

Endogenous Spatial Differentiation with Vertical Contracting

Frago Kourandi and Nikolaos Vettas*

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Abstract

We examine a linear city model with duopoly in the upstream and downstream level. We set up a five-stage game where location and pricing decisions of both upstream and downstream firms are determined. We show that the unique equilibrium outcome is maximum differentiation by upstream and downstream firms. Apart from the standard "demand" and "price competition" effect when firms change their locations, there is also a third force that affects the wholesale prices that upstream firms charge. The interaction of these forces give the equilibrium result. Under price discrimination by the upstream firms, wholesale and final prices reduce and the equilibrium locations move towards the centre of the line. Price discrimination may be not anticompetitive, as it further reduces the social transportation cost for some parameter values.

JEL Classification: L13; R32

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*Kourandi: Department of Economics, Athens University of Economics and Business, 76 Patision Str., Athens 10434, Greece; e-mail: kourandi@aueb.gr.
Vettas: Department of Economics, Athens University of Economics and Business, 76 Patision Str., Athens 10434, Greece and CEPR, UK; e-mail: nvettas@aueb.gr.

1 Introduction

A very significant part of trade in markets is at the wholesale level, between wholesalers (or manufacturers) and retailers. Recently, a growing literature has been examining aspects of oligopoly competition in such important markets, among upstream and downstream (vertically linked) firms. Such aspects include the type of contracts, exclusivity of relations, resale price maintenance and other. One of the important ingredients of the problem, which however has received thus far only very little attention, in the literature, is product differentiation (at the upstream or downstream level). In this paper we set up a model to study horizontal product differentiation decisions of vertically linked firms. Our work contributes to the product differentiation literature by considering the location and price decision of firms that do not directly sell to the final consumers. In particular we enrich the location literature *a' la* Hotelling by considering endogenous locations and (then) price decisions by upstream and downstream firms and we enrich the vertical contracting literature by considering the role of (endogenous) product differentiation.

As usual a "location" can be understood either as geographical location or as a ("horizontal") choice of product characteristics. The main questions that can be answered in a framework that combines vertical and product differentiation elements refer to the implications of vertical pricing rules for the location choices (e.g. how does the ability of wholesalers to price discriminate among retailers affect the equilibrium outcome) and the role that upstream and/or downstream differentiation plays for vertical trade and final prices. We endogenize location and pricing decisions both for the upstream and the downstream firms. We lay partial attention to the ability of upstream firms to sell to only one retailer or to all retailers, thus covering the whole market. We also study the implications of the ability of each wholesaler to charge different wholesale prices to each retailer. Interestingly, when there is the possibility of price discrimination with upstream duopoly and downstream duopoly, the price that each upstream firm sets matters even if it is not a price that is being accepted in equilibrium by that retailer because it affects the equilibrium price set by the rival upstream firm to that downstream firm, a factor that affects the price that can be charged to the other retailer. Thus, whereas in a simple Hotelling model price discrimination among final consumers implies that each pricing problem is independent of the other (each buyer will simply buy from the seller that offers the best total deal), when there is price discrimination from wholesalers to retailers the problem

becomes much richer since the retailers problems are linked through the final demand for each of them which in turn depends on the wholesale prices that are being selected. In other words, even though final consumers have "unit demands" as in any classic Hotelling model, demand at the wholesale level is not unit demand, instead the (endogenous) portion of the downstream sales for each retailer becomes the retailer's demand to the wholesalers. As a result the demand elasticities get modified in the presence of vertical contracting.

The remainder of the paper is as follows. Related literature is presented in Section 2. Section 3 sets up the basic model. Section 4 presents the equilibrium. Section 5 modifies the basic model so that upstream firms can price discriminate and Section 6 concludes.

2 Related literature

After the classic paper of Hotelling (1929) an extensive and significant literature on horizontal differentiation has emerged. Horizontal differentiation can take the form of pure spatial differentiation or the form of different product characteristics. Hotelling (1929) suggests the principle of minimal differentiation while subsequent work by D'Aspremont *et al.* (1979) suggests the principle of maximal differentiation by stressing the price competition effect neglected by Hotelling (1929).

Many variants of the standard linear city model has been studied. Ziss (1993) examines the D'Aspremont *et al.* model with heterogeneous production technologies. If marginal cost difference is sufficient small, a price and location equilibrium exists in which both firms enter and maximum differentiation emerges. Quality considerations in a spatial model are studied by Vettas (2003) and Vettas and Christou (2005) where firms are horizontally as well as vertically differentiated. Nevertheless, few papers examine the vertical chain interactions in a horizontal differentiation framework. In the "real world" almost all market structures have an important vertical element with the upstream firm to supply the downstream firms. Marginal production cost of the downstream firms are endogenously determined in the vertical chain. Thus, location choices by the upstream and/or downstream firms are affected by the transactions in a vertical environment.

Gupta *et al.* (1994) assume that an upstream monopoly sets the wholesale price based on its observation of the locations chosen by the downstream firms

and that the downstream firms can price discriminate. Beladi *et al.* (2008) study the case of an upstream monopoly and a downstream duopoly where two-part tariffs are signed and the downstream firms cannot produce all varieties demanded. We consider in contrast a model with two upstream and two downstream firms that pay linear wholesale prices. A location-price equilibrium is analysed by Brekke and Straume (2004) where upstream firms bargain on the input prices with the downstream firms. Allain (2002) and Laussel (2006) examine the situation where two upstream firms are exogenously brand differentiated and two downstream firms are exogenously spatially differentiated on the line, thus, consumers face four different products and are distributed on a rectangle. In our paper consumers care only about the spatial differentiation and we further endogenize the location choices in both levels assuming no bargaining for the input prices.

We set up a model where two upstream firms and two downstream firms are located on the unit line. Location decisions are taken before wholesale and retail prices are determined. This is commonly observed in the literature as location choices have longer run characteristics than pricing. Consumers have preferences only on the downstream differentiated products. The main task is to examine the equilibrium horizontal differentiation chosen by the upstream and downstream firms when linear wholesale prices are signed. The unique equilibrium is the two pairs of upstream and downstream firms to locate at the two endpoints when upstream firms cannot price discriminate among the downstream firms, that is, the D'Aspremont *et al.* (1979) result. A type of double marginalization emerges with both upstream and downstream firms having positive profit margins. Final prices are higher than the vertical integration case.

In models of horizontal differentiation without upstream firms, there are two opposite forces, one pushing firms close to each other, the "demand" effect, and one to the opposite direction, the "price competition" effect. In our model there is a third force affecting the wholesale prices and the transportation cost that the downstream firms pay when moving to the centre of the line. The interaction of these forces give the equilibrium result.

In the case where upstream firms can price discriminate among the downstream firms, competition becomes more intense pushing the wholesale prices down to the difference in the transportation costs. The retailers are indifferent where to buy from. We assume that the upstream firms are located at the two opposite endpoints and find that the wholesale and final prices under price dis-

crimination are lower than without price discrimination. For some transportation parameter values the total transportation cost under price discrimination is also lower.

Previous work¹ by Matsushima (2004) has also studied the two upstream and two downstream firm structure on the line and find equilibrium locations that depend on the transportation cost parameters. Our model differs in that upstream firms do not restrict wholesale prices to be equal to the rival firm's transportation cost and we also study the case where upstream firms cannot price discriminate among downstream firms. Furthermore, we assume that the downstream firms pay the quadratic transportation costs per unit of input and not the opposite when supplied by the upstream firms. We present a sequential location choice by the upstream firms initially and the downstream firms thereafter, as opposed to the simultaneous symmetric location choices by Matsushima (2004).

3 The model

Consider two downstream firms, X and Y, producing a homogeneous product. There is a linear city of length 1 where consumers are uniformly distributed on the $[0,1]$ line and have unit demands. Firm X is located at point x and firm Y at point $1 - y$, with $x, y \in [0, 1]$. Without loss of generality, it is sufficient to consider only the case where $1 - x - y \geq 0$, that is, firm X is to the left of firm Y on the unit line (see Figure 1). Consumers pay transportation costs quadratic in distance. A consumer located at point z pays transportation cost $t(x - z)^2$ when purchasing a product from firm X and $t(1 - y - z)^2$ when purchasing a product from firm Y. We assume that the basic reservation value of each consumer is high enough so as each consumer purchases one unit of the product. This is a common assumption in the product differentiation literature.

There are two upstream firms, A and B, each producing the same physical product for the downstream firms. The marginal production cost is the same for the two suppliers. Without loss of generality we normalize it to zero. Firm A is located at point a and firm B at point $1 - b$ on the line, with $a, b \in [0, 1]$. Again, it is sufficient to consider only the case where $1 - a - b \geq 0$, that is, firm A is to

¹Dobson and Waterson (1996) study the exclusivity in an exogenously differentiated successive duopoly in upstream and downstream level with consumers facing four varieties. Inderst and Shaffer (2007) analyse the impact of retail mergers on product variety in a non-Hotelling type differentiated model.

the left of firm B (see Figure 1). Downstream firms face the most simple fixed proportions technology. We assume one unit of the upstream firms' product becomes one unit of final good. Downstream firms pay linear wholesale prices to the upstream firms and incur transportation costs quadratic in distance. For example, firm X pays a wholesale price w_A and unit transportation cost $\tau(x-a)^2$ when is supplied by upstream firm A. Downstream firms pay transportation costs per unit of product they purchase. These costs may be real transportation costs (for example unit transportation costs that depend on the weight or the volume of the product) or may be transformation costs necessary to convert one unit of the upstream firm to one unit of final good. Consumers do not have different preferences on product A or product B, product is homogeneous, that is, they care only about the downstream differentiation. The timing of the game is as follows:

- (1) Upstream firms A and B simultaneously choose their locations a, b
- (2) Downstream firms X and Y simultaneously choose their locations x, y
- (3) Upstream firms A and B simultaneously set the linear contract terms w_A, w_B . Notice that each upstream firm sets unique wholesale price, the same for both downstream firms. There is no price discrimination.
- (4) Having observed the wholesale prices, each downstream firm chooses its supplier between A and B. Then simultaneously set their product prices p_X, p_Y . There is no price discrimination also in the downstream level.
- (5) Having observed the firms' locations and the product prices, each consumer purchases one unit of the product from one of the downstream firms X and Y.

We proceed backwards, solving for the subgame perfect Nash equilibria.

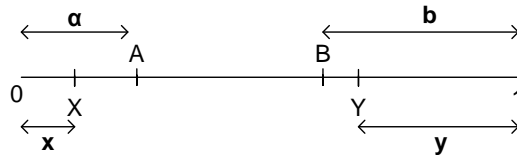


Figure 1: Upstream and downstream locations (example)

4 Equilibrium analysis

4.1 Consumers choice and retail prices

Given the firms' locations and the wholesale prices, we calculate the demand for the downstream firms X and Y. Let z be the demand of firm X and $1 - z$ the demand of firm Y. The indifferent consumer z determines these demands using the following expression

$$p_X + t(x - z)^2 = p_Y + t(1 - y - z)^2 \quad (1)$$

The location of the indifferent consumer depends on the firms' locations, product prices and the transportation cost parameter t , that is, $z = z(p_X, p_Y, x, y, t)$; for simplicity we suppress the arguments of this function, so

$$z = \frac{1 + x - y}{2} + \frac{p_Y - p_X}{2t(1 - x - y)} \quad (2)$$

Taking into account the possibility that all consumers may choose one of the firms to be supplied, the firms' profit functions are

$$\begin{aligned} \Pi_X &= (p_X - f_X)z \\ \Pi_Y &= (p_Y - f_Y)(1 - z) \end{aligned}$$

where f_X (respectively f_Y) is the total marginal cost, that is, the wholesale price and the transportation cost of firm X (respectively Y) and z is equal to

$$z = \begin{cases} 1 & \text{if } \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \geq 1 \\ \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} & \text{if } 0 < \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} < 1 \\ 0 & \text{if } \frac{1+x-y}{2} + \frac{p_Y-p_X}{2t(1-x-y)} \leq 0 \end{cases} \quad (3)$$

The equilibrium price and profit for firm X (respectively we calculate for firm Y) are as follows

$$p_X^* = \begin{cases} f_Y - t(1 - x - y)(1 + y - x) & \text{if } f_X \leq \overline{f_X} \\ \frac{1}{3} (t(1 - x - y)(3 + x - y) + f_Y + 2f_X) & \text{if } \overline{f_X} < f_X < \widehat{f_X} \\ f_X & \text{if } f_X \geq \widehat{f_X} \end{cases} \quad (4)$$

and

$$\Pi_X^* = \begin{cases} f_Y - t(1-x-y)(1+y-x) - f_X & \text{if } f_X \leq \overline{f_X} \\ \frac{(t(1-x-y)(3+x-y)+f_Y-f_X)^2}{18t(1-x-y)} & \text{if } \overline{f_X} < f_X < \widehat{f_X} \\ 0 & \text{if } f_X \geq \widehat{f_X} \end{cases} \quad (5)$$

where

$$\begin{aligned} \widehat{f_X} &= f_Y + t(1-x-y)(3-y+x) \\ \overline{f_X} &= f_Y - t(1-x-y)(3-x+y) \end{aligned}$$

The results from this stage are the same as Ziss (1993) where he solves the linear city model with two firms facing different marginal production costs. When firm X has a low marginal cost (lower than the critical value $\overline{f_X}$) is a downstream monopolist as firm Y has zero demand. The opposite is true when firm X has a high marginal cost (higher than the critical value $\widehat{f_X}$). In this paper we want to examine a vertical structure with a duopoly in the downstream level where there is strategic interaction when choosing optimal locations. We study the case where there is no downstream monopolist and both firms X and Y are active in the retail market. So from equation (5) we study the case where $\overline{f_X} < f_X < \widehat{f_X}$.

4.2 Wholesale prices

In this stage each upstream firm is competing with his rival for the demand of the downstream firms by setting its wholesale price. When the total price offered by one upstream firm, that is, the wholesale price plus the transportation cost is lower (higher) compared to the price of the rival upstream firm for both downstream firms, then this upstream firm takes the whole (zero) demand. Another case is firm A to supply firm X and firm B to supply firm Y. There is no case where firm A supplies firm Y and firm B supplies firm X, as X lies to the left of firm Y and A to the left of firm B with each upstream firm setting one wholesale price for the two downstream firms (no price discrimination)². We summarize the demand function for firm A (similarly we calculate the expressions for firm B)

² As $1-x-y \geq 0$ and $1-a-b \geq 0$ it is not possible to get $w_A + \tau(a-x)^2 > w_B + \tau(1-b-x)^2$ and $w_A + \tau(1-y-a)^2 < w_B + \tau(b-y)^2$ simultaneously.

$$D_A = \begin{cases} 1 & \text{if } w_A + \tau(a-x)^2 \leq w_B + \tau(1-b-x)^2 \text{ and } w_A + \tau(1-y-a)^2 \leq w_B + \tau(b-y)^2 \\ z & \text{if } w_A + \tau(a-x)^2 < w_B + \tau(1-b-x)^2 \text{ and } w_A + \tau(1-y-a)^2 > w_B + \tau(b-y)^2 \\ 0 & \text{if } w_A + \tau(a-x)^2 \geq w_B + \tau(1-b-x)^2 \text{ and } w_A + \tau(1-y-a)^2 \geq w_B + \tau(b-y)^2 \end{cases}$$

But as $\min(w_B + \tau(1-b-x)^2 - \tau(a-x)^2, w_B + \tau(b-y)^2 - \tau(1-y-a)^2) = w_B + \tau(b-y)^2 - \tau(1-y-a)^2$
and $\max(w_B + \tau(1-b-x)^2 - \tau(a-x)^2, w_B + \tau(b-y)^2 - \tau(1-y-a)^2) = w_B + \tau(1-b-x)^2 - \tau(a-x)^2$ we get

$$D_A = \begin{cases} 1 & \text{if } w_A \leq w_B + \tau(b-y)^2 - \tau(1-y-a)^2 \\ z & \text{if } w_B + \tau(b-y)^2 - \tau(1-y-a)^2 < w_A < w_B + \tau(1-b-x)^2 - \tau(a-x)^2 \\ 0 & \text{if } w_A \geq w_B + \tau(1-b-x)^2 - \tau(a-x)^2 \end{cases} \quad (6)$$

The profit function of firm A is as follows

$$\Pi_A = \begin{cases} w_A & \text{if } w_A \leq w_B + \tau(b-y)^2 - \tau(1-y-a)^2 \\ w_A z & \text{if } w_B + \tau(b-y)^2 - \tau(1-y-a)^2 < w_A < w_B + \tau(1-b-x)^2 - \tau(a-x)^2 \\ 0 & \text{if } w_A \geq w_B + \tau(1-b-x)^2 - \tau(a-x)^2 \end{cases} \quad (7)$$

where $z = \frac{2t(1-x-y)(y-x)+f_y-f_x}{6t(1-x-y)} + \frac{1}{2}(1+x-y)$ from the previous stage (equation (3) and (4)).

Upstream firms maximize their profits with respect to the linear contract terms. The reaction function for firm A is

$$w_A = \begin{cases} 0 & \text{if } w_B \leq \widehat{w}_B \\ \frac{1}{2}w_B + \frac{t(x-y+3)(1-x-y)-\tau(a+b-x-y)(a-b-x+y)}{2} & \text{if } \widehat{w}_B < w_B < \overline{w}_B \\ w_B + \tau(b-y)^2 - \tau(1-y-a)^2 & \text{if } w_B \geq \overline{w}_B \end{cases}$$

where

$$\begin{aligned} \widehat{w}_B &= t(x-y+3)(1-x-y) - \tau((a+b-x-y)(a-b-x+y) + 2(b+x-1)^2 - 2(a-x)^2) \\ \overline{w}_B &= t(x-y+3)(1-x-y) - \tau((a+b-x-y)(a-b-x+y) - 2(a+y-1)^2 + 2(b-y)^2) \end{aligned}$$

This gives the equilibrium wholesale price

$$w_A = \begin{cases} \frac{\tau(b-y)^2 - \tau(1-y-a)^2}{3} & \text{if } 2t(x-y)(1-x-y) \leq C \\ \frac{\tau(x+y-b-a)(a-b-x+y)+t(y-x-9)(x+y-1)}{3} & \text{if } C < 2t(x-y)(1-x-y) < D \\ 0 & \text{if } 2t(x-y)(1-x-y) \geq D \end{cases}$$

where

$$\begin{aligned} C &= \tau(6a + 6y - 4ax - 6ay - 2by - a^2 + b^2 + 2x^2 - 2y^2 - 3) \\ D &= \tau(-6b - 6x + 4by + 6bx + 2ax - a^2 + b^2 + 2x^2 - 2y^2 + 3) \end{aligned}$$

The constraint $\overline{f_X} < f_X < \widehat{f_X}$ should also be satisfied. In the case where firm A takes the whole demand $t(3-x+y) > \tau(x-y-2a+1)$ and $t(3-y+x) > \tau(y-x+2a-1)$ should be satisfied. In the case where firm X is supplied by firm A and firm Y by firm B we need $t(3-x+y) > \tau(x-y-2a+1)$ and $t(3-y+x) > \tau(y-x-2b+1)$. In the final case where firm B takes the whole demand the location choices should satisfy $t(3-x+y) > \tau(x-y+2b-1)$ and $t(3-y+x) > \tau(y-x-2b+1)$.

4.3 Downstream (retailers) locations

Downstream firms simultaneously choose their locations to maximise their profits. The profit function of firm X is

$$\Pi_X = \begin{cases} \frac{(1-x-y)(t(3+x-y)+\tau(x-2a-y+1))^2}{18t} & \text{if } 2t(x-y)(1-x-y) \leq C \\ \frac{(\tau(a-b-x+y)(a+b-x-y)+t(x-y+9)(x+y-1))^2}{162t(1-x-y)} & \text{if } C < 2t(x-y)(1-x-y) < D \\ \frac{(1-x-y)(t(3+x-y)+\tau(x+2b-y-1))^2}{18t} & \text{if } 2t(x-y)(1-x-y) \geq D \end{cases}$$

When $2t(x-y)(1-x-y) \leq C$ the optimal locations of the downstream firms that satisfy also the constraint $\overline{f_X} < f_X < \widehat{f_X}$ are $(x, y) = \left(\frac{4a\tau-t}{4(t+\tau)}, \frac{4\tau(1-a)-t}{4(t+\tau)}\right)$. When both suppliers have positive demand, from the first order conditions of the downstream firms and by the fact that the profit margin is positive we find two potential equilibrium locations $(x_1, y_1) = \left(\frac{1+a-b}{2}, \frac{1+b-a}{2}\right)$ and $(x_2, y_2) = \left(\frac{(16a\tau^2(1-a-b)+4t\tau(17a+6b-a^2+b^2-7)-63t^2)}{4(t+\tau)(4\tau(1-a-b)+9t)}, \frac{(16b\tau^2(1-a-b)+4t\tau(17b+6a-b^2+a^2-7)-63t^2)}{4(t+\tau)(4\tau(1-a-b)+9t)}\right)$. Notice that for the first pair $x_1 = 1 - y_1$, that is, X and Y are located at the same point, which is not an equilibrium point. Given $x_1 = \frac{1+a-b}{2}$ the best

response of firm Y is not y_1 . The second pair of locations (x_2, y_2) is positive if the numerators of x_2 and y_2 are positive. If the numerators of x_2 and y_2 are negative the equilibrium locations may be $(x'_2, y'_2) = (0, 0)$. The equilibrium locations should satisfy $C < 2t(x - y)(1 - x - y) < D$ and $\overline{f_X} < f_X < \widehat{f_X}$. Finally, when $2t(x - y)(1 - x - y) \geq D$ the optimal locations satisfying the constraint $\overline{f_X} < f_X < \widehat{f_X}$ are $(x, y) = \left(\frac{4\tau(1-b)-t}{4(t+\tau)}, \frac{4b\tau-t}{4(t+\tau)} \right)$.

4.4 Upstream locations

In this stage upstream firms choose their locations on the unit line. We find that there are no equilibrium locations that satisfy $2t(x - y)(1 - x - y) \leq C$ or $2t(x - y)(1 - x - y) \geq D$. Therefore there are no upstream monopolists. In the case where $C < 2t(x - y)(1 - x - y) < D$ there are no equilibrium locations (a, b) such that (x_2, y_2) is positive. Unique equilibrium is $(a, b) = (0, 0)$ with $(x'_2, y'_2) = (0, 0)$. For these optimal locations we need $3t > \tau$ so as $\overline{f_X} < f_X < \widehat{f_X}$ is satisfied. The transportation cost t of the consumers is higher than the $\frac{1}{3}$ of the transportation cost τ that pay the downstream firms when they are supplied by the upstream firms. Next propositions summarize the equilibrium outcome, states the social cost in equilibrium and the optimal location when firms are vertically integrated.

Proposition 1 *The equilibrium outcome of the five-stage game is:*

$$a = b = 0, x = y = 0, w_A = w_B = 3t, p_X = p_Y = 4t, q_X = q_Y = \frac{1}{2}, \Pi_X = \Pi_Y = \frac{1}{2}t, \Pi_A = \Pi_B = \frac{3}{2}t \text{ for } 3t > \tau$$

Proposition 2 *Social cost is equal to $\frac{1}{12}t$ in equilibrium. As the transportation cost parameter increases the social cost increases too.*

In equilibrium social cost is equal to the transportation cost of the consumers as this is a unit demand model and the downstream firms are located at the same point of their suppliers.

$$SC = \int_0^{\frac{1}{2}} t(z - x)^2 dz + \int_{\frac{1}{2}}^1 t(1 - y - z)^2 dz = \int_0^{\frac{1}{2}} tz^2 dz + \int_{\frac{1}{2}}^1 t(1 - z)^2 dz = \frac{1}{12}t$$

Proposition 3 *If firms are vertically integrated (firm X with her supplier A and firm Y with her supplier B) we end up to the D'Aspremont et al model as wholesale prices are zero and the two vertically integrated pair of firms locate at the two endpoints (each vertically integrated pair of firms acts as one firm).*

Now let's discuss the above results. As the transportation cost parameter t of the consumers increases, product differentiation matters more and consumers pay higher travel costs. The wholesale and retail prices and the profits of the upstream and downstream firms increase. In the extreme case where t is zero there is no product differentiation and the bertrand result emerges. Notice that in equilibrium the retail price p equals the wholesale price w plus the transportation cost t , that is, the marginal cost of the downstream firm plus the transportation cost of the consumers. This is also the equilibrium price presented at the D'Aspremont *et al* (1979) model where the prices equal the marginal production cost c of the firms plus the travel parameter t ($p = c + t$). But if the production cost c is zero (as we assume) the D'Aspremont *et al* final prices equal the transportation cost parameter t ($p = c + t = t$). But in our model even though the marginal production cost is zero final prices are higher than the transportation cost t due to the existence of the intermediaries.

Remark 1 *A type of double marginalization emerges with both upstream and downstream firms having positive profit margins. Final prices are higher than the vertical integration case.*

The transportation cost parameter τ that affects the transportation cost paid by the downstream firms when supplied by the upstream firms does not affect the equilibrium prices and profits as retailers are located at the same point of their suppliers. But in equilibrium τ should not be high enough (τ is lower than $3t$), otherwise, the retail market may be monopolised. If τ is very high then the marginal cost of the retailers is high and this may exclude a retailer from the market (get zero demand given the location of the firms). The aim of this paper is to explore the location choices of a vertical environment when there is competition in the downstream level. So by assuming that τ is not very high we focus in the case where there is a duopoly in the retail market.

In models of horizontal differentiation with two firms on the line and no upstream firms, there are two opposite forces, one pushing firms close to each other to obtain higher demand and one in the opposite direction to reduce the price competition. In our formulation there is also a third force. Given the locations of the other three firms, if the downstream firm moves to the centre of the line this affects the total marginal cost that it pays, ie the wholesale price plus the transportation cost. This effect may be positive or negative depending on the location of the upstream firms. For example given the location of firm A, B and Y (a, b, y) if retailer X moves to right (increase x), this may increase or may decrease the total marginal cost that it pays depending on the

location of his supplier A ($\frac{d(w_A + \tau(a-x)^2)}{dx} = -\frac{2}{3}(t(x+4) + 2\tau(a-x))$). If firm A is located to the right of firm X ($a > x$) then $\frac{d(w_A + \tau(a-x)^2)}{dx} < 0$ and as x increases the marginal cost of firm X decreases which is a positive effect. So by increasing x marginal cost is reduced, demand is increased (demand effect) and prices fall (price competition effect). But if firm A is located to the left of firm X ($a < x$), the sign of the third effect $\frac{d(w_A + \tau(a-x)^2)}{dx}$ depends on the values of the cost parameters. If $\frac{d(w_A + \tau(a-x)^2)}{dx} > 0$ this means that as x increases total marginal cost increase too, which may lead to an increase in the retail price and a reduction in the demand. Therefore as firm X moves to the centre is ambiguous if the demand effect is positive and the price competition effect negative. In equilibrium maximum differentiation is obtained ($a = b = 0, x = y = 0$) which means that the forces pushing the firms (X compared to Y and A compared to B) close to each other are dominated by the forces pushing firms to the endpoints.

Finally, social optimum locations are briefly discussed here. The social planner minimises the total transportation cost $\int_0^{\frac{1}{2}} t(z-x)^2 dz + \int_{\frac{1}{2}}^1 t(1-y-z)^2 dz + \tau(x-a)^2 + \tau(y-b)^2$.

Remark 2 *The social optimum locations are $x^* = y^* = \frac{1}{4}, a^* = b^* = \frac{1}{4}$*

Firm X is located at the same point of her supplier A and firm Y at the same point of her supplier B, but not at the endpoints. The social cost is $SC^* = \frac{1}{48}t$ lower than in equilibrium. These social optimum locations also result from the classic D'Aspremont *et al* model where there are no upstream firms. Here the two pairs upstream-downstream firm are located at the same point and the downstream firms pay zero transportation costs.

5 Price discrimination

Suppose now that upstream firms can charge different wholesale prices among the two downstream firms. The game remains the same with the only difference that now upstream firm j charges the downstream firm i a wholesale price w_i^j . We proceed backwards solving for the subgame perfect equilibrium. The retail pricing stage remains the same as in Section 4. Final prices are given by equation (4). These prices depend on the total marginal cost of the retailers f_X and f_Y .

Next the four wholesale prices are determined. Upstream firm A supplies both retailers if the total prices that she charges (wholesale prices plus transportation costs) firm X and firm Y are lower than those of firm B. So the demand

for firm A has two components D_X^A that equals z (indifferent consumer or the demand of firm X) and D_Y^A that equals $1 - z$. The demand for firm B is zero ($D_X^B = D_Y^B = 0$). Upstream firm A supplies only firm X and upstream firm B supplies firm Y if the total prices that they charge to firm X and Y respectively are lower than their rival. Similarly, there is the case where firm A supplies firm Y and firm B supplies firm X, as well as, the case where firm B supplies both retailers. The demand functions are the following.

$$\begin{aligned}
D_X^A &= \begin{cases} z & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ z & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \\ 0 & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ 0 & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \end{cases} \\
D_Y^A &= \begin{cases} 1 - z & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ 0 & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \\ 1 - z & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ 0 & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \end{cases} \\
D_X^B &= \begin{cases} 0 & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ 0 & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \\ z & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ z & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \end{cases} \\
D_Y^B &= \begin{cases} 0 & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ 1 - z & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \\ 0 & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ 1 - z & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \end{cases}
\end{aligned}$$

Hence, the profit function of the upstream firms are

$$\begin{aligned}
\Pi_A &= \begin{cases} w_X^A z + w_Y^A (1 - z) & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ w_X^A z & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \\ w_Y^A (1 - z) & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ 0 & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \end{cases} \\
\Pi_B &= \begin{cases} 0 & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ w_Y^B (1 - z) & \text{if } w_X^A + \tau(a - x)^2 \leq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \\ w_X^B z & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \leq w_Y^B + \tau(b - y)^2 \\ w_X^B z + w_Y^B (1 - z) & \text{if } w_X^A + \tau(a - x)^2 \geq w_X^B + \tau(1 - b - x)^2 \ \& \ w_Y^A + \tau(1 - y - a)^2 \geq w_Y^B + \tau(b - y)^2 \end{cases}
\end{aligned}$$

where $z = \left(\frac{1}{6} \frac{2t(y-x)(1-x-y)+f_Y-f_X}{t(1-x-y)} + \frac{1+x-y}{2} \right)$ is the demand of firm X.

Before calculating the optimal wholesale prices some interesting questions arise. When firm A supplies firm X and firm B supplies firm Y, does the wholesale price w_Y^A that charges firm A to the retailer that does not supply goes to zero (the marginal production cost) due to the bertrand price competition? Or does firm A charge a positive w_Y^A so as to induce firm B to set a higher w_Y^B that will lead to a reduction in the demand of firm B and an increase in the demand of firm A? Notice that the wholesale price that firm A charges to the retailer that finally does not supply, affects the demand functions. But as firms are symmetric, does firm B think in the same way as firm A?

We find that the equilibrium wholesale prices are equal to the difference in the transportation costs of the retailers. This means that the retailers are indifferent in purchasing from firm A or firm B³. Price competition is very intense when price discrimination is possible and wholesale prices are set to the lowest level. Firms neither have an incentive to further decrease their wholesale prices as this leads to lower profits (lower wholesale prices with the same demand) nor to increase their wholesale prices as this does not increase their profits. If one firm priced above this amount, the rival firm could price at this amount and serve all demand. Notice also that the wholesale prices that upstream firms charge to the downstream firms that do not supply go down to zero. The equilibrium wholesale prices are given by the following expressions.

$$w_X^A = \begin{cases} \tau(1-b-x)^2 - \tau(a-x)^2 & \text{if } \tau(1-b-x)^2 - \tau(a-x)^2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$w_Y^A = \begin{cases} \tau(b-y)^2 - \tau(1-y-a)^2 & \text{if } \tau(b-y)^2 - \tau(1-y-a)^2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$w_X^B = \begin{cases} \tau(a-x)^2 - \tau(1-b-x)^2 & \text{if } \tau(a-x)^2 - \tau(1-b-x)^2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$w_Y^B = \begin{cases} \tau(1-y-a)^2 - \tau(b-y)^2 & \text{if } \tau(1-y-a)^2 - \tau(b-y)^2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

For example, when firm A supplies firm X and firm B supplies firm Y the wholesale prices are $w_X^A = \tau(1-b-x)^2 - \tau(a-x)^2$, $w_Y^A = 0$, $w_X^B = 0$, $w_Y^B = \tau(1-y-a)^2 - \tau(b-y)^2$. Notice that there is no case where firm A supplies firm Y and firm B supplies firm X, that is, retailers cannot be supplied by the farrest

³We assume that indifferent retailers are supplied by the closest supplier (efficient outcome).

supplier (cannot get $w_Y^A = \tau(b-y)^2 - \tau(1-y-a)^2 > 0$ and $w_X^B = \tau(a-x)^2 - \tau(1-b-x)^2 > 0$ simultaneously given that X lies to the left of Y). So we study the equilibrium location choices when firm A supplies both retailers, when A supplies X and B supplies Y, and when B supplies both retailers (respectively the equilibrium locations in the next expressions).

The downstream firms choose the profit maximizing locations⁴.

$$x = \begin{cases} \frac{4\tau-t-4b\tau}{4(t+\tau)} & \text{if } b < \frac{-3t-2\tau+2a\tau+4b\tau+2at}{2(t+\tau)} \\ \frac{4t\tau(-2a-3b+a^2-b^2+4)-16\tau^2(b-1)(a+b-1)-3t^2}{4(t+\tau)(3t-4\tau(1-a-b))} & \text{if } L < b < K \\ \frac{4a\tau-t}{4(t+\tau)} & \text{if } b > \frac{3t+2\tau-2a\tau+2at}{2(t+\tau)} \end{cases}$$

$$y = \begin{cases} \frac{4b\tau-t}{4(t+\tau)} & \text{if } b < \frac{-3t-2\tau+2a\tau+4b\tau+2at}{2(t+\tau)} \\ \frac{(4t\tau(-a^2-3a+b^2-2b+4)-16\tau^2(a-1)(a+b-1)-3t^2)}{4(t+\tau)(3t-4\tau(1-a-b))} & \text{if } L < b < K \\ \frac{4\tau-t-4a\tau}{4(t+\tau)} & \text{if } b > \frac{3t+2\tau-2a\tau+2at}{2(t+\tau)} \end{cases}$$

where $K = \frac{(t^2(6a+9)+2t\tau(7a+10b+4ab+2a^2+2b^2-9))+8\tau^2(a+b-1)(a+2b-1)}{2(t+\tau)(3t-4\tau(1-a-b))}$ and $L = \frac{(t^2(6a-9)+2t\tau(-11a-8b+4ab+2a^2+2b^2+9))-8\tau^2(a-1)(a+b-1)}{2(t+\tau)(3t-4\tau(1-a-b))}$

No equilibrium locations (a, b) exist such that only one upstream firm is active in the wholesale market. That is, there are no equilibrium pairs (a, b) that satisfy either $b < \frac{-3t-2\tau+2a\tau+4b\tau+2at}{2(t+\tau)}$ or $b > \frac{3t+2\tau-2a\tau+2at}{2(t+\tau)}$ with supplier A or B serving the whole demand. This is the same result as under no price discrimination. Firms are symmetric, so there is no case one of them to be an upstream monopolist. Hence, the last case is to calculate the equilibrium upstream locations when firm A supplies firm X and firm B supplies firm Y.

If we assume that upstream firms are located at the two opposite endpoints then

Proposition 4 *Suppose that $a = b = 0$ then⁵*

$$x = y = \begin{cases} \frac{4\tau-t}{4(t+\tau)} & \text{if } 1.0149\tau < t < 4\tau \\ 0 & \text{if } t > 4\tau \end{cases}$$

with profits

$$\Pi_X = \Pi_Y = \begin{cases} \frac{(3t-2\tau)t}{4(t+\tau)} & \text{if } 1.0149\tau < t < 4\tau \\ \frac{1}{2}t & \text{if } t > 4\tau \end{cases},$$

$$\Pi_A = \Pi_B = \begin{cases} \frac{(3t-2\tau)\tau}{4(t+\tau)} & \text{if } 1.0149\tau < t < 4\tau \\ \frac{1}{2}\tau & \text{if } t > 4\tau \end{cases}$$

⁴ As in the previous section we examine the case where both downstream firms are active in the market. So the inequality $\widehat{f}_X < f_X < \widehat{f}_X$ should be satisfied.

⁵ For $t < 1.0149\tau$ we prove that there are no location equilibrium in pure strategies.

and prices

$$w_{AX} = w_{BY} = \begin{cases} \frac{(3t-2\tau)\tau}{2(t+\tau)} & \text{if } 1.0149\tau < t < 4\tau \\ \tau & \text{if } t > 4\tau \end{cases}, w_{AY} = w_{BX} = 0,$$

$$p_X = p_Y = \begin{cases} \frac{(33t\tau-16\tau^2+24t^2)t}{16(t+\tau)^2} & \text{if } 1.0149\tau < t < 4\tau \\ t + \tau & \text{if } t > 4\tau \end{cases}$$

Remark 3 Under price discrimination wholesale and final prices decrease compared to the case where upstream firms charge the same price to the retailers⁶. Also, the social cost is equal to

$$SC = \begin{cases} \frac{96\tau^3-20t\tau^2-28t^2\tau+13t^3}{48(t+\tau)^2} & \text{if } 1.0149\tau < t < 4\tau \\ \frac{1}{12}t & \text{if } t > 4\tau \end{cases}$$

The two downstream firms are not located at the two endpoints in contrast to the case where firms cannot price discriminate for $1.0149\tau < t < 4\tau$. Firms move toward the centre of the line. When $1.633\tau < t < 4\tau$ the social cost under price discrimination is lower than the non-price discrimination case. The opposite is true when $1.0149\tau < t < 1.633\tau$.

Remark 4 Price discrimination may be not anticompetitive as it intensifies price competition at the wholesale level which leads to lower final prices for all values of t and equilibrium locations are closer to the social optimal when $1.633\tau < t < 4\tau$.

6 Conclusion

We have studied a horizontal differentiation model where upstream and downstream firms are located on the unit line. The equilibrium outcome is the D'Aspremont result of maximum product differentiation. Upstream firms locate at the two opposite endpoints and downstream firms locate at the same point of their suppliers. The transportation cost parameter τ of the downstream firms need to be lower than the transportation cost parameter t of the consumers so as to ensure a duopoly in the retail market. The equilibrium locations are not the social optimum. The social planner would not choose the extreme points for the upstream and downstream firms. When there is the possibility of price

⁶The difference in the wholesale and final prices under price discrimination and no price discrimination equals $\Delta w = \begin{cases} \frac{3t\tau+2\tau^2+6t^2}{2(t+\tau)} & \text{if } 1.0149\tau < t < 4\tau \\ 3t - \tau & \text{if } t > 4\tau \end{cases}$ and $\Delta p = \begin{cases} \frac{5(19t\tau+16\tau^2+8t^2)t}{16(t+\tau)^2} & \text{if } 1.0149\tau < t < 4\tau \\ 3t - \tau & \text{if } t > 4\tau \end{cases}$ respectively. Both expressions are positive.

discrimination, the price competition is more intense and the wholesale prices are equal to the difference in the transportation costs of the retailers. Wholesale prices and final prices reduce compared to the case where firms cannot price discriminate. Price discrimination changes the optimal location choices and under most parameters values of t price discrimination reduces social cost.

An interesting extension of the model would be to allow the consumers to have different preferences for the products of the upstream firms. That is, final consumers will face four different products, product ji produced by upstream firm j and sold by downstream firm i . What are the equilibrium locations (or product characteristics)? Also, different contract types may affect the location choices. Does the equilibrium outcome change if we assume that upstream firms set non-linear contract terms?

While our model cannot be fully general one, we believe that the examination of the issues described in this paper will shed light to aspects of the market structure and the strategic interaction between the firms that corresponds to numerous real-world cases. Nevertheless, the interplay between product differentiation and vertical relationships should be further examined.

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