Long-run PPP under the presence of near-to-unit roots:  
The case of the British Pound-US Dollar rate

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Abstract

Empirical tests typically provide evidence that the British Pound-US Dollar exchange rate and the relative Wholesale Price Index contain exact unit roots and exhibit cointegration. However, the cointegrating vector is significantly different from [1,-1], thus raising doubts on the validity of the Purchasing Power Parity (PPP) hypothesis. Following Elliott (1998), we show that if the exchange rate and relative price series contain near-to-unit roots in the context of a bivariate system, then any inference on the 'cointegrating' vector and consequently on PPP, which is based on standard cointegration estimation methods, will be misleading. We then argue that the existing evidence against the PPP hypothesis in the British Pound-US Dollar market can be attributed to the finite sample bias of the standard cointegration estimators, arising from an endogenous and 'nearly' nonstationary regressor. We also show that when robust procedures are employed the evidence favors the PPP hypothesis.

**JEL classification:** F31, C22, C32.  
**Keywords:** Purchasing Power Parity, near-to-unit roots, cointegration.

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1. Introduction

Historically, Purchasing Power Parity (PPP) provides the simplest explanation of long-run exchange rate determination, according to which the equilibrium exchange rate between the domestic and foreign currency equals the ratio between domestic and foreign prices. Numerous studies have attempted to test this hypothesis: the early period at which “the results have confirmed the theory with an unexpected accuracy” (Cassel, 1916, p. 62) was succeeded by a ‘time-dependent’ degree of skepticism. Competing hypotheses such as “PPP is always and continuously true” (Frenkel, 1978), “PPP holds in the long run but not in the short run” (Terborgh, 1926, Dornbush, 1978), “PPP does not hold even in the long run” (Balassa, 1964), “PPP works better during inflation than under monetary stability” (Frenkel, 1981), “PPP is valid when interpreted more broadly than is usual” (Yeager, 1958), were all found to be supported by the relevant empirical evidence. Naturally, the rate at which empirical work was accumulating has been proportional to the rate at which statistical techniques were developing. The last thirty years have witnessed an explosion of the empirical literature on PPP, which now exhibits a great variety in terms not only of statistical methods, datasets, sample frequencies, but, unfortunately, in terms of results as well.

Regarding the employment of ‘modern’ statistical methods, the first period of testing for PPP starts in the beginning of the seventies. The theory was tested within the ‘stationary’ framework, where the OLS estimator along with its various extensions was the obvious choice. The arsenal of statistical tools expanded rapidly after the end of eighties, when the concept of cointegrated regressions was put forward. In the context of the new statistical paradigm and the often-cited non-stationarity of exchange rates and prices, the PPP hypothesis became equivalent to two conditions. First, the exchange rate and the relative price index should be cointegrated. Second, the cointegration parameter should equal one. The first is a necessary condition, without which the second one becomes irrelevant. In the context of cointegration the second condition is important as well, because if the cointegrating vector between the nominal exchange rate and the relative price is not [1,-1], then the real exchange rate is not identical to the cointegration error. This is not as innocuous as it might look at first sight. If the nominal exchange rate and the relative price index are cointegrated with the unique cointegrating vector being [1, -0], then the vector [1,-1] is not a cointegrating one and the real exchange rate is not stationary. In other words, the validity of the
PPP theory, under the assumption that the nominal exchange rate and relative price contain unit roots, critically depends on whether $\theta = 1$.

Regarding the evidence on these conditions, it is by now accepted that the non-stationarity of the series at hand and the existence of cointegration seem to hold, provided that long data spans are employed. The longest available sample for exchange rates and relative prices, which is also employed in the present study, comes from the UK-US pair of countries and covers more than two centuries of data. In general, empirical studies on the existence of PPP between UK-US find that the British Pound-US Dollar (BP-USD) exchange rate and the Wholesale Price Index (WPI) series are non-stationary and cointegrated. However, as regards the second condition, the direct estimates from this market on the cointegration parameter do not seem to be in the vicinity of unity. On the contrary, all studies seem to agree that the point estimate of the corresponding parameter on the relative price index for the UK-US market is less than unity. Interestingly, this piece of evidence appears robust to the choice of the cointegration estimator and the data span utilized.

Apparently, the bulk of the literature agrees that the commonly employed tests fail to reject the null hypothesis of unit roots in both the exchange rate and relative prices and proceeds in testing for PPP with cointegration analysis. This approach, however, runs the risk of producing misleading inferences on $\theta$ if the exchange rate and relative price series contain near-to-unit roots. This risk comes from a combination of two factors. First the ‘near-to-unit root’ case is almost observationally equivalent to the ‘exact unit root’ case since the existing unit root tests have very low power for near-to-unit root alternatives. Second, under near-to-unit roots all the standard cointegration estimators suffer from large biases in finite samples, which in turn produce severe size distortions. Elliot (1998) who was the first to study the robustness of the usual cointegration estimators in a near-to-unit root environment, reports empirical sizes exceeding 50% when slowly mean-reverting processes are approximated by exact unit roots.

The preceding discussion gives rise to the following question concerning the value of the slope

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1For a discussion of this approach within the cointegration framework and the implications for the real exchange rate see MacDonald (1993) and the survey of Sarno and Taylor (2002).

2This dataset has also been employed by Lothian and Taylor (1996, 2000) and Cuddington and Liang (2000) to test the time series properties of the real exchange rate.

coefficient, $\theta$, in a regression of exchange rates on relative prices. Is it possible that the accumulated evidence on $\theta < 1$ for the UK-US pair arises from the application of cointegration estimators on time series that contain near-to-unit roots? If yes, then it is possible that the rejection of the PPP hypothesis is attributable merely to the bias, $\bar{\theta}$, produced by the employed estimators, provided that $\bar{\theta}$ is negative and such that $\bar{\theta} - 1 = \tilde{\theta}$. We argue that the evidence is consistent with this scenario: PPP holds remarkably well under the assumption that nominal exchange rates and relative prices contain roots equal to 0.9, a value which is hardly detectable by standard unit root tests. The results from the estimation and the hypothesis testing on $\theta$, combined with Monte Carlo results on the direction and magnitude of the bias, confirm that in such a case the cointegration parameter equals one, thus favoring the PPP hypothesis as opposed to the case of exact unit roots.

In turn, under the near-to-unit roots scenario the estimates of $\theta$ obtained by a method that is robust to the presence of near-to-unit roots should produce more favorable evidence to the PPP hypothesis compared to those produced by the standard cointegration estimators. We therefore employ the Optimal Median Unbiased (OMUB) estimator recently proposed by Eliasz (2005), which is robust to the presence of near-to-unit roots in the regressors, and indeed we obtain estimates of $\theta$ that are insignificantly different from unity.\footnote{We thank a referee for pointing out to us the estimator proposed by Eliasz (2005).} In particular, the 95% confidence interval for the cointegration parameter, obtained by OMUB as well as those produced by an alternative robust procedure proposed by Wright (2000), do include unity, as opposed to the intervals produced by the standard cointegration estimators.

We close the introduction by noting that our findings support the validity of long-run PPP in the BP/USD market advocated by Lothian and Taylor (1997), although our approach has a different starting point. In particular, Lothian and Taylor (1997) have shown that the power of unit root tests in the BP/USD real exchange rate is very high when 200 annual sample points are considered. This reinforces their previous findings (Lothian and Taylor, 1996) that the real exchange rate is a stationary but slowly mean reverting process, which supports the long-run PPP hypothesis for the BP/USD market.\footnote{Alternative unit root tests that utilize the concept of fractional integration have also provided evidence against the unit root null; see, for example, Diebold et al. (1991) and Cheung and Lai (1993).} Our approach adopts a bivariate interdependent setup that allows us to estimate (rather than impose) the coefficient on the price differential by taking into
account the potential endogeneity between the exchange rate and the relative price index.

The rest of the paper is organized as follows: Section 2 summarizes the impact of near-to-unit roots on the consistency of ordinary cointegration estimators, commonly employed for testing the PPP hypothesis. Section 3 presents the estimation results on the PPP parameter and assesses their accuracy by means of Monte Carlo simulations. It also contains estimates of the PPP coefficient along with confidence intervals for this parameter obtained by means of robust procedures. Finally, section 4 concludes the paper.

2. Estimator’s choice under exact versus near-to-unit roots

Let \( \zeta_t \) and \( u_t \) be two bivariate processes, with \( \zeta_t = [y_t, x_t]^\top \) and \( u_t = [u_{1t}, u_{2t}]^\top \). We assume that \( u_t \) is an \( I(0) \) process and the generating mechanism for \( y_t \) is given by the system:

\[
y_t = \theta x_t + u_{1t} \tag{1}
\]

\[
x_t = \rho x_{t-1} + u_{2t} \tag{2}
\]

If \( \rho = 1 \) equations (1) - (2) yield the triangular representation of cointegration, put forward by Phillips (1988). On the other hand, if \( |\rho| < 1 \) then the system is stationary. We are particularly interested in the case where \( \rho \) is less than but very close to unity.

Next, we assume that \( u_t \) follows a bivariate VAR(1) process,

\[
\begin{pmatrix}
u_{1t} \\
u_{2t}
\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12} \\a_{21} & a_{22}\end{pmatrix} \begin{pmatrix}u_{1t-1} \\
u_{2t-1}
\end{pmatrix} + \begin{pmatrix}e_{1t} \\
e_{2t}
\end{pmatrix} \tag{3}
\]

where \( \begin{pmatrix}e_{1t} \\
e_{2t}
\end{pmatrix} \sim \text{IN}\left[0, \begin{pmatrix}\sigma_{11} & \sigma_{12} \\\sigma_{12} & \sigma_{22}\end{pmatrix}\right] \). Both eigenvalues of the matrix \( A = [a_{ij}] \), \( i, j = 1, 2 \) are assumed to be less than one in modulus and \( \Sigma = [\sigma_{ij}] \), \( i, j = 1, 2 \) is positive definite.

When \( \rho = 1 \), the Data Generation Process (DGP), defined by (1) - (3), is a cointegrated process. In such a case, the hypothesis of interest \( \theta = 1 \) may be tested by means of standard cointegration procedures such as the Fully Modified Least Squares (FMLS) procedure of Phillips and Hansen (1990), or in the context of parametric Autoregressive Distributed Lag (ADL) regressions. The
FMLS procedure is asymptotically optimal regardless of the values of the parameters $a_{12}$, $a_{21}$ and $\sigma_{12}$ that determine the temporal and contemporaneous correlation of the errors. On the other hand, ADL is optimal only when $a_{21} = 0$. If there are feedbacks from the regression error to the error that drives the regressor, i.e. when $a_{21} \neq 0$, the ADL models have to be augmented by leads of the regressor in order to achieve asymptotic optimality (see Phillips and Loretan, 1991, Saikonnen, 1991, Stock and Watson, 1993). We refer to the resulting estimator (test-statistic) as the Dynamic OLS (DOLS) estimator (test-statistic). When $\rho < 1$, the process is covariance stationary. In this case, the OLS-based t-statistic, corrected for possible autocorrelation in $u_{1t}$, may be used for testing the hypothesis $\theta = 1$ only when $a_{12} = \sigma_{12} = 0$. If any of these parameters is different from zero, the "exogeneity" status of $x_t$ is lost, OLS becomes inconsistent and instrumental-variables procedures should be employed instead.

If $\rho$ is close to but less than unity ($\rho \approx 1$), then the standard cointegration methods fail to produce reliable inferences on $\theta$. Elliot (1998) has demonstrated that all the commonly employed cointegration estimators produce large biases and suffer from severe size distortions, when slowly mean reverting processes are approximated by ones with unit roots (see also the Monte Carlo results in the next section). These results have important consequences for the estimation of $\theta$. As is well known, for sample sizes usually employed in empirical applications, the power of the usual unit root tests is less than 30% in rejecting a false unit root null for alternatives that lie in the interval $[0.90,1)$ (see Phillips and Xiao, 1998). Therefore, if unit root pre-testing has wrongly led to the conclusion that the series at hand are $I(1)$ and cointegrated, then inference on $\theta$ based on the usual single-equation cointegration methods, such as the DOLS or the FMLS estimators, as well as system-based estimators, such as the Johansen (1992) estimator (JOH), is likely to be misleading. The preceding discussion suggests that the case $\rho \approx 1$ requires special treatment. To this end, robust procedures that work quite well when the observed series have near-to-unit roots have recently been proposed in the literature and are analyzed below.

3. Empirical results and Monte Carlo simulations

In this section, the implications described above shall be investigated in the context of PPP. In particular, the extent of inference distortions in testing for PPP by means of standard cointegration
techniques when the series follow near-to-unit root processes shall be examined for the UK-US pair of countries. Our data set consists of annual data on the BP-USD exchange rate and the WPI of UK and US covering the period 1790-2006.\(^6\)

### 3.1. Implications of near-to-unit roots for PPP tests in the BP-USD market

We start the analysis by testing for the presence of a unit root in each of the series under consideration. As expected, the standard Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are unable to reject the unit-root null for both the nominal exchange rate and the relative price series. However, when the ADF or PP tests are applied on the residuals from the static regression of \(y_t\) on \(x_t\), the unit-root null is easily rejected, even at the 1\% level. The stationarity of residuals is consistent with two alternative scenarios on the time series properties of \(y_t\) and \(x_t\):

First, both series are \(I(1)\) and cointegrated; second, both series are stationary (or asymptotically stationary) with roots so close to unity that the usual tests fail to reject the false null.

According to the first scenario, one can attempt to estimate the parameter of interest by various cointegration estimators. The latter include \(OLS, JOH, ADL, DOLS\), as well as various versions of \(FMLS\) estimators that involve consistent estimators of \(\Omega\) and \(\Delta\). While any consistent estimates of \(\Omega\) and \(\Delta\) will produce the same asymptotics, Phillips and Hansen (1990) consider a class of kernel estimators that necessarily generate positive semi-definite estimators of \(\Omega\) in finite samples (see Andrews 1991). This set includes the kernels, employed in the present study, namely the Bartlett, Parzen and Quadratic Spectral (QS) ones. Moreover, the implementation of the \(FMLS\) estimation method requires selection of the optimal bandwidth parameter \(S_T^+\), which determines the number of autocovariances used in the estimation of the covariance matrices. We employ two methods of estimating the optimal bandwidth. The first (\(FMLS-A\)), suggested by Andrews (1991), is parametric in nature since it involves specification of simple time series models for \(u_{1t}\) and \(u_{2t}\). The second (\(FMLS-NW\)), suggested by Newey and West (1994), is non-parametric since it does not make any specific assumptions on the structure of \(u_t\), but instead utilizes truncated sums of the sample autocovariances to obtain estimates of \(S_T^+\). In the present study we utilize the QS kernel, and both the Andrews (1991) and Newey and West (1994) procedures for the estimation of the

\(^6\)The data are obtained from Global Financial Data, Annual Worksheets. See the data appendix in Lothian and Taylor (1996) for a detailed description of the dataset.
optimal bandwidth.

Table 1 contains the estimation results and the results of the relevant tests under the PPP hypothesis of \( \theta = 1 \) along with the estimated confidence intervals produced by each of these estimators, and computed as the point estimates \( \pm 1.96 \) standard errors. The point estimates on \( \theta \) provided by the OLS, JOH, ADL, DOLS, FMLS-A and FMLS-NW estimators are very close to each other and lie in the neighborhood of 0.65, which is a typical point estimate of the PPP coefficient for the UK-US pair of countries reported in cointegration studies. The associated t-statistics for OLS, FMLS-A, FMLS-NW and JOH massively reject the null hypothesis of \( \theta = 1 \), thus pointing towards unambiguous rejection of PPP. Moreover, none of the confidence intervals for \( \theta \) produced by the above-mentioned estimators contains unity; for example, the 95% confidence intervals of DOLS and FMLS-A are [0.40, 0.81] and [0.50, 0.91], respectively. These results can be considered as \textit{prima facie} evidence against the joint hypothesis of PPP and \( \rho = 1 \). However, if the maintained hypothesis \( \rho = 1 \) is false, then rejection of the joint hypothesis may be solely attributed to \( \rho \) being less than (but near to) unity.

To investigate this possibility we first examine the finite-sample performance of the cointegration estimators under study for various values of the autoregressive parameter \( \rho \). This set of experiments aims at investigating whether the presence of a near-to-unit root produces large negative biases, proportional to \( (1 - \rho) \), which shift the distribution of the t-statistics to the left, thus causing large deviations between the empirical and asymptotic critical values for testing \( \theta = 1 \).

We conduct various Monte Carlo experiments with data generated by equations (1) to (3). The model parameters are calibrated from the exchange rate and relative price data. In particular, we assume that \( \theta = 1, a_{11} = 0.62, a_{12} = 0.17, a_{21} = -0.098, a_{22} = 0 \). The elements of the covariance matrix \( \Sigma \) are estimated to be \( \sigma_{11} = 0.00065, \sigma_{12} = -0.00034 \) and \( \sigma_{22} = 0.0004 \). Concerning the key parameter \( \rho \), we initially consider the near-to-unit root scenario by setting \( \rho = 0.9 \) and then compare it with the exact unit root case, \( \rho = 1 \).

The aforementioned estimators are then assessed on the basis of the following characteristics:

(a) The average bias \( \hat{\theta} - \theta \).

(b) The 2.5% and 9.75% points of the empirical distributions of the corresponding t-statistics\(^7\)

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\(^7\)For the case of the JOH estimator, the 5% point of the empirical distribution of the likelihood ratio statistic is
(c-i) The percent rejections at the asymptotic 5% level.

(c-ii) The coverage rates of confidence intervals for $\theta = 1$.

The results, reported in Table 2, can be summarized as follows:

(i) When $\rho = 0.9$, the bias, $\tilde{\theta}$, for all the ordinary cointegration estimators, namely $OLS$, $JOH$, $ADL$, $DOLS$, $FMLS-A$, and $FMLS-NW$, is negative. As a consequence, the associated t-statistic distributions are shifted to the left, thus producing critical values that are vastly different than those predicted by asymptotic theory. The resulting size distortions are severe with the probability of rejecting the true hypothesis $\theta = 1$ being equal to 100%, 81.4% and 92.4% for the $OLS$, $JOH$ and $FMLS-A$ respectively.

(ii) The confidence intervals produced by the aforementioned estimators have effective coverage rates far from 95%, thus implying that the proportion of confidence intervals covering the value $\theta = 1$ is far less than 95% for all the estimation procedures under consideration. These results explain the empirical results of Table 1, according to which none of the standard cointegration methods produces a confidence interval that contains unity.

(iii) Contrary to the near-to-unit root case, all the cointegration procedures perform quite well when the regressor contains an exact unit root.

To further examine the effects of near-to-unit roots, we focus on the $ADL$ estimator and compute the median length of the confidence intervals, the median lower and upper end points of the confidence intervals, and the coverage rates of the $ADL$ procedure, for values of $\rho$ that belong to the interval $[1, 0.8]$. The results, presented in Table 3, show that for $\rho = 1$, the median length of the $ADL$ confidence interval is relatively small and the coverage rate is very close to its nominal rate of 95%. When $\rho$ decreases, the median length increases and the coverage rate decreases at a fast rate. Moreover, both the median lower and upper end points decrease monotonically with $\rho$. For example, for $\rho = 0.9$ the median length and coverage rate are 0.219 and 30.2%, respectively, whereas the median lower and upper end points are 0.745 and 0.971 respectively. Our results imply that for $\rho = 0.9$ we face a 70% probability to end up with a confidence interval that does not contain the true value $\theta = 1$. Moreover, this interval is likely to be a rather uninformative one, since its length

\footnote{8Similar results are obtained for the rest of the cointegration estimators under study.}
is expected to be rather long. These results suggest that a slight deviation of \( \rho \) from unity case is enough to increase dramatically the probability that the constructed confidence interval does not contain the true value \( \theta = 1 \).

### 3.2. Robust Procedures

The analysis so far suggests that the PPP hypothesis can be reconciled with the empirical evidence provided that the nominal exchange rate and relative price series do not possess exact unit roots. Under this scenario, we expect that the employment of a procedure that is robust to the presence of near-to-unit roots will provide evidence consistent with the PPP hypothesis. Such procedures were proposed in the literature soon after the severe problems in asymptotic inference, caused by nearly nonstationary regressors, became known. Stock and Watson (1996), proposed a test for the hypothesis \( \theta = \theta_0 \) with some demonstrable optimality properties. Wright (2000) developed an intuitively appealing procedure that jointly tests a hypothesis concerning the cointegration parameter \( \theta \) and the null hypothesis of cointegration. This procedure (WJ) defines confidence intervals of \( \theta \) as the set of values of \( \theta \) for which the null hypothesis of cointegration is not rejected. The general principle underlying this procedure is that if the hypothesized cointegrating coefficient \( \theta = \theta_0 \) is false then the linear combination \( y_t - \theta_0 x_t \) will be local to \( I(1) \) and the test will reject the null asymptotically. However, Jansson and Moreira (2006) argued that both of the above-mentioned procedures are asymptotically biased and put forward alternative testing procedures which enjoy asymptotic optimality. More specifically, in the context of the predictive regression model (where \( x_t \) is replaced by \( x_{t-1} \) in (1)), and building on the work of Stock and Watson (1996), these authors proved the existence of a uniformly most powerful test among a subset of the class of tests considered by Stock and Watson (1996). Eliasz (2005) combined the optimal testing procedures of Jansson and Moreira (2006) with the theory of optimal median unbiased estimation in the presence of nuisance parameters of Pfanzagl (1979) and obtained an optimal estimator of \( \theta \) (\( \hat{\theta}_{OMUB} \)) along with optimal confidence intervals.

For the case under study, the \( \hat{\theta}_{OMUB} = 0.796 \) whereas the 95% confidence interval is \([0.744, 1.023]\). The corresponding confidence interval, computed by the WJ method is \([0.582, 1.24]\). The estimated intervals contain unity, although their lengths are rather long. In order to shed more
light on whether this evidence is supportive for the PPP hypothesis, we conduct additional Monte
Carlo experiments aiming at investigating the performance of the OMUB and WJ procedures under
exact and near-to-unit roots. Table 4 presents the coverage rates of the OMUB and WJ confidence
intervals for alternative values of $\rho$ in the interval $[1, 0.8]$. The results from these experiments may
be summarized as follows: The effective coverage rates of the OMUB and WJ confidence intervals
are much closer to their nominal value of 95% than the cointegration estimators, for all the values
of $\rho$ under consideration. For the exact unit root case, $\rho = 1$, the coverage rate of OMUB is
93.6%, whereas that of WJ is less accurate being equal to 78.2%. For the stationary cases, $\rho < 1$,
both procedures exhibit some deviations between the nominal and effective coverage rates. These
deviations, however, are very small compared to those produced by the standard cointegration
procedures. For example, for $\rho = 0.9$, the effective coverage rate of OMUB is 84.4% whereas for
the same value of $\rho$, the ADL coverage rate hardly exceeds 30%.

To sum up, the Monte Carlo analysis favors the PPP hypothesis $\theta = 1$ in conjunction with the
hypothesis that the relative price series contain a root approximately equal to 0.9. The estimates
of $\theta$ via standard cointegration methods are around 0.68, whereas the corresponding Monte Carlo
average bias is around -0.20. Moreover, the length, lower and upper end points of confidence
intervals produced by the standard cointegration estimators are very close to the corresponding
numbers obtained from the Monte Carlo simulations for $\rho = 0.9$. In contrast, when $\theta$ is estimated
by a robust method, such as OMUB, the point estimate gets closer to one, whereas the confidence
intervals produced by the robust methods do contain unity.

4. Conclusions

This paper has provided some new evidence on the validity of the PPP hypothesis. It has shown
how inferences on PPP might be totally misleading when there is uncertainty about the stationarity
properties of the nominal exchange rate and the relative price index. In particular, when these series
have a root that is close -but not equal- to unity and there exists long-run simultaneity between the
regression error and the error that drives the relative price, the ordinary cointegration estimators
exhibit large biases and tend to over-reject the true null hypothesis. The extent of this over-rejection
depends on the departure of the largest root in the relative price from unity and on the degree of
long-run simultaneity. Moreover, the confidence intervals produced by the standard cointegration estimators have coverage rates far below the corresponding nominal ones.

Our simulation results have shown that PPP for the UK-US pair of countries can be reconciled with the empirical evidence provided that the value of the autoregressive root, $\rho$, of the relative price series is around 0.9 equals and the long-run simultaneity is negative. The bias of the ordinary cointegration estimators is then found to be equal to the difference between the corresponding point estimates and unity. Moreover, the Monte Carlo confidence intervals produced by the cointegration estimators for $\rho = 0.9$ resemble very closely the confidence intervals constructed from the real data. Further evidence in favor of the PPP hypothesis is offered by two robust to near-to-unit roots procedures, under which the point estimates of the PPP parameter $\theta$ are closer to unity and the estimated confidence intervals do contain unity. The overall evidence suggests that the scenario $(\theta = 1, \rho < 1)$ appears to be more strongly supported than the scenario $(\theta < 1, \rho = 1)$.

We close the paper by noting that the near non-stationarity of financial variables has been put forward as an explanation of other empirical puzzles and open issues in asset markets. For instance, Roll and Yan (2000) have shown that the famous ‘forward premium puzzle’ can be explained if the nominal and forward exchange rates are allowed to follow a nearly non-stationary process. Interestingly, a plausible estimate of the autoregressive coefficient is found to be in the range between 0.8 and 0.9, which is remarkably close to the value needed to rehabilitate PPP for the UK-US pair of countries within the current context. In conjunction with the theoretical basis for the presence of a near random walk in the exchange rate put forward by Engel and West (2005), it appears that our evidence on the validity of long-run PPP in the BP-USD market under the presence of near-to-unit roots offers another potential route for supporting long-run PPP for the UK-US pair of countries. Hence, the view of Taylor and Taylor (2004) that confidence to long-run PPP emerges again after three decades of overall skepticism seems to be further reinforced.

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References


Table 1. Estimation results and tests for the PPP hypothesis ($\theta=1$) using the asymptotic critical values

<table>
<thead>
<tr>
<th>Cointegration estimator</th>
<th>Estimate</th>
<th>Standard error</th>
<th>$t$-statistic</th>
<th>95% confidence intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>0.696</td>
<td>0.021</td>
<td>-33.14</td>
<td>[0.65, 0.74]</td>
</tr>
<tr>
<td>JOH</td>
<td>0.601</td>
<td>0.089</td>
<td>-6.75</td>
<td>[0.43, 0.77]</td>
</tr>
<tr>
<td>ADL</td>
<td>0.602</td>
<td>0.129</td>
<td>-4.66</td>
<td>[0.35, 0.85]</td>
</tr>
<tr>
<td>DOLS</td>
<td>0.605</td>
<td>0.107</td>
<td>-5.65</td>
<td>[0.40, 0.81]</td>
</tr>
<tr>
<td>FMLS-A</td>
<td>0.702</td>
<td>0.106</td>
<td>-6.62</td>
<td>[0.50, 0.91]</td>
</tr>
<tr>
<td>FMLS-NW</td>
<td>0.689</td>
<td>0.092</td>
<td>-7.48</td>
<td>[0.51, 0.87]</td>
</tr>
</tbody>
</table>

Notes:
1) See the text for the definition of the estimators.
2) The ‘Fully modified’ estimators utilize a Quadratic Spectral kernel. Similar results are obtained when Bartlett or Parzen kernels are employed.
3) The bandwidth parameter is selected via the parametric (FMLS-A) and non-parametric (FMLS-NW) procedures of Andrews (1991) and Newey and West (1994) respectively.
Table 2. Monte Carlo results

<table>
<thead>
<tr>
<th>Cointegration estimator</th>
<th>Bias</th>
<th>(t_{0.025})</th>
<th>(t_{0.975})</th>
<th>Size</th>
<th>Coverage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho = 0.9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>OLS</strong></td>
<td>-0.26</td>
<td>-9.74</td>
<td>-4.64</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td><strong>JOH</strong></td>
<td>-0.17</td>
<td>-5.51</td>
<td>-0.83</td>
<td>81.4</td>
<td>18.6</td>
</tr>
<tr>
<td><strong>ADL</strong></td>
<td>-0.14</td>
<td>-4.85</td>
<td>-0.59</td>
<td>69.8</td>
<td>30.2</td>
</tr>
<tr>
<td><strong>DOLS</strong></td>
<td>-0.16</td>
<td>-5.30</td>
<td>-0.83</td>
<td>78.8</td>
<td>21.2</td>
</tr>
<tr>
<td><strong>FMLS-A</strong></td>
<td>-0.10</td>
<td>-5.92</td>
<td>-1.43</td>
<td>92.4</td>
<td>7.6</td>
</tr>
<tr>
<td><strong>FMLS-NW</strong></td>
<td>-0.12</td>
<td>-5.61</td>
<td>-1.31</td>
<td>89.0</td>
<td>11.6</td>
</tr>
<tr>
<td>(\rho = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>OLS</strong></td>
<td>0.019</td>
<td>-2.84</td>
<td>0.95</td>
<td>15.05</td>
<td>84.9</td>
</tr>
<tr>
<td><strong>JOH</strong></td>
<td>0.006</td>
<td>-2.08</td>
<td>1.99</td>
<td>5.9</td>
<td>94.1</td>
</tr>
<tr>
<td><strong>ADL</strong></td>
<td>0.005</td>
<td>-1.98</td>
<td>1.92</td>
<td>5.1</td>
<td>94.9</td>
</tr>
<tr>
<td><strong>DOLS</strong></td>
<td>0.005</td>
<td>-1.97</td>
<td>1.94</td>
<td>5.0</td>
<td>95</td>
</tr>
<tr>
<td><strong>FMLS-A</strong></td>
<td>0.006</td>
<td>-2.05</td>
<td>1.96</td>
<td>5.1</td>
<td>94.9</td>
</tr>
<tr>
<td><strong>FMLS-NW</strong></td>
<td>0.006</td>
<td>-2.04</td>
<td>1.97</td>
<td>5.1</td>
<td>94.9</td>
</tr>
</tbody>
</table>

Notes:
1) The parameters of the DGP are calibrated from the BP-USD exchange rate and UK-US relative price data (210 observations). The results are based on 10,000 replications.
2) The term ‘size’ refers to percent rejections at the asymptotic 5% level.
3) The term "coverage rate" refers to the proportion of confidence intervals, computed as \(\widehat{\theta} \pm 1.96 \times (s.e.(\widehat{\theta}))\), that contain the value \(\theta = 1\).
### Table 3. Length, lower-end and upper-end points, and coverage rates of the ADL confidence intervals

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Median length</th>
<th>Median lower-end point</th>
<th>Median upper-end point</th>
<th>Coverage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>0.0757</td>
<td>0.9651</td>
<td>1.0464</td>
<td>97.0</td>
</tr>
<tr>
<td>$0.98$</td>
<td>0.1160</td>
<td>0.9135</td>
<td>1.0357</td>
<td>84.4</td>
</tr>
<tr>
<td>$0.96$</td>
<td>0.1450</td>
<td>0.8688</td>
<td>1.0207</td>
<td>66.6</td>
</tr>
<tr>
<td>$0.94$</td>
<td>0.1723</td>
<td>0.8265</td>
<td>1.0050</td>
<td>52.0</td>
</tr>
<tr>
<td>$0.92$</td>
<td>0.1973</td>
<td>0.7856</td>
<td>0.9883</td>
<td>38.6</td>
</tr>
<tr>
<td>$0.90$</td>
<td>0.2198</td>
<td>0.7458</td>
<td>0.9710</td>
<td>30.2</td>
</tr>
<tr>
<td>$0.88$</td>
<td>0.2420</td>
<td>0.7068</td>
<td>0.9530</td>
<td>23.2</td>
</tr>
<tr>
<td>$0.86$</td>
<td>0.2605</td>
<td>0.6685</td>
<td>0.9346</td>
<td>17.6</td>
</tr>
<tr>
<td>$0.84$</td>
<td>0.2800</td>
<td>0.6307</td>
<td>0.9158</td>
<td>12.6</td>
</tr>
<tr>
<td>$0.82$</td>
<td>0.2979</td>
<td>0.5934</td>
<td>0.8967</td>
<td>10.0</td>
</tr>
<tr>
<td>$0.80$</td>
<td>0.3154</td>
<td>0.5564</td>
<td>0.8773</td>
<td>7.8</td>
</tr>
</tbody>
</table>

**Note:**
See the text for the definition of the ADL estimator.
Table 4. Coverage rates of the robust procedures

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>OMUB</th>
<th>WJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>93.6</td>
<td>78.2</td>
</tr>
<tr>
<td>0.98</td>
<td>88.0</td>
<td>79.4</td>
</tr>
<tr>
<td>0.96</td>
<td>84.0</td>
<td>80.2</td>
</tr>
<tr>
<td>0.94</td>
<td>84.2</td>
<td>80.6</td>
</tr>
<tr>
<td>0.92</td>
<td>83.4</td>
<td>80.6</td>
</tr>
<tr>
<td>0.90</td>
<td>84.4</td>
<td>80.6</td>
</tr>
<tr>
<td>0.88</td>
<td>82.8</td>
<td>81.0</td>
</tr>
<tr>
<td>0.86</td>
<td>85.2</td>
<td>81.2</td>
</tr>
<tr>
<td>0.84</td>
<td>86.2</td>
<td>81.4</td>
</tr>
<tr>
<td>0.82</td>
<td>86.6</td>
<td>81.8</td>
</tr>
<tr>
<td>0.80</td>
<td>86.6</td>
<td>82.4</td>
</tr>
</tbody>
</table>

Note:
See the text for the definition of the OMUB and WJ estimators.