is laxer as the marginal damage is partly internalized (REE-P). We

[23]The proof of flatness is presented in Appendix C.

[24]See Introduction: Greenstone (2002), Henderson (1996), Elbers and Withagen (2004).

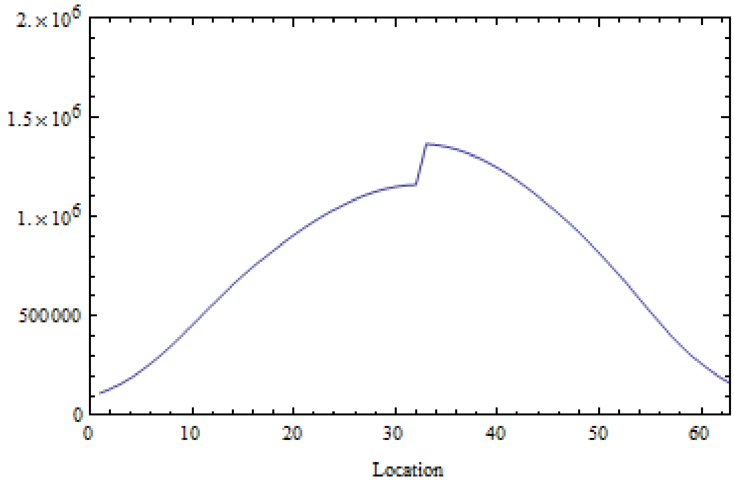


Figure 9: The Distribution of Economic Activity: Region A (left): $\psi = 1.5$ and Region B (right): $\psi = 0.9$ (with $\beta = 0.06$, $\delta = 2$ and $\zeta = 2$)

let region A enforce strict measures in order to protect the environment ($\psi = 1.5$) and region B adopt less severe environmental regulations. The rest of the structure of the model remains the same. Machinery is now available at the boundary between the two regions, at $\bar{r} = \pi$. As a result, this point still has an advantage: there is no transportation cost for machinery. In that context, it's interesting to study how environmental policy affects the distribution and the relocation of economic activity in both regions.

In Figures 8 and 9, we use $\psi = 0.3$ and $\psi = 0.9$ respectively for region B. It is obvious that the different environmental policies lead to a relocation of pollution industries between the two regions. Thus, there is a high clustering of economic activity at the right of $r = \pi$, and a lower one on the left side. In that way, firms locating in region B try to avoid the increased cost of emissions in region A. The more lenient the environmental policy in region B is, the higher its concentration of economic activity is compared to region A's.

This result can be associated with the "pollution haven hypothesis", according to which polluting industries have a tendency to relocate to areas with less stringent environmental regulations. In other words, countries or regions with weak environmental regulations provide a "haven" to polluting firms which come from countries with strict environmental laws. A few empirical methods have been used, so far, to test whether the hypothesis is confirmed. Earlier studies, based on cross section data, failed to prove the pollution haven hypothesis. However, more recent studies using panel data and fixed-effect models to control for unobserved heterogeneity and instrumental variables to control for simultaneity, and taking into account other factors affecting trade and investment flows, lead to more confirming results.[25] According to Ben Kheder and Zugravu (2008), the reason for the lack of robust empirical proof for the pollution haven hypothesis is that the stringency of the environmental regulations is not the only factor determining the location decisions of firms. Specifically, other factors, such as the endowments of qualified human capital and physical capital, the weak institutions, the high corruption level, the lack of civil freedoms' and the property rights' protection, do play an important role in the relocation of pollution industries. So, by applying a geographic economy model to French firm-level data and considering many of these factors, they succeeded in confirming the essential role played by environmental regulations in determining firm's location. What is also very important is that this effect was reinforced for the most polluting firms, in the sense that these firms are more probable to relocate after the enforcement of strict environmental regulations.

6 Conclusion

Our model consists of a single city - of length S - in which firms are free to choose where to locate. The city has a nonuniform internal structure because of externalities in production, transportation cost of the machinery input and environmental policy. Specifically, when firms take location decisions, they consider certain facts. First, labor at each location will be more productive if there is a high concentration of labor at nearby locations. This is the assumption of knowledge spillovers. Second, the transportation of machinery is costly and its cost depends on the distance. Finally, the use of emissions as an input in the production process induces the environmental regulator to adopt some kind of environmental policy. The stringency and, therefore, the cost of this policy for the firms is an increasing function of aggregate emissions at each spatial point. If all firms decide

[25] For a review of the literature on the pollution haven hypothesis, see Brunnermeier and Levinson (2004) and Taylor (2004).

to locate around the city centre, they will benefit from positive knowledge spillovers and avoid paying a high transportation cost for machinery. So, these forces promote agglomeration. On the other hand, if they locate at that point, the concentration of emissions will be high and they will be obliged to pay higher taxation. As a result, environmental policy impedes agglomeration. The trade-off between centripetal and centrifugal forces determines both the REE and the optimal concentration of economic activity. Comparing the equilibrium and the optimal outcomes, we derived and characterized optimal spatial policies.

The results of our analysis can be summarized in the following way. When the external effect is more localized (δ is high), firms have a strong desire to locate near other producers. When the transportation cost of machinery increases, firms move closer to the city centre. Furthermore, when the environmental policy is strict, the distribution of production becomes flatter. In other words, firms have fewer incentives for agglomeration and concentration of economic activity. We also observed that environmental policy tends to discourage the clustering of economic activity that would occur in its absence. Finally, we showed that if there are two bordering regions with different environmental policies, the region which has enforced more lenient environmental regulation will provide a "haven" for polluting industries.

Using the present model, we can study many aspects of the internal structure of cities under environmental policy. A possible extension could be to study the dynamic problem of location decisions of firms. This can be done by considering pollution accumulation and capital accumulation over time. In that way, we could explain the structure of cities not only across space, but also across time.[26] What is also interesting and appealing is the empirical work based on models of New Economic Geography. As far as we know, there is a limited number of empirical papers relevant to the literature.[27] In that context, we could test the effects of environmental policy on urban structure. These thoughts are left for future research.

[26] The theory of optimal control and spatial heterogeneity, analysed by Brock and Xepapadeas (2008), will help us solve the problem.

[27] For example, Ioannides et al. (2008) study the effects of information and communication technologies on urban structure.

References

Alonso, W. (1964). *Location and Land Use*, Cambridge: Harvard University Press.

Ben Kheder, S. and Zugravu N. (2008), "The Pollution Haven Hypothesis: A Geographic Economy Model in a Comparative Study", *Fondazione Eni Enrico Mattei*, Nota di Lavoro 73.2008.

Bottazzi L. and Peri G. (2003), "Innovation and Spillovers in Regions: Evidence from European Patent Data", *European Economic Review*, 47:687-710.

Brock, W. (1977), "A Polluted Golden Age", in *Economics of Natural and Environmental Resources*, V. Smith (ed.), New York: Gordon and Breach, 441-462.

Brock, W. and Xepapadeas, A. (2008), "Diffusion-Induced Instability and Pattern Formation in Infinite Horizon Recursive Optimal Control", *Journal of Economic Dynamics and Control*, 32, 2745-2787.

Brunnermeier, S. and Levinson, A. (2004), "Examining the Evidence on Environmental Regulations and Industry Location", *Journal of Environment and Development*, 13, 6-41.

Carlino, G.A., Chatterjee, S. and Hunt R.M. (2007), "Urban Density and the Rate of Invention", *Journal of Urban Economics*, 61:389-419.

Elbers, C. and Withagen, C. (2004), "Environmental Policy, Population Dynamics and Agglomeration", *Contributions to Economic Analysis & Policy*, 3(2) : Article 3.

Forslid, R. and Ottaviano, G.I. (2003), "An Analytically Solvable Core-Periphery Model", *Journal of Economic Geography*, 3, 229-240.

Fujita, M., Krugman, P. and Venables, A. J. (1999). *The Spatial Economy.* Cambridge: MIT Press.

Fujita, M., Krugman, P., and Mori, T. (1999), "On the Evolution of Hierarchical Urban Systems", *European Economic Review*, 43, 209-251.

Fujita, M. and Mori, T. (1996), "The Role of Ports in the Making of Major Cities: Self Agglomeration and Hub-effect", *Journal of Development Economics*, 49, 93-120.

Fujita, M. and Thisse, J-F. (2002). *Economics of Agglomeration: Cities, Industrial Location and Regional Growth.* Cambridge University Press.

Greenstone, M. (2002), "The Impacts of Environmental Regulations on Industrial Activity: Evidence from the 1970 and the 1977 Clean Air Act Ammendments and the Census of Manufactures", *Journal of Political Economy*, 110(6), 1175-219.

Henderson, J.V. (1974), "The Sizes and Types of Cities", *American Economic Review*, 64, 640-656.

Henderson, J.V. (1996), "Effects of Air Quality Regulation", *American Economic Review*, 86, 789-813.

Ioannides, Y., Overman, H., Rossi-Hansberg E. and Schmidheiny K. (2008), "The Effect of Information and Communication Technologies On Urban Structure", *Economic Policy*, 201-242.

Keller, W. (2002), "Geographic Localization of International Technology Diffusion", *American Economic Review*, 92(1):120-142.

Krugman, P. (1991), "Increasing Returns and Economic Geography", *Journal of Political Economy*, 99, 483-499.

Krugman, P. (1993a), "On the Number and Location of Cities", *European Economic Review*, 37, 293-298.

Krugman, P. (1993b), "First Nature, Second Nature and Metropolitan Location", *Journal of Regional Science*, 33, 129-144.

Krugman, P. (1998), "Space: The Final Frontier", *Journal of Economic Perspectives*, 12, 161-174.

Lange, A., and Quaas, M. (2007), "Economic Geography and the Effect of Environmental Pollution on Agglomeration", *The B.E. Journal of Economic Analysis & Policy*, 7(1), Article 52.

Lucas, R.E. (2001), "Externalities and Cities", *Review of Economic Dynamics*, 4, 245-274.

Lucas, R.E. and Rossi-Hansberg, E. (2002), "On the Internal Structure of Cities", *Econometrica*, 70(4), 1445-1476.

Maleknejad, K., Aghazadeh, N. and Rabbani, M. (2006), "Numerical Solution of Second Kind Fredholm Integral Equations System by Using a Taylor-Series Expansion

Method", *Applied Mathematics and Computation*, 175, 1229-1234.

Moiseiwitsch, B.L. (2005). *Integral Equations*, Dover Publications, New York.

OECD (2006), "Competitive Cities in the Global Economy", *OECD Territorial Reviews.*

Polyanin D. and Manzhirov V. (1998). *Handbook of Integral Equations*, CRC Press.

Rauscher, M. (1994), "On Ecological Dumping", *Oxford Economic Papers*, 46: 822-840.

Rauscher, M. (2008), "Concentration, Separation and Dispersion: Economic Geography and the Environment", Paper presented at the 2008 Conference of European Association of Environmental and Resource Economists in Gothenberg, Sweden.

Rossi-Hansberg, E. (2005), "A Spatial Theory of Trade", *American Economic Review*, 95(5), 1464-1491.

Tahvonen, O. and Kuuluvainen, J. (1993), "Economic Growth, Pollution and Renewable Resources", *Journal of Environmental Economics and Management*, 24, 101-118.

Taylor S. (2004), "Unbundling the Pollution Haven Hypothesis", *Advances in Economic Analysis & Policy*, 4(2), Article 8.

van Marrewijk, C. (2005), "Geographical Economics and the Role of Pollution on Location", *ICFAI Journal of Environmental Economics*, 3: 28-48.

von Thünen, J. (1826). *The Isolated State*, English edn, London, Pergamon Press.

Xepapadeas, A. (2005), "Economic Growth and the Environment", Chapter 23, 1220-1271, in the *Handbook of Environmental Economics*, Vol. 3: Economywide and International Environmental Issues edited by Karl-Goran Mäler and Jeffery Vincent, a volume in the series *Handbooks in Economics*, edited by Kenneth Arrow and Michael D. Intrilligator, Elsevier Publishers.

Appendix A:*Solving a system of second kind Fredholm integral equations, following the modified Taylor-series expansion method (Maleknejad et al., 2006).*

The Rational Expectations Equilibrium: For the solution of the problem, we need to take logs of 2.10-2.12. Then the FOC become:

$$\ln p + \ln a + \gamma\delta \int_0^S e^{-\delta(r-s)^2} \ln(L(s))ds + (a-1)\ln L(r) + b\ln K(r) + c\ln E(r) = \ln w$$

$$\ln p + \ln b + \gamma\delta \int_0^S e^{-\delta(r-s)^2} \ln(L(s))ds + a\ln L(r) + (b-1)\ln K(r) + c\ln E(r)$$
$$= \ln p_K + \beta(r-\overline{r})^2$$

$$\ln p + \ln c + \gamma\delta \int_0^S e^{-\delta(r-s)^2} \ln(L(s))ds + a\ln L(r) + b\ln K(r) + (c-1)\ln E(r)$$
$$= \ln\psi + (\phi-1) \int_0^S e^{-\zeta(r-s)^2} \ln(E(s))\, ds$$

Setting $\ln L = y$, $\ln K = x$ and $\ln E = \varepsilon$, we obtain the following system:

$$\gamma\delta \int_0^S e^{-\delta(r-s)^2} y(s)ds + (a-1)y(r) + bx(r) + c\varepsilon(r) = \ln w - \ln p - \ln a$$

$$\gamma\delta \int_0^S e^{-\delta(r-s)^2} y(s)ds + ay(r) + (b-1)x(r) + c\varepsilon(r) = \ln p_K + \beta(r-\overline{r})^2 - \ln p - \ln b$$

$$\gamma\delta \int_0^S e^{-\delta(r-s)^2} y(s)ds + ay(r) + bx(r) + (c-1)\varepsilon(r) + (1-\phi) \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s)\, ds$$
$$= \ln\psi - \ln p - \ln c$$

We transform the system in order to obtain a system of second kind Fredholm integral equations with symmetric kernels:

$$\underbrace{\begin{pmatrix} \gamma\delta & 0 \\ \gamma\delta & 0 \\ \gamma\delta & 1-\phi \end{pmatrix} \begin{pmatrix} \int_0^S e^{-\delta(r-s)^2} y(s)ds \\ \int_0^S e^{-\zeta(r-s)^2} \varepsilon(s)ds \end{pmatrix} + \begin{pmatrix} \ln a + \ln p - \ln w \\ \ln p + \ln b - \ln p_K - \beta(r-\overline{r})^2 \\ \ln c + \ln p - \ln\psi \end{pmatrix}}_{\text{B}}$$

$$= \underbrace{\begin{pmatrix} 1-a & -b & -c \\ -a & 1-b & -c \\ -a & -b & 1-c \end{pmatrix}} \underbrace{\begin{pmatrix} y(r) \\ x(r) \\ \varepsilon(r) \end{pmatrix}}$$

A Z

$$B = AZ \Rightarrow A^{-1}B = Z \text{ where } A^{-1} = \begin{pmatrix} \frac{1-b-c}{1-a-b-c} & \frac{b}{1-a-b-c} & \frac{c}{1-a-b-c} \\ \frac{a}{1-a-b-c} & \frac{1-a-c}{1-a-b-c} & \frac{c}{1-a-b-c} \\ \frac{a}{1-a-b-c} & \frac{b}{1-a-b-c} & \frac{1-a-b}{1-a-b-c} \end{pmatrix}$$

From $A^{-1}B = Z$, we derive the following system of second kind Fredholm integral equations:

$$\begin{aligned}
\frac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}y(s)ds + \frac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_1(r) &= y(r) \quad \text{(A1)} \\
\frac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}y(s)ds + \frac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_2(r) &= x(r) \quad \text{(A2)} \\
\frac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}y(s)ds + \frac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\varepsilon(s)ds + g_3(r) &= \varepsilon(r) \quad \text{(A3)}
\end{aligned}$$

where:

$$\begin{aligned}
g_1(r) &= \tfrac{1}{1-a-b-c}\{(1-b-c)\ [\ln a + \ln p - \ln w] + \\
&\quad b\ [\ln p + \ln b - \ln p_K - \beta(r-\bar{r})^2] + c\ [\ln c + \ln p - \ln \psi]\} \\
g_2(r) &= \tfrac{1}{1-a-b-c}\{a\ [\ln a + \ln p - \ln w] + \\
&\quad (1-a-c)\ [\ln p + \ln b - \ln p_K - \beta(r-\bar{r})^2] + c\ [\ln c + \ln p - \ln \psi]\} \\
g_3(r) &= \tfrac{1}{1-a-b-c}\{a\ [\ln a + \ln p - \ln w] + \\
&\quad b\ [\ln p + \ln b - \ln p_K - \beta(r-\bar{r})^2] + (1-a-b)\ [\ln c + \ln p - \ln \psi]\}
\end{aligned}$$

Taylor-series expansions can be made for the solutions $y(s)$ and $\varepsilon(s)$:

$$\begin{aligned}
y(s) &= y(r) + y'(r)(s-r) + \frac{1}{2}y''(r)(s-r)^2 \\
\varepsilon(s) &= \varepsilon(r) + \varepsilon'(r)(s-r) + \frac{1}{2}\varepsilon''(r)(s-r)^2
\end{aligned}$$

Substituting the expansions into the integrals of the system (A1)-(A3), we get:

$$
\begin{aligned}
y(r) \;&=\; \tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}\,\{y(r)+y'(r)(s-r)+\frac{1}{2}y''(r)(s-r)^2\}\,ds + \\
&\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\{\varepsilon(r)+\varepsilon'(r)(s-r)+\frac{1}{2}\varepsilon''(r)(s-r)^2\}\,ds+g_1(r)
\end{aligned} \tag{A4}
$$

$$
\begin{aligned}
x(r) \;&=\; \tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}\,\{y(r)+y'(r)(s-r)+\frac{1}{2}y''(r)(s-r)^2\}\,ds + \\
&\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\{\varepsilon(r)+\varepsilon'(r)(s-r)+\frac{1}{2}\varepsilon''(r)(s-r)^2\}\,ds+g_2(r)
\end{aligned} \tag{A5}
$$

$$
\begin{aligned}
\varepsilon(r) \;&=\; \tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}\,\{y(r)+y'(r)(s-r)+\frac{1}{2}y''(r)(s-r)^2\}\,ds + \\
&\tfrac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}\{\varepsilon(r)+\varepsilon'(r)(s-r)+\frac{1}{2}\varepsilon''(r)(s-r)^2\}\,ds+g_3(r)
\end{aligned} \tag{A6}
$$

Rewriting the equations we have:

$$
\begin{aligned}
g_1(r) =& \left[1-\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}ds\right]y(r) - \left[\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}(s-r)ds\right]y'(r) \\
&-\left[\frac{1}{2}\,\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}(s-r)^2ds\right]y''(r) - \left[\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}ds\right]\varepsilon(r) - \\
&\left[\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}(s-r)ds\right]\varepsilon'(r) - \left[\frac{1}{2}\,\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}(s-r)^2ds\right]\varepsilon''(r)
\end{aligned} \tag{A7}
$$

$$
\begin{aligned}
g_2(r) =& \, x(r) - \left[\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}ds\right]y(r) - \left[\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}(s-r)ds\right]y'(r) \\
&-\left[\frac{1}{2}\,\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}(s-r)^2ds\right]y''(r) - \left[\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}ds\right]\varepsilon(r) - \\
&\left[\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}(s-r)ds\right]\varepsilon'(r) - \left[\frac{1}{2}\,\tfrac{c(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}(s-r)^2ds\right]\varepsilon''(r)
\end{aligned} \tag{A8}
$$

$$g_3(r) = -\left[\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}ds\right]y(r) - \left[\tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}(s-r)ds\right]y'(r) \tag{A9}$$
$$-\left[\frac{1}{2}\ \tfrac{\gamma\delta}{1-a-b-c}\int_0^S e^{-\delta(r-s)^2}(s-r)^2ds\right]y''(r) + \left[1 - \tfrac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}ds\right]\varepsilon(r) -$$
$$\left[\tfrac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}(s-r)ds\right]\varepsilon'(r) - \left[\frac{1}{2}\ \tfrac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S e^{-\zeta(r-s)^2}(s-r)^2ds\right]\varepsilon''(r)$$

If the integrals in equations A7-A9 can be solved analytically, then the bracketed quantities are functions of r alone. So A7-A9 become a linear system of ordinary differential equations that can be solved, if we use an appropriate number of boundary conditions.

To construct boundary conditions we differentiate A1 and A3:

$$\begin{aligned} y'(r) &= \tfrac{\gamma\delta}{1-a-b-c}\int_0^S -2\delta\ (r-s)\ e^{-\delta(r-s)^2}\ y(s)\ ds + \\ &\quad \tfrac{c(1-\phi)}{1-a-b-c}\int_0^S -2\zeta\ (r-s)\ e^{-\zeta(r-s)^2}\ \varepsilon(s)\ ds + g_1'(r) \end{aligned} \tag{A10}$$

$$\begin{aligned} y''(r) &= \tfrac{\gamma\delta}{1-a-b-c}\int_0^S \left[-2\delta + 4\delta^2\ (r-s)^2\right]\ e^{-\delta(r-s)^2}\ y(s)\ ds + \\ &\quad \tfrac{c(1-\phi)}{1-a-b-c}\int_0^S \left[-2\zeta + 4\zeta^2\ (r-s)^2\right]\ e^{-\zeta(r-s)^2}\ \varepsilon(s)\ ds + g_1''(r) \end{aligned} \tag{A11}$$

$$\begin{aligned} \varepsilon'(r) &= \tfrac{\gamma\delta}{1-a-b-c}\int_0^S -2\delta\ (r-s)\ e^{-\delta(r-s)^2}\ y(s)\ ds + \\ &\quad \tfrac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S -2\zeta\ (r-s)\ e^{-\zeta(r-s)^2}\ \varepsilon(s)\ ds + g_3'(r) \end{aligned} \tag{A12}$$

$$\begin{aligned} \varepsilon''(r) &= \tfrac{\gamma\delta}{1-a-b-c}\int_0^S \left[-2\delta + 4\delta^2\ (r-s)^2\right]\ e^{-\delta(r-s)^2}\ y(s)\ ds + \\ &\quad \tfrac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S \left[-2\zeta + 4\zeta^2\ (r-s)^2\right]\ e^{-\zeta(r-s)^2}\ \varepsilon(s)\ ds + g_3''(r) \end{aligned} \tag{A13}$$

We substitute $y(r)$ and $\varepsilon(r)$ for $y(s)$ and $\varepsilon(s)$ in equations A10 - A13:

$$\mathbf{y}'(\mathbf{r}) = \left[\frac{\gamma\delta}{1-a-b-c}\int_0^S -2\delta\ (r-s)\ e^{-\delta(r-s)^2}\ ds\right]\mathbf{y}(\mathbf{r}) + \left[\frac{c(1-\phi)}{1-a-b-c}\int_0^S -2\zeta\ (r-s)\ e^{-\zeta(r-s)^2}\ ds\right]\boldsymbol{\varepsilon}(\mathbf{r}) + g_1'(r) \tag{A14}$$

$$\mathbf{y}''(\mathbf{r}) = \left[\frac{\gamma\delta}{1-a-b-c}\int_0^S \left[-2\delta + 4\delta^2\ (r-s)^2\right]\ e^{-\delta(r-s)^2}\ ds\right]\mathbf{y}(\mathbf{r}) + \left[\frac{c(1-\phi)}{1-a-b-c}\int_0^S \left[-2\zeta + 4\zeta^2\ (r-s)^2\right]\ e^{-\zeta(r-s)^2}\ ds\right]\boldsymbol{\varepsilon}(\mathbf{r}) + g_1''(r) \tag{A15}$$

$$\boldsymbol{\varepsilon}'(\mathbf{r}) = \left[\frac{\gamma\delta}{1-a-b-c}\int_0^S -2\delta\ (r-s)\ e^{-\delta(r-s)^2}\ ds\right]\mathbf{y}(\mathbf{r}) + \left[\frac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S -2\zeta\ (r-s)\ e^{-\zeta(r-s)^2}\ ds\right]\boldsymbol{\varepsilon}(\mathbf{r}) + g_3'(r) \tag{A16}$$

$$\boldsymbol{\varepsilon}''(\mathbf{r}) = \left[\frac{\gamma\delta}{1-a-b-c}\int_0^S \left[-2\delta + 4\delta^2\ (r-s)^2\right]\ e^{-\delta(r-s)^2}\ ds\right]\mathbf{y}(\mathbf{r}) + \left[\frac{(1-a-b)(1-\phi)}{1-a-b-c}\int_0^S \left[-2\zeta + 4\zeta^2\ (r-s)^2\right]\ e^{-\zeta(r-s)^2}\ ds\right]\boldsymbol{\varepsilon}(\mathbf{r}) + g_3''(r) \tag{A17}$$

In equations (A14-A17), we observe that $y'(r)$, $y''(r)$, $\varepsilon'(r)$, $\varepsilon''(r)$ are functions of $y(r)$, $\varepsilon(r)$, $g_1'(r)$, $g_1''(r)$, $g_3'(r)$, $g_3''(r)$. Substituting them into (A7), (A8) & (A9), we have a linear system of three algebraic equations that can be solved using Mathematica.

Appendix B: The same method of modified Taylor-series expansion was used in order to solve for the social optimum. The FOC for the social optimum, (3.3-3.5), contain one extra term compared to the FOC for the REE. The FOC with respect to $L(r)$ (3.3) contains the ratio $\frac{\vartheta z(r)}{\vartheta L(r)}$, which is equal to: $\frac{\vartheta z(r)}{\vartheta L(r)} = \delta\frac{1}{L(r)}\int_0^S e^{-\delta(r-s)^2}ds$. Also, the FOC with respect to $E(r)$ (3.5) contains the term $\frac{\vartheta X(r)}{\vartheta E(r)} = \frac{1}{E(r)}\ e^{\int_0^S\left[e^{-\zeta(r-s)^2}\ln E(s)\right]ds}\int_0^S e^{-\zeta(s-r)^2}ds$. Using these two terms, we follow the method analysed in Appendix A to find the optimal solution.

Appendix C: *Transformation of the system (2.13) to a single Fredholm equation of 2nd kind* (Polyanin and Manzhirov, 1998).

We define the functions $Y(r)$ and $G(r)$ on $[0, 3S]$, where $Y(r) = y_i(r - (i-1)S)$ and $G(r) = g_i(r - (i-1)S)$ for $(i-1)S \leq r \leq iS$.[28] Next, we define the kernel $C(r, s)$ on the square $[0, 3S] \times [0, 3S]$ as follows: $C(r, s) = k_{ij}(r - (i-1)S,\ s - (j-1)S)$ for $(i-1)S \leq r \leq iS$ and $(j-1)S \leq s \leq jS$.

So, the system (2.13) can be rewritten as the single Fredholm equation:

$Y(r) - \frac{1}{1-a-b-c} \int_0^{3S} C(r, s)\, Y(s)\, ds = G(r)$, where $0 \leq r \leq 3S$.

If the kernels $k_{ij}(r, s)$ are square integrable on the square $[0, S] \times [0, S]$ and $g_i(r)$ are square integrable on $[0, S]$, then the kernel $C(r, s)$ is square integrable on the new square: $[0, 3S] \times [0, 3S]$ and $G(r)$ is square integrable on $[0, 3S]$.

Appendix D: Figures 6 and 7: *Proof of the flatness.*

In order to measure flatness, we use the concept of curvature. Curvature is the amount by which a geometric object deviates from being flat, or straight in the case of a line. To measure curvature of a line we can use the approximation:

$$\kappa \approx \left| \frac{d^2 q}{dr^2} \right|$$

where in our case $q(r) = \exp(\gamma z(r)) L(r)^a K(r)^b E(r)^c$, is the production function. We use Mathematica to measure the curvature of lines in figures 6 and 7. In figure 6, at the point $r = \pi$, the dotted line has $\kappa(\pi) = 168, 174$, the dashed line has $\kappa(\pi) = 190, 340$ and the solid line has $\kappa(\pi) = 248, 240$. In figure 7, at the point $r = \pi$, the dotted line has $\kappa(\pi) = 360, 077$, the dashed line has $\kappa(\pi) = 425, 289$ and the solid line has $\kappa(\pi) = 608, 352$. The flattest curve is the one with the lowest curvature value, ie the dotted line (in both cases).

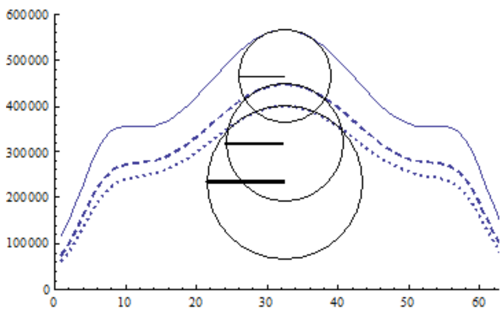
Another way to measure the curvature at a specific point is to use the approach of the osculating circle. According to it, from any point of any curve, where the curvature is non-zero, there is a unique circle which most closely approximates the curve near that point. This is the osculating circle at that point. The radius (R) of the osculating circle

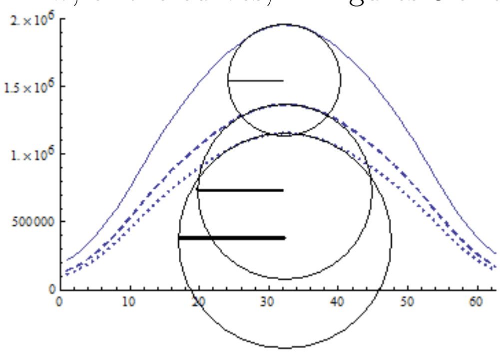
[28] We assume that $y_1 = y$, $y_2 = x$ and $y_3 = \varepsilon$, so as to follow the notation of our model.

determines the curvature at that point in the following way:

$$\kappa = \frac{1}{R}$$

So, we draw the osculating circles at point $r = \pi$, of the curves, in Figures 6 and 7.





Left Figure (6): Let R_{11} be the radius of the osculating circle of the solid line, R_{12} be the radius of the osculating circle of the dashed line and R_{13} be the radius of the osculating circle of the dotted line, then it is obvious that $R_{11} < R_{12} < R_{13}$. Also, if the corresponding curvatures are $\kappa_{11} = \frac{1}{R_{11}}$, $\kappa_{12} = \frac{1}{R_{12}}$ and $\kappa_{13} = \frac{1}{R_{13}}$, then $\kappa_{11} > \kappa_{12} > \kappa_{13}$.

Right Figure (7): Let R_{21} be the radius of the osculating circle of the solid line, R_{22} be the radius of the osculating circle of the dashed line and R_{23} be the radius of the osculating circle of the dotted line, then it is obvious that $R_{21} < R_{22} < R_{23}$. Also, if the corresponding curvatures are $\kappa_{21} = \frac{1}{R_{21}}$, $\kappa_{22} = \frac{1}{R_{22}}$ and $\kappa_{23} = \frac{1}{R_{23}}$, then $\kappa_{21} > \kappa_{22} > \kappa_{23}$.

As a result, in both figures, the dotted line is the flattest curve.