

Federation Formation with Endogenous Tax Determination

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Abstract

We consider n regions, each with a discrete project in its jurisdiction. Every region can freely either form a federation with some other regions or undertake its project all alone. Regions are motivated to participate in the federation by the fact that they fail to benefit from project externalities otherwise. Federation members decide by majority voting simultaneously on which projects to be funded and how to split their costs. Whenever the federation achieves some surplus, voting circles emerge and we do have an equilibrium. A unique equilibrium can be reached when the federation achieves no surplus.

JEL D71, H11, H23, H41

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1 Introduction

In the last few decades we have witnessed various countries participating in federations, such as NAFTA and EU. There is a number of advantages in participating in these organizations: Federation members can usually trade in better terms than non-members¹. Additional gains arise from free goods, capital and labour mobility. Also, federation's availability on resources is higher than individual countries. Thus, the federation can exploit economies of scale. Finally, it is easier to realize and manage the external benefits from regional public goods.

In this paper we focus on this latter issue. For instance, consider the construction of a new port in Portugal. The fact that this regional public good is located in Portugal does not mean that Portugal is the only one benefited from it. Spain, France, even Germany may obtain some benefit from cheaper and

¹We know from the standard literature that when taxes decrease dead weight losses are lower.

faster transfer of goods from and to countries on the other side of the Atlantic, Africa etc. If Portugal is not in a federation, constructs the port alone and the other countries do not have positive externalities (as they have to pay to use the port's services). On the other hand, if Portugal participates in a federation, all members contribute to the construction of the port. and benefit in return from the externalities.

There are two issues concerning the establishment of a federation: Federation participants benefit from the fact that local governments may find it hard to coordinate to internalize interjurisdictional externalities or exploit economies of scale in the provision of regional public goods. On the other hand, entering a federal system reduces ones responsiveness to the preferences of his citizens' in the choice of quality, quantity and choice of the supplied public good. Initially, the assumption made by the literature was that if a regional public good were provided centrally, it would be provided at the same quantity per capita across all members. This assumption provided a sense of equal treatment among regions. After all, there are political constraints on the central government that typically prevent it from providing more generous outputs to some jurisdictions. Moreover, asymmetry of information arises as the information available to the central and local government is not the same. As local governments are closer to their constituencies, they have a superior knowledge of preferences, demands and local conditions. The conclusion drawn is the well know Oates' Decentralization Theorem, which states that there is an efficient level of decentralization of the provision of a public good, where the additional benefit from less policy uniformity is balanced by the loss due to less efficient internalization of externalities (Oates (1972)).

Bolton and Roland (1997) extend the policy uniformity outcomes to other instruments, such as taxes. Since then, there have been efforts to embody political choice mechanisms. However, the policy uniformity assumption was heavily criticized as is neither theoretically satisfactory (it lacks micro foundations) nor empirically (Boadway and Wildasin (1984), Costello (1993)). This literature suggests that if the central government were permitted to choose different levels of public goods for each jurisdiction, it would choose the surplus maximizing level for each jurisdiction. Thus, the centralized system would produce at least as much surplus as a decentralized system and strictly more if externalities are present. Bolton and Roland (1996) and Ellingsen (1998) depart from the policy uniformity assumption but obtain de facto uniformity. Besharov (2002) exclaims that there might be cases where policy uniformity is the Pareto optimal policy. Models also departing from policy uniformity but embodying political decision mechanisms were studied by Besley and Coate (2003), Lockwood (2002), Alesina et al (2005).

The work closest to the present paper is by Lockwood (2002). Lockwood studies an odd number of regions, each with a discrete project in its jurisdiction. All regions ex-ante act either under decentralization or centralization. With decentralization, each project is funded by regional resources and only if its local benefit exceeds its cost. Under centralization on the other hand, delegates from all regions, decide about which projects to fund. Every region participates

to the cost of projects undertaken with an exogenous and positive cost share. He further imposes some minimal rules on the legislature. Specifically, delegates first propose alternatives (bundles of projects) for consideration, and then all proposed alternatives are voted on, according to an amendment agenda. The equilibrium outcome is a set of projects chosen for funding.

Under centralization, the model has benefit-insensitivity of the equilibrium. Specifically, the set of projects undertaken is independent to local benefits. Centralization means that decisions are less responsive to regional preferences. Furthermore, decentralization is found to be more efficient than centralization if net externalities are zero. Centralization is more efficient if project net externalities are positive and high enough. However, when regions are ε -homogenous, centralization strictly Pareto-dominates decentralization.

We extend the above analysis by allowing n regions to decide endogenously to either form a federation with some other regions or undertake their projects all alone. Federation participants endogenously decide on which projects to fund and how should costs be shared, through majority voting. We find that all projects with non-negative net benefit for the federation are funded, thus our model also has benefit-insensitivity. In cases where the federation achieves some surplus, voting circles emerge and we do not have an equilibrium. A unique equilibrium can be reached when there is no surplus to share, in other words when the regions are indifferent between participating in the federation or not.

2 The model

We consider $N = 1, 2, \dots, n$ regions², each populated with a mass of unity identical citizens. Each $i \in N$ has a discrete project in its jurisdiction, that we call it project i , which costs $c_i \in \mathfrak{R}^{++}$. Project i generates benefit $b_i \in \mathfrak{R}$ for the citizens of i and also some externality or external benefit $e_{ji} \in \mathfrak{R}$ to citizens of region $j \in N, \forall j \neq i$, with $e_{ii} = 0$. Region i can freely either form a federation with some other regions or undertake project i all alone. We call $F \subseteq N$ the set of regions that enter the federation. The federation can undertake one or more projects but only from those that belong to F .

Members of N are motivated to participate in the federation by the fact that they fail to benefit from externalities otherwise. On the other hand, all $i \in F$ are obligated to contribute in federation costs. To make this more clear, we present the utility function of i . We are able to define such a utility function, as we assume that all residents of region i have identical preferences over the undertaken projects.

Let i be in the federation that constructs a set of P projects. Then, i has utility function of linear form:

$$u_i(P, \lambda_i^P) = \begin{cases} b_i + \sum_{j \in P} e_{ij} - \lambda_i^P \sum_{j \in P} c_j & \text{if } i \in P, \\ \sum_{j \in P} e_{ij} - \lambda_i^P \sum_{j \in P} c_j & \text{if } i \notin P, \end{cases} \quad (1)$$

²with n either odd or common

where $\lambda_i^P \in \mathfrak{R}^+$ is the share of the total construction cost $\sum_{j \in P} c_j$, region i contributes to the federation.

Moreover, region $i \in N \setminus F$ also has utility function of linear form,

$$v_i = \begin{cases} b_i - c_i & \text{if } b_i \geq c_i, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Equation (2) arises from (1) where i constructs its own project with the use of own resources to finance it, $\lambda_i^{\{i\}} = 1$. Notice that i chooses not to undertake project i if its cost, c_i , exceeds the benefit generated for citizens of i , b_i . Furthermore, equation (2) implies that if $i \in N \setminus F$, externalities from federation's projects fail to enter its utility function. This latter feature of our model needs some justification.

Let us take for example, California, Arizona and Mexico. California and Arizona belong to USA and pay some taxes to the federal government, whereas Mexico does not. The federal government distributes tax revenues among federation members for project funding. Hence, if California had a project to be undertaken, Arizona would be contributing to its cost. Mexico however, is not obligated to pay.

Consider California's project, which could be either a tax free trading policy or a new highway or even a new airport. It is fine for Arizona to benefit from this project, but when Mexico does so, a free riding problem emerges. We can safely claim that there are ways for California to eliminate externalities targeted for Mexico. Actually, it has incentive to do so. The tax free trading policy for example, could be restricted to USA members only, the highway may have tolls installed on the borders with Mexico and the airport could charge special import taxes on goods manufactured outside the USA, etc.

3 Funding decision

Federation participants simultaneously determine not only which projects to undertake, but also the way costs are shared. We denote this decision by (P, λ^P) . Its first part, P , answers the question "Which projects to undertake?", while its second part, λ^P , is the $1 \times F$ dimensional vector of cost shares for $i \in F$ and provides the answer to the question "How to split the costs?". We restrict our attention to feasible decisions: for a given P ,

$$\sum_{i \in F} \lambda_i^P = 1.$$

We assume that two decisions $(P, \lambda^P), (P', \lambda^{P'})$ differ when either $P \neq P'$ or $\lambda^P \neq \lambda^{P'}$ or both.

Let us define the preference relations of our model. Region $i \in F$ (weakly) prefers (P, λ^P) to $(P', \lambda^{P'})$, $(P, \lambda^P) \succeq_i (P', \lambda^{P'})$, when utility gains for i under

(P, λ) are at least as high as under $(P', \lambda^{P'})$: $u_i((P, \lambda_i^P) \geq u_i((P', \lambda_i^{P'}))$. In analogy, when some majority of regions within the federation finds (P, λ^P) at least as beneficial as $(P', \lambda^{P'})$ the federation (weakly) prefers (P, λ^P) to $(P', \lambda^{P'})$: $(P, \lambda^P) \succeq (P', \lambda^{P'})$. Preferences are assumed to consist common knowledge and $\forall i \in N$ have full information.

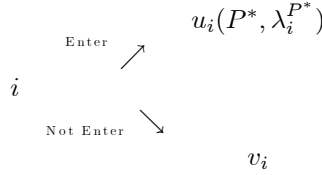
We denote the federation's equilibrium (P^*, λ^{P^*}) and assume that it is determined by majority voting. Only regions belonging to the federation have the right to vote. For example, if $F = \{1, 2, 3\}$ and the following ranking holds:

$$\begin{aligned} 1 & : (P, \lambda^P) \succ_1 (P', \lambda^{P'}), \\ 2 & : (P, \lambda^P) \succ_2 (P', \lambda^{P'}), \\ 3 & : (P', \lambda^{P'}) \succ_3 (P, \lambda^P), \end{aligned}$$

$\forall (P, \lambda^P), (P', \lambda^{P'})$ feasible, $(P, \lambda^P) \neq (P', \lambda^{P'})$, the federation would undertake $P^* = P$ and the costs will be shared according to $\lambda^{P^*} = \lambda^P$.

3.1 The game

Members of N play the following game: Each i simultaneously decides either to enter the federation or not and $\lambda_i^{P^*}$, its contribution to the federation's total cost. If i decides to enter, obtains utility level of $u_i(P^*, \lambda_i^{P^*})$. If not, it gains utility equal to v_i .



Since we assume that participation in the federation is not coercive, region i choose to enter the federation only if it gains higher utility than v_i .

Corollary 1 *Region i will enter the federation only if $u_i(P^*, \lambda_i^{P^*}) \geq v_i$.*

In other words, v_i could be considered as i 's exit option from the federation. It consists the utility level region i can guarantee to its citizens.

4 Majority or colluding regions

Throughout our study, what is of outmost importance is not only which projects are to be funded, but also which are the regions in majority. We use the set $M \subset F$ to denote these regions.

Superscripts applied on a project set P stand for the majority regions or colluding regions. For instance, $\{1\}^{12}$ means that the federation undertakes only project 1 and regions 1, 2 are in majority. Whenever the colluding regions

are identical to the projects undertaken, superscripts are omitted. Hence, under $\{1, 2\}$ the federation funds both projects 1, 2 and also 1, 2 are in majority.

Given F we can determine the minimum number of regions required to consist a majority and write the set of project sets, \mathbf{P}_F . \mathbf{P}_F includes a number of project sets equal to

$$\frac{f!}{(f-m)!m!} \sum_{x=1}^f \frac{f!}{(f-x)!x!}$$

where f, m are the cardinalities of F, M respectively. $\frac{f!}{(f-m)!m!}$ is the number of possible majorities and $\sum_{x=1}^f \frac{f!}{(f-x)!x!}$ the number of project combinations available to the federation.

Example 2 We take $F = \{1, 2\}$. Then,

$$\frac{2!}{(2-2)!2!} \sum_{x=1}^2 \frac{2!}{(2-x)!x!} = 3$$

and $\mathbf{P}_F = \{\{1\}^{12}, \{2\}^{12}, \{1, 2\}\}$.

Example 3 Similarly, for $F = \{1, 2, 3\}$,

$$\frac{3!}{(3-2)!2!} \sum_{x=1}^3 \frac{3!}{(3-x)!x!} = 21$$

and

$$\begin{aligned} \mathbf{P}_F = & \{\{1\}^{12}, \{1\}^{13}, \{1\}^{23}, \{2\}^{12}, \{2\}^{13}, \{2\}^{23}, \{3\}^{12}, \{3\}^{13}, \{3\}^{23}, \\ & \{1, 2\}, \{1, 2\}^{13}, \{1, 2\}^{23}, \{1, 3\}^{12}, \{1, 3\}, \{1, 3\}^{23}, \{2, 3\}^{12}, \{2, 3\}^{13}, \{2, 3\}, \\ & \{1, 2, 3\}^{12}, \{1, 2, 3\}^{13}, \{1, 2, 3\}^{23}\}.^3 \end{aligned}$$

The majority regions are important because since $(P^*, \boldsymbol{\lambda}^{P^*})$ are determined by majority voting, those m regions are the ones that choose $\boldsymbol{\lambda}^{P^*}$. More specifically, they manipulate $\boldsymbol{\lambda}^{P^*}$ towards their own interest, as to maximize the utility of their citizens. To put it differently, we assume that the majority yields utility as high as possible for its members. This is done by consuming the utility of the minority. Under $\{1\}^{12}$ for instance, 1 and 2 will choose $\lambda_1^{\{1\}^{12}}, \lambda_2^{\{1\}^{12}}$ to be as low as possible and charge the minority, region 3, a $\lambda_3^{\{1\}^{12}}$ as high as possible.

Determination of $\boldsymbol{\lambda}$ has to take into account two facts. First, for a $(P, \boldsymbol{\lambda}^P)$, $\boldsymbol{\lambda}^P$ has to be such that resources are enough for P to be feasible:

$$\sum_{i \in F} \lambda_i^P \sum_{i \in P} c_i = \sum_{i \in P} c_i \Leftrightarrow \sum_{i \in F} \lambda_i^P = 1. \quad (3)$$

Second, the majority has to take into account that the minority has an exit option available, given by equation (2). The utility level the majority allows the minority to achieve cannot be lower than the exit option, otherwise the minority will not enter the federation. Thus, under $\{1\}^{12}$ we must have that $\lambda_1^{\{1\}^{12}} + \lambda_2^{\{1\}^{12}} + \lambda_3^{\{1\}^{12}} = 1$ and $u_3(\{1\}^{12}, \lambda_3^{\{1\}^{12}}) = v_3$.⁴

5 Which projects will be funded?

We define the net benefit the federation gains from a project i as follows:

Definition 4 *The net benefit, $NB_i \in \mathfrak{R}$, the federation gains from project i , is equal to*

$$NB_i = b_i + \sum_{j \in F} e_{ji} - c_i. \quad (4)$$

The above value is crucial in our model because it determines whether the specific project will be included in P^* or not, as stated by the following Proposition:

Proposition 5 *A project i is included in P^* if and only if*

$$NB_i \geq 0.$$

6 Majorities and voting circles

As far as Proposition 5 is concerned, the majority was held fixed, and we proved that any project with non-negative net benefit will be included in P^* . In this section we let the regions in majority vary and find that voting circles emerge. These circles are due to the vulnerability of some (and possibly all) project sets.

Proposition 6 *For $P', P \in \mathbf{P}_F$ with different majorities, $P' \succ P$ if*

$$\sum_{i \in P'} NB_i - \sum_{i \in F} v_i > 0. \quad (5)$$

What is striking is that when majorities vary, for some pairs of project sets, $P' \succ P$ and $P \succ P'$.

Example 7 *We take $F = 1, 2, 3$ and*

Project	b_i	c_i	e_{i1}	e_{i2}	e_{i3}
1	2	0.5		0.5	0.3
2	2	1	0.4		0.3
3	2	1	0.4	0.5	

⁴We note that the concept we are studying is different from the concept of the Nash equilibrium. In this game, any pair P and λ^P that satisfies $\sum_{i \in F} \lambda_i^P = 1$ and $u_i(P, \lambda_i^P) \geq v_i$ is one of the infinite Nash Equilibria.

We verify that $NB_1 = 2.3, NB_2 = 2, NB_3 = 1.6$ are all positive and according to Proposition 5, P^* will include all the available projects. Moreover, $v_1 = 1.5, v_2 = v_3 = 1$,

$$\begin{aligned} \sum_{i=1}^3 NB_i - \sum_{i=1}^3 v_i &= 2.3 + 2 + 1.6 - 1.5 - 1 - 1 = 2.4 > 0, \\ \sum_{i=1}^2 NB_i - \sum_{i=1}^3 v_i &= 2.3 + 2 - 1.5 - 1 - 1 = 0.8 > 0, \\ \sum_{i=1,3} NB_i - \sum_{i=1}^3 v_i &= 2.3 + 1.6 - 1.5 - 1 - 1 = 0.4 > 0 \text{ and} \\ \sum_{i=2}^3 NB_i - \sum_{i=1}^3 v_i &= 2 + 1.6 - 1.5 - 1 - 1 = 0.1 > 0. \end{aligned}$$

Then,

$$\sum_{i=1}^3 NB_i - \sum_{i=1}^3 v_i > 0$$

which by Proposition 6 implies that $\{1, 2, 3\}^{12}$ is majority preferred to any other project set.

Voting circles emerge because some (possible all) project sets are vulnerable to other project sets. For instance, we take $\{1, 2, 3\}^{12}$ of the latter example where $F = 1, 2, 3$ and $\forall i, NB_i > 0$ thus $\forall i \in P^*$. Proposition 6 suggests that if $\sum_{i=1}^3 NB_i - \sum_{i=1}^3 v_i > 0$ holds, $\{1, 2, 3\}^{12}$ beats all other project sets. Since 3 is in minority under $\{1, 2, 3\}^{12}$, is it the case that by entering the federation will obtain utility level no more than v_3 ? Aren't there any counter suggestions to $\{1, 2, 3\}^{12}$? In other words, can 3 break the collusion of 1,2?

For the collusion to be broken it is necessary to exist a deviation, either with 1 or 2, more profitable than $\{1, 2, 3\}^{12}$. Following Proposition 6, the inequality

$$\sum_{i=1,3} NB_i - \sum_{i=1}^3 v_i > 0 \tag{6}$$

implies that $\{1, 3\}$ is preferred to any other project set, $\{1, 2, 3\}^{12}$ included:

$$\{1, 2, 3\}^{12} \prec \{1, 3\}.$$

As neither majorities nor λ have to stay fixed, 3 suggests $\{1, 3\}$ to 1, a project set less costly than $\{1, 2, 3\}^{12}$ and with a different cost sharing scheme,

$$\begin{aligned} u_1(\{1, 3\}, \lambda'_1) &> u_1(\{1, 2, 3\}^{12}, \lambda_1) \\ u_2(\{1, 3\}, \lambda'_2) &< u_2(\{1, 2, 3\}^{12}, \lambda_2) . \\ u_3(\{1, 3\}, \lambda'_3) &> u_3(\{1, 2, 3\}^{12}, \lambda_3) \end{aligned}$$

Under $\{1, 3\}$, both 1, 3 obtain greater utility levels for their citizens (than under $\{1, 2, 3\}^{12}$) and $u_2(\{1, 3\}, \lambda'_2) = v_2$. We should not forget though that a fixed majority will fund all the available projects that satisfy $NB_i > 0$, as stated in Proposition 5. Hence,

$$\{1, 3\} \prec \{1, 2, 3\}^{13}.$$

Moreover, with (6) holding, 2 also has a profitable deviation to suggest to 1 as to gain utility level greater than v_2 ,

$$\{1, 2, 3\}^{13} \prec \{1, 3\}^{12}.$$

Finally, by using again Proposition 5,

$$\{1, 3\}^{12} \prec \{1, 2, 3\}^{12}.$$

We write the above in a single line as to demonstrate one of the many circles emerging in this example:

$$\{1, 2, 3\}^{12} \prec \{1, 3\} \prec \{1, 2, 3\}^{13} \prec \{1, 3\}^{12} \prec \{1, 2, 3\}^{12} \dots$$

Avoiding these circles on \mathbf{P}_F is necessary to obtain an equilibrium for this game,

Lemma 8 *This game has an equilibrium if Proposition 6 is satisfied only for $P \in \mathbf{P}_F$ that include $\forall i \in F$ such that $NB_i \geq 0$.*

In other words, we demand that the $\frac{f!}{(f-m)!m!}$ project sets that include the most profitable projects (the ones that satisfy $NB_i \geq 0$), beat in majority voting $\forall P \in \mathbf{P}_F$.

7 Equilibrium

Even though Lemma 7 guarantees that voting circles among the $\frac{f!}{(f-m)!m!}$ project sets and all the other project sets will be avoided, Proposition 6 implies that generally exist voting circles among these $\frac{f!}{(f-m)!m!}$ project sets.

Example 9 *We consider $F = 1, 2, 3$ and,*

Project	b_i	c_i	e_{i1}	e_{i2}	e_{i3}
1	2	0.5		0.5	0.05
2	2	1	0.4		0.05
3	2	1	0.4	0.5	

For these projects $NB_i > 0$, so P^ will include all projects. Lemma 7 is also satisfied. Hence, P^* is one of the following project sets : $\{1, 2, 3\}^{12}, \{1, 2, 3\}^{13}, \{1, 2, 3\}^{23}$.*

But since Proposition 6 holds for all these three project sets, $\sum_{i=1}^3 NB_i - \sum_{i=1}^3 v_i > 0$, voting circles emerge even for them: $\{1, 2, 3\}^{12} \prec \{1, 2, 3\}^{13} \prec \{1, 2, 3\}^{12} \prec \{1, 2, 3\}^{23}$ etc.

Hence, whenever the federation achieves some positive surplus, $\sum_{i \in P} NB_i - \sum_{i \in F} v_i > 0$, there does not exist a single majority of regions which can dominate. The game cannot reach a unique equilibrium since for every (P^M, λ^M) there exist another $(P^{M'}, \lambda^{M'})$ with different majority and λ for which $P^{M'} \succ P^M$.

There is however a case in which the federation can reach a unique equilibrium:

Proposition 10 *The game has a unique equilibrium (P^*, λ^{P^*}) , including all projects that satisfy Proposition 5, when Lemma 7 is satisfied, there exists a λ^{P^*} such that*

$$u_i(P^*, \lambda_i^{P^*}) = v_i, \forall i \in F$$

and

$$\sum_{i \in P^*} NB_i = \sum_{i \in F} v_i.$$

For the above equilibrium to exist, we need to have at least one project non-positive externalities and no surplus to share among the federation participants.

Example 11 (A negative externality) *We consider $F = 1, 2, 3$ with*

Project	b_i	c_i	e_{i1}	e_{i2}	e_{i3}
1	2	1		0.1	0.2
2	2	1	0.2		-0.8
3	2	1	0.2	0.1	

Proposition 5 is satisfied for all these projects, hence, all will be included in P^ . $\sum_{i \in F} NB_i = \sum_{i \in F} v_i$ and the only $\lambda^{\{1,2,3\}}$ that satisfies the feasibility of $P^* = \{1, 2, 3\}$ is $\lambda^{\{1,2,3\}} = (\frac{1.3}{3}, \frac{0.4}{3}, \frac{1.3}{3})$. Any other cost distribution does not create enough resources for $\{1, 2, 3\}$ to be funded or leads at least one region not entering the federation. The utility levels achieved are*

$$\begin{aligned} u_1(\{1, 2, 3\}, \frac{1.3}{3}) &= 1 = v_1, \\ u_2(\{1, 2, 3\}, \frac{0.4}{3}) &= 1 = v_2, \\ u_3(\{1, 2, 3\}, \frac{1.3}{3}) &= 1 = v_3. \end{aligned}$$

The unique equilibrium is: $(\{1, 2, 3\}, (\frac{1.3}{3}, \frac{0.4}{3}, \frac{1.3}{3}))$.

Example 12 (Two negative externalities) *Similarly,*

Project	b_i	c_i	e_{i1}	e_{i2}	e_{i3}
1	2	1		0.1	0.2
2	2	1	0.2		-0.2
3	2	1	-0.4	0.1	

and unique equilibrium is $(\{1, 2, 3\}, (\frac{1.3}{3}, \frac{1}{3}, \frac{0.7}{3}))$.

8 Appendix

Proof of Proposition 5. Without loss of generality, we take $F = 1, 2, \dots, f$, fix two random project sets P', P with the same majority. $P' \supset P$ and P' includes only one additional project in comparison to P , project k . We prove that $P' \succeq P$ if project k 's net benefit is not negative and $P' \succ P$ if project k 's net benefit is strictly positive. In the table below we note the utility level each region achieves under P', P :

$$\begin{array}{rcl}
 & P' & P \\
 1 : & b_1 + \sum_{i \in P'} e_{1i} - \lambda_1^{P'} \sum_{i \in P'} c_i & \geq b_1 + \sum_{i \in P} e_{1i} - \lambda_1^P \sum_{i \in P} c_i \\
 2 : & b_2 + \sum_{i \in P'} e_{2i} - \lambda_2^{P'} \sum_{i \in P'} c_i & \geq b_2 + \sum_{i \in P} e_{2i} - \lambda_2^P \sum_{i \in P} c_i \\
 & \vdots & \vdots \\
 m-1 : & \sum_{i \in P'} e_{m-1i} - \lambda_{m-1}^{P'} \sum_{i \in P'} c_i & \geq \sum_{i \in P} e_{m-1i} - \lambda_{m-1}^P \sum_{i \in P} c_i \\
 m : & \sum_{i \in P'} e_{mi} - \lambda_m^{P'} \sum_{i \in P'} c_i & \geq \sum_{i \in P} e_{mi} - \lambda_m^P \sum_{i \in P} c_i \\
 m+1 : & b_{m+1} + \sum_{i \in P'} e_{m+1i} - \lambda_{m+1}^{P'} \sum_{i \in P'} c_i & = b_{m+1} + \sum_{i \in P} e_{m+1i} - \lambda_{m+1}^P \sum_{i \in P} c_i \\
 & \vdots & \vdots \\
 k : & b_k + \sum_{i \in P'} e_{ki} - \lambda_k^{P'} \sum_{i \in P'} c_i & = \sum_{i \in P} e_{ki} - \lambda_k^P \sum_{i \in P} c_i \\
 & \vdots & \vdots \\
 f : & \sum_{i \in P'} e_{fi} - \lambda_f^{P'} \sum_{i \in P'} c_i & = \sum_{i \in P} e_{fi} - \lambda_f^P \sum_{i \in P} c_i
 \end{array}$$

(Table 1)

where regions $M = 1 \dots m$ are in majority under both P', P . As all $i \in F \setminus M$ consists minority either under P or P' (notice, both collusions are against $F \setminus M$'s interest) will be left with the minimum utility possible anyhow,

$$u_i(P', \lambda_i^{P'}) = u_i(P, \lambda_i^P) = v_i, \forall i \in F \setminus M.$$

We add those $f - m$ equations up, eliminate the commons and obtain

$$b_k + \sum_{i \in F \setminus M} e_{ik} = \sum_{i \in F \setminus M} \lambda_i^{P'} \sum_{i \in P'} c_i - \sum_{i \in F \setminus M} \lambda_i^P \sum_{i \in P} c. \quad (7)$$

Also, we take into account that both P', P have to be feasible. Thus, from (3),

$$\begin{aligned}
 \sum_{i \in F} \lambda_i^{P'} \sum_{i \in P'} c_i &= \sum_{i \in P'} c_i \Leftrightarrow \\
 \lambda_m^{P'} \sum_{i \in P'} c_i &= \sum_{i \in P'} c_i - \sum_{i=1}^{m-1} \lambda_i^{P'} \sum_{i \in P'} c_i - \sum_{i \in F \setminus M} \lambda_i^{P'} \sum_{i \in P'} c_i \Leftrightarrow
 \end{aligned}$$

$$\lambda_m^{P'} \sum_{i \in P'} c_i = \sum_{i \in P'} c_i - \lambda_{m-1}^{P'} \sum_{i \in P'} c_i - \sum_{i=1}^{m-2} \lambda_i^{P'} \sum_{i \in P'} c_i - \sum_{i \in F \setminus M} \lambda_i^{P'} \sum_{i \in P'} c_i \quad (8)$$

and similarly for $\lambda_m^P \sum_{i \in P} c_i$

$$\lambda_m^P \sum_{i \in P} c_i = \sum_{i \in P} c_i - \lambda_{m-1}^P \sum_{i \in P} c_i - \sum_{i=1}^{m-2} \lambda_i^P \sum_{i \in P} c_i - \sum_{i \in F \setminus M} \lambda_i^P \sum_{i \in P} c_i \quad (9)$$

Straight: Next, we use the last two inequalities from Table 1 as follows:

$$\begin{aligned} m-1: \quad & \sum_{i \in P'} e_{m-1i} - \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \geq \sum_{i \in P} e_{m-1i} - \lambda_{m-1}^P \sum_{i \in P} c_i \Leftrightarrow \\ m: \quad & \sum_{i \in P'} e_{mi} - \lambda_m^{P'} \sum_{i \in P'} c_i \geq \sum_{i \in P} e_{mi} - \lambda_m^P \sum_{i \in P} c_i \end{aligned}$$

eliminate the commons

$$\begin{aligned} m-1: \quad & e_{m-1k} - \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \geq -\lambda_{m-1}^P \sum_{i \in P} c_i \Leftrightarrow \\ m: \quad & e_{mk} - \lambda_m^{P'} \sum_{i \in P'} c_i \geq -\lambda_m^P \sum_{i \in P} c_i \end{aligned}$$

use (9) and (10) in replacing $\lambda_m^{P'} \sum_{i \in P'} c_i, \lambda_m^P \sum_{i \in P} c_i$

$$\begin{aligned} m-1: \quad & e_{m-1k} - \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \geq -\lambda_{m-1}^P \sum_{i \in P} c_i \\ m: \quad & e_{mk} - \sum_{i \in P'} c_i + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \geq -\sum_{i \in P} c_i + \lambda_{m-1}^P \sum_{i \in P} c_i + \sum_{i=1}^{m-2} \lambda_i^P \sum_{i \in P} c_i \Leftrightarrow \\ & + \sum_{i=1}^{m-2} \lambda_i^{P'} \sum_{i \in P'} c_i + \sum_{i \in F \setminus M} \lambda_i^{P'} \sum_{i \in P'} c_i \geq \sum_{i \in F \setminus M} \lambda_i^P \sum_{i \in P} c_i \\ m-1: \quad & e_{m-1k} - \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \geq -\lambda_{m-1}^P \sum_{i \in P} c_i \\ m: \quad & e_{mk} - \sum_{i \in P'} c_i + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i + \sum_{i=1}^{m-2} \lambda_i^{P'} \sum_{i \in P'} c_i \geq -\sum_{i \in P} c_i + \lambda_{m-1}^P \sum_{i \in P} c_i + \sum_{i=1}^{m-2} \lambda_i^P \sum_{i \in P} c_i \Leftrightarrow \\ & + \sum_{i \in F \setminus M} \lambda_i^{P'} \sum_{i \in P'} c_i - \sum_{i \in F \setminus M} \lambda_i^P \sum_{i \in P} c_i \end{aligned}$$

and (8)

$$\begin{aligned} m-1: \quad & \lambda_{m-1}^{P'} \sum_{i \in P} c_i \geq -e_{m-1k} + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \\ m: \quad & b_k + \sum_{i \in F \setminus M} e_{ik} + e_{mk} - \sum_{i \in P'} c_i + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \geq \lambda_{m-1}^P \sum_{i \in P} c_i \\ & + \sum_{i=1}^{m-2} \lambda_i^{P'} \sum_{i \in P'} c_i + \sum_{i \in P} c_i - \sum_{i=1}^{m-2} \lambda_i^P \sum_{i \in P} c_i \end{aligned}$$

Hence, for λ_{m-1}^P to exist

$$\begin{aligned} b_k + e_{mk} + \sum_{i \in F \setminus M} e_{ik} - \sum_{i \in P'} c_i + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \\ + \sum_{i=1}^{m-2} \lambda_i^{P'} \sum_{i \in P'} c_i + \sum_{i \in P} c_i - \sum_{i=1}^{m-2} \lambda_i^P \sum_{i \in P} c_i \end{aligned} \geq -e_{m-1k} + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i .$$

Following, we take the above inequality along with region's $m-2$ utility levels:

$$\begin{aligned} m-2 : \quad & \lambda_{m-2}^P \sum_{i \in P} c_i \geq -e_{m-2k} + \lambda_{m-2}^{P'} \sum_{i \in P'} c_i \\ & b_k + e_{mk} + \sum_{i \in F \setminus M} e_{ik} - \sum_{i \in P'} c_i + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \Leftrightarrow \\ & + \sum_{i=1}^{m-2} \lambda_i^{P'} \sum_{i \in P'} c_i + \sum_{i \in P} c_i - \sum_{i=1}^{m-2} \lambda_i^P \sum_{i \in P} c_i \geq -e_{m-1k} + \lambda_{m-1}^{P'} \sum_{i \in P'} c_i \\ \\ m-2 : \quad & \lambda_{m-2}^P \sum_{i \in P} c_i \geq -e_{m-2k} + \lambda_{m-2}^{P'} \sum_{i \in P'} c_i \\ & b_k + e_{m-1k} + e_{mk} + \sum_{i \in F \setminus M} e_{ik} - c_k \Leftrightarrow \\ & + \sum_{i=1}^{m-2} \lambda_i^{P'} \sum_{i \in P'} c_i - \sum_{i=1}^{m-2} \lambda_i^P \sum_{i \in P} c_i \geq 0 \\ \\ m-2 : \quad & \lambda_{m-2}^P \sum_{i \in P} c_i \geq -e_{m-2k} + \lambda_{m-2}^{P'} \sum_{i \in P'} c_i \\ & b_k + e_{m-1k} + e_{mk} + \sum_{i \in F \setminus M} e_{ik} - c_k \Leftrightarrow \\ & + \lambda_{m-2}^{P'} \sum_{i \in P'} c_i + \sum_{i=1}^{m-3} \lambda_i^{P'} \sum_{i \in P'} c_i - \sum_{i=1}^{m-3} \lambda_i^P \sum_{i \in P} c_i \geq \lambda_{m-2}^P \sum_{i \in P} c_i \\ & \vdots \\ & b_k + \sum_{i \in M} e_{ik} + \sum_{i \in F \setminus M} e_{ik} - c_k \geq 0 \Leftrightarrow \\ & NB_k \geq 0. \end{aligned}$$

Opposite: In analogy we can prove that if project k has non-negative net benefit, a fixed majority will choose to undertake it. ■

Proof of Proposition 6. It suffices to show that for any $P', P \in \mathbf{P}_F$ with different majorities, $P' \succ P$ when $\sum_{i \in P'} NB_i - \sum_{i \in F} v_i > 0$. Without loss of generality, we set $F = 1, 2, 3$, $P' = \{1, 2\}$, $P = \{2, 3\}$ and write the utility gains

in the following table:

$$\begin{array}{l}
\{1, 2\} \qquad \qquad \qquad \{2, 3\} \\
1 : \quad b_1 + e_{12} - \lambda_1^{\{1,2\}} A > \sum_{i \neq 1}^n e_{1i} - \lambda_1^{\{2,3\}} B \\
2 : \quad b_2 + e_{21} - \lambda_2^{\{1,2\}} A > b_2 + e_{23} - \lambda_2^{\{2,3\}} B \\
3 : \quad \sum_{i \neq 3}^n e_{3i} - \lambda_3^{\{1,2\}} A < b_3 + e_{32} - \lambda_3^{\{2,3\}} B
\end{array} \tag{Table 2}$$

where $A = c_1 + c_2$ and $B = c_2 + c_3$. Notice that under $\{1, 2\}$ region 3 is a minority, meaning that will be left with:

$$u_3(\{1, 2\}, \lambda_3^{\{1,2\}}) = v_3$$

or

$$\sum_{i \neq 3}^n e_{3i} - \lambda_3^{\{1,2\}} A = v_3 \Leftrightarrow \lambda_3^{\{1,2\}} A = \sum_{i \neq 3}^n e_{3i} - v_3,$$

whereas under $\{2, 3\}$, 1 is the region that will yields $u_1(\{2, 3\}, \lambda_1^{\{2,3\}}) = v_1$:

$$u_1(\{2, 3\}, \lambda_1^{\{2,3\}}) = v_1$$

or

$$\sum_{i \neq 1}^n e_{1i} - \lambda_1^{\{2,3\}} B = v_1 \Leftrightarrow \lambda_1^{\{2,3\}} B = \sum_{i \neq 1}^n e_{1i} - v_1.$$

Furthermore, notice that

$$u_2(\{1, 2\}, \lambda_2^{\{2,3\}}) \geq v_2 \Leftrightarrow b_2 + e_{23} - \lambda_2^{\{2,3\}} B \geq v_2$$

or

$$\lambda_2^{\{2,3\}} B \leq b_2 + e_{23} - v_2.$$

This has to hold, otherwise region 2's utility is lower than its exit option and exiting the federation is promising. In what follows we use the first and second inequality from Table 2 above in solving for $\lambda_2^{\{2,3\}}$:

$$\begin{array}{l}
b_1 + e_{12} - \lambda_1^{\{1,2\}} A > \sum_{i \neq 1}^n e_{1i} - \lambda_1^{\{2,3\}} B \\
b_2 + e_{21} - \lambda_2^{\{1,2\}} A > b_2 + e_{23} - \lambda_2^{\{2,3\}} B
\end{array} \Leftrightarrow$$

expand $\sum_{i \neq 1}^n e_{1i}$

$$\begin{array}{l}
b_1 + e_{12} - \lambda_1^{\{1,2\}} A > e_{12} + e_{13} - \lambda_1^{\{2,3\}} B \\
e_{21} - \lambda_2^{\{1,2\}} A > e_{23} - \lambda_2^{\{2,3\}} B
\end{array} \Leftrightarrow$$

$$\begin{aligned} b_1 - \lambda_1^{\{1,2\}} A &> e_{13} - \lambda_1^{\{2,3\}} B \\ e_{21} - \lambda_2^{\{1,2\}} A &> e_{23} - \lambda_2^{\{2,3\}} B \end{aligned} \Leftrightarrow$$

use $\sum_{i=1}^n \lambda_i = 1$ on $\lambda_1^{\{1,2\}}$ and $\lambda_1^{\{2,3\}} B = \sum_{i \neq 1}^n e_{1i} - v_1$

$$\begin{aligned} b_1 - A + \lambda_2^{\{1,2\}} A + \lambda_3^{\{1,2\}} A &> e_{13} - \sum_{i \neq 1}^n e_{1i} + v_1 \\ e_{21} - \lambda_2^{\{1,2\}} A &> e_{23} - \lambda_2^{\{2,3\}} B \end{aligned}$$

replace $\lambda_3^{\{1,2\}} A = \sum_{i \neq 3}^n e_{3i} - v_3$

$$\begin{aligned} b_1 - A + \lambda_2^{\{1,2\}} A + \sum_{i \neq 3}^n e_{3i} - v_3 &> e_{13} - \sum_{i \neq 1}^n e_{1i} + v_1 \\ e_{21} - \lambda_2^{\{1,2\}} A &> e_{23} - \lambda_2^{\{2,3\}} B \end{aligned} \Leftrightarrow$$

$$\begin{aligned} \lambda_2^{\{1,2\}} A + &> e_{13} - \sum_{i \neq 1}^n e_{1i} + v_1 - b_1 + A - \sum_{i \neq 3}^n e_{3i} + v_3 \\ e_{21} - e_{23} + \lambda_2^{\{2,3\}} B &> \lambda_2^{\{1,2\}} A \end{aligned} \Leftrightarrow$$

$$e_{21} - e_{23} + \lambda_2^{\{2,3\}} B > \lambda_2^{\{1,2\}} A > e_{13} - \sum_{i \neq 1}^n e_{1i} + v_1 - b_1 + A - \sum_{i \neq 3}^n e_{3i} + v_3.$$

For such a $\lambda_2^{\{1,2\}}$ to exist:

$$e_{21} - e_{23} + \lambda_2^{\{2,3\}} B > e_{13} - \sum_{i \neq 1}^n e_{1i} + v_1 - b_1 + A - \sum_{i \neq 3}^n e_{3i} + v_3$$

or

$$\lambda_2^{\{2,3\}} B > e_{13} - e_{12} - e_{13} + v_1 - b_1 + A - e_{31} - e_{32} + v_3 - e_{21} + e_{23} \Leftrightarrow$$

$$\lambda_2^{\{2,3\}} B > -e_{12} + v_1 - b_1 + A - e_{31} - e_{32} + v_3 - e_{21} + e_{23}.$$

Also, we recall $\lambda_2^{\{2,3\}} B \leq b_2 + e_{23} - v_2$. Hence, for such a $\lambda_2^{\{2,3\}}$ to exist

$$b_2 + e_{23} - v_2 > -e_{12} + v_1 - b_1 + A - e_{31} - e_{32} + v_3 - e_{21} + e_{23} \Leftrightarrow$$

or

$$b_2 - v_2 > -e_{12} + v_1 - b_1 + A - e_{31} - e_{32} + v_3 - e_{21} \Leftrightarrow$$

rewrite

$$b_1 + e_{21} + e_{31} - v_1 + b_2 + e_{12} + e_{32} - v_2 - A - v_3 > 0.$$

We replace $A = c_1 + c_2$

$$b_1 + e_{21} + e_{31} - v_1 + b_2 + e_{12} + e_{32} - v_2 - c_1 - c_2 - v_3 > 0,$$

and rearrange as to reveal $NB_i = b_i + \sum_{j \neq i} e_{ji} - c_i$ for 2 and 3 :

$$b_1 + e_{21} + e_{31} - v_1 - c_1 + b_2 + e_{12} + e_{32} - v_2 - c_2 - v_3 > 0 \Leftrightarrow$$

$$NB_1 - v_1 + NB_2 - v_2 - v_3 > 0 \Leftrightarrow$$

$$\sum_{i \in P'} NB_i + \sum_{i \in F} v_i > 0.$$

■

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