Capital Maintenance and Depreciation over the Business Cycle*

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Abstract

This paper develops and estimates a stochastic general equilibrium model with capital maintenance, which affects endogenously the depreciation rate of capital. The estimate of maintenance series is found to track survey-based measures for Canada quite closely and to generate the procyclical pattern of maintenance observed in the data. We use our model estimates to infer the time profile of equipment capital depreciation in Canadian and US manufacturing. The depreciation rate is estimated to be volatile and highly procyclical in both countries.


Keywords: real business cycle, endogenous capital depreciation, maintenance.

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*An online Appendix of the paper is available at www.eui.eu/Personal/Pappa/research.html
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1 Introduction

Casual empiricism suggests that expenditures on capital maintenance constitute an integral part of the capital accumulation process. Broadly, outlays on capital maintenance cover the “deliberate utilization of all resources that preserve the operative state of capital goods” (Bitros, 1976). As pointed out by Feldstein and Foot back in 1971, surveys on planned investment in the US for the period 1949-68 suggest that roughly one half of 'gross' investment aimed at maintaining the operative state of capital goods ('replacement and modernization') as opposed to 'new' investment ('expansion'). Capital maintenance is, thus, directly related to capital depreciation and many authors have studied the optimal maintenance level at the firm level with the depreciation rate modeled as an endogenous function of maintenance outlays.1

McGrattan and Schmitz (1999) were the first to provide a detailed picture on the size of aggregate capital maintenance using evidence from the Canadian survey on Capital and Repair Expenditures, which is the only source of aggregate long-run data on capital expenditures in newly purchased assets ('new' investment) and maintenance available at the national level. According to this survey, total (private and public) maintenance and repair expenditures in Canada amounted on average to around 6.3% of GDP for the period 1956-93. This number is roughly equal to one third of spending on 'new' investments and, when compared to other ‘engines of growth’, is somewhat lower than education spending (6.8% of GDP), but far above the average spending on R&D (1.4% of GDP) over the same period, suggesting that maintenance expenditures are ‘too big to ignore’.

This paper develops and estimates a Dynamic Stochastic General Equilibrium (DSGE) model, in which capital maintenance together with capital utilization affect endogenously the capital depreciation rate. Our model is found to perform well in replicating key features of the data and allows us to derive the time profile of endogenous capital depreciation in a general equilibrium setup. Several studies have attempted to estimate the depreciation rate, mainly in US manufacturing, using various single or multi-equation econometric

1See, among others, Schmalensee (1974), Nickell (1978), Schworm (1979) and Parks (1979) for early contributions in this literature. Also, some empirical studies at the sectoral level have confirmed that capital deterioration is affected by maintenance expenditures; see Nelson and Caputo (1997) and the references cited therein for a brief survey of the empirical findings.
approaches (see Epstein and Denny, 1980; Hulten and Wykoff, 1981a, 1981b; Nadiri and Prucha, 1996a, 1996b; Jorgenson, 1996; Oliner, 1996; Huang and Diewert, 2011). Most of these papers find that the depreciation rate is fairly stable and that a constant depreciation rate may be a valid approximation for calibration exercises. On the other hand, Tevlin and Whelan (2003) point out that the rapid depreciation of computing equipment that took place in the 1990s led to a rise of the estimated depreciation rate for aggregate equipment capital. This assessment is confirmed by Doms et al. (2004) and Geske et al. (2007). Our results complement the findings in those studies: the implied depreciation rate for equipment capital in Canadian and US manufacturing displayed substantial volatility and a highly procyclical pattern over the last 50 years.

What generates the difference in our estimate relative to previous ones is the behavior of capital maintenance. While investment spending can be typically obtained from fixed non-residential private investment on property, plant and equipment in national accounts, and capital outlays from panel data for two-digit or plant-level manufacturing firms (in the US Compustat Industrial database), capital maintenance is mainly undertaken by employees. Hence there are no recorded market transactions. Moreover, maintenance and repair services purchased by firms in the market are typically treated as transactions involving intermediate goods. Thus, although maintenance activities are included in measured real output, their magnitude cannot be recovered by standard sources, like national accounting systems. Given the scarcity of available estimates for maintenance, we use the ‘Capital and Repair Expenditures,’ survey, which covers the period 1956-2005, to obtain proxies for maintenance and ‘new’ investment of equipment capital in the Canadian manufacturing sector. According to this data, total expenditures in ‘new’ investment and maintenance was on average 16.7% of manufacturing output, with the average share of maintenance over total investment amounting to 36.1% and accounting for 6% of output and 4.9% of the capital stock. Turning to the cyclical properties of the data, we observe that maintenance expenditures are procyclical. Figures 1a and 1b plot spending on capital maintenance and the associated maintenance to capital ratio (henceforth, $MK$ ratio), and manufacturing output. Both measures of maintenance are strongly procyclical in agreement with the evidence reported by McGrattan and Schmitz (1999).²

²Descriptive statistics point towards a contemporaneous correlation between maintenance and $MK$
We set up an otherwise standard Real Business Cycle (RBC) model in which capital outlays comprise, apart from ‘new’ investment that adds directly to the capital stock, maintenance expenditures that affect the capital decay rate. We also employ a general specification for the depreciation function that embeds the effect of capital utilization on depreciation, as in Burnside and Eichenbaum (1996), and its interactions with capital maintenance. The structural parameters of the model are estimated with Bayesian techniques using aggregate Canadian manufacturing data for output, capacity utilization, total investment, consumption and hours worked as observables for the period 1956-2005. The model generates estimates for capital maintenance expenditures that mimic reasonably well the cyclical behavior of actual survey-based series for Canada. Given the success of the model for Canada we also obtain consistent estimates for capital maintenance in the US over the period 1958-2009, a period for which there has been no systematic data collection on this type of outlays. We then use these estimates to obtain the time profile of the depreciation rate of equipment capital in Canadian and US manufacturing over the business cycle.

To the best of our knowledge very few DSGE macroeconomic models have attempted to endogenize maintenance outlays. Early contributions to this literature can be found in Licandro and Puch (2000) and Collard and Kollintzas (2000). In both studies maintenance moves countercyclically, which contradicts the stylized facts presented in Figures 1a and 1b. Collard and Kollintzas (2000) consider two types of labor that can be used in production and maintenance, respectively. Since higher productivity causes labor in production activities to be more efficient, maintenance activities may fall after a total factor productivity (henceforth TFP) shock, but, at the same time, higher output efficiency ratio with output of 0.63 and 0.60, respectively. This correlation seems to be higher in the first part of the sample: for the period 1956-1983 the corresponding correlation coefficients amount to 0.85 and 0.86.

We note that the US Census Bureau has added in the Annual Survey of Manufacturers entries on Repair and Maintenance services of buildings and/or machinery for the years 2007, 2008, and 2009. The definition includes payments on purchased services for all maintenance and repair work on buildings and equipment. Payments made to other establishments of the same company and for repair and maintenance of any leased property also are included. Excluded are extensive repairs or reconstruction that was capitalized, which is considered capital expenditures, costs incurred directly by the establishment in using its own work force to perform repairs and maintenance work, and repairs and maintenance provided by the building or machinery owner as part of the rental contract. ‘New’ investments and maintenance account on average for 8.7% of total (equipment and structures) US manufacturing output, with maintenance amounting to 20.9% of total investment.
releases labor towards maintenance activities. In equilibrium the first effect dominates and maintenance is countercyclical driving depreciation rates up during booms. In our specification the impact of technology shocks on maintenance depends solely on its interaction with capital utilization. Licandro and Puch (2000) argue that maintenance should be countercyclical because it is cheaper for firms to repair and maintain machines in recessions. They formalize this argument by assuming that the cross derivative of the depreciation function with respect to maintenance and utilization is positive. Our estimates instead suggest that the sign of this derivative is negative.

Some papers have investigated issues that affect the trade offs discussed in this paper. For example, Whelan (2002) and Tevlin and Whelan (2003) study how investment and depreciation respond to technology shocks, a feature that is crucial to determine the magnitude of the depreciation rate, particularly in high-technology sectors like computing. Boucekkine and Ruiz-Tamarit (2003) and Saglam and Veliov (2008) have studied how depreciation reacts to different types of technology shocks. Boucekkine et al. (2009) distinguish between endogenous age-related depreciation, which depends on optimal capital utilization when new capital goods arrive, and endogenous capital scrapping. They find that the scrapping rate drops when neutral technical progress accelerates while age-related depreciation remains unaffected, whereas both age-related depreciation and scrapping rates increase with investment-specific technical progress. In our model, a TFP shock raises capital utilization, maintenance and depreciation, while an investment-specific shock reduces the price of investment, leading to higher capital utilization and depreciation. When maintenance is considered, the fall in the price of investment increases the relative price of maintenance and agents find it optimal to decrease maintenance expenditures, which further raises the depreciation rate. Boucekkine et al. (2010) examine the short-run responses of investment and maintenance and find that they move in the same direction following technology shocks, thus, suggesting that they act complementary to each other.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses the results from the Bayesian estimation and presents the model dynamics. Section 4 presents the estimates for the time profile of capital depreciation in Canada and
the US. Finally, section 5 concludes.

2 The model

2.1 Households

The economy is inhabited by infinitely lived agents who derive utility from consumption, $C_t$, and disutility from hours worked, $h_t$, at each period $t$. The present-value utility of the household is given by:

$$E \sum_{t=0}^{\infty} \beta^t \eta^u_t \left[ \frac{C_{t}^{1-\sigma}}{1-\sigma} - \lambda_n \eta^h_t \frac{h_{t}^{1+\theta_n}}{1+\theta_n} \right]$$

(1)

where $\sigma > 0$ is the risk aversion coefficient, $\theta_n > 0$ determines the supply elasticity of hours, and $\lambda_n > 0$ is a preference parameter. Parameter $\beta$ is a subjective discount factor with $0 < \beta < 1$ and E is the expectation operator. $\eta^u_t$ and $\eta^h_t$ represent a preference shock and a labor supply shock, respectively; both shocks are assumed to follow an AR(1) process with i.i.d. Normal error term: $\log(\eta^u_t/\eta^u_{t-1}) = \rho_u \log(\eta^u_{t-1}/\eta^u_{t-2}) + \epsilon^u_t$ and $\log(\eta^h_t/\eta^h_{t-1}) = \rho_h \log(\eta^h_{t-1}/\eta^h_{t-2}) + \epsilon^h_t$, where $\epsilon^u_t, \epsilon^h_t$, are Normal i.i.d. shocks. We introduce both a labor supply and a preference shock in our model because the literature has shown that both are important to reproduce the dynamics of business cycle fluctuations (see e.g. Smets and Wouters, 2007).

The representative household owns the capital stock and receives income from renting the effective capital stock (capital services), $U_t K_t$, to the firm at a rate $r_t$, where $U_t$ is the utilization rate of the capital stock $K_t$ and from working at a wage rate $w_t$. The household allocates her income stream between consumption $C_t$, ‘new’ investment $I_t$, and capital maintenance $M_t$:

$$C_t + I_t + M_t \leq w_t h_t + r_t U_t K_t$$

(2)

The rate at which capital depreciates depends positively on its utilization and negatively on maintenance expenditures. ‘New’ investment, $I_t$ is related to the capital stock
accumulation by:

\[ Z_t I_t = K_{t+1} - \left(1 - \delta \left( U_t, \frac{M_t}{K_t} \right) \right) K_t + v \left( \frac{K_{t+1}}{K_t} \right) K_t \]  (3)

where \( \delta(.) \) is the depreciation function and \( v(.) \) is a function of gross investment regulating capital adjustment costs. The variable \( Z_t \) denotes an investment-specific technology shock that represents technological advances either in the investment good (like technology advances) or in the process for producing it, thus affecting the real price of investment. Greenwood et al. (1997, 2000) have shown that technology shocks involving investment-specific rather than neutral technological change can be a major source of the business cycle. Fisher (2006) shows that the combined impact of neutral and investment-specific shocks is important in explaining fluctuations of output and labor in the US with investment-specific shocks mattering more than TFP shocks. As a result, including investment-specific shocks in the model is crucial for studying the dynamics of maintenance. We let \( \log(Z_t/Z) = \rho_z \log(Z_{t-1}/Z) + \epsilon^z_t \), where \( \epsilon^z_t \) is an i.i.d. Normal error.

We specify the capital adjustment costs function to be of the form:

\[ v \left( \frac{K_{t+1}}{K_t} \right) = \frac{b}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 \]  (4)

where \( b > 0 \) is a parameter measuring the degree of capital adjustment costs.

### 2.2 The depreciation function

As in McGrattan and Schmitz (1999), we assume that depreciation is a decreasing function of maintenance expenditure, so that as maintenance services per unit of the capital stock increase, the rate at which capital depreciates decreases. Following Greenwood et al. (1988) and Burnside and Eichenbaum (1996) we also allow depreciation to be an increasing function of capital utilization. Hence, the depreciation function is parameterized as:

\[ \delta \left( U_t, \frac{M_t}{K_t} \right) = \xi \left[ \psi U_t^\phi + (1 - \psi) e^{-\gamma \frac{M_t}{K_t}} \right]^\theta \]  (5)
where $\phi, \gamma \geq 0$. Given the trade-off between the production benefits and the depreciation costs of capital utilization, the agent will, in general, not find it optimal to fully utilize the capital stock. Under our assumptions there is also a trade-off in allocating resources between ‘new’ investment $I_t$ and capital maintenance $M_t$, which will be determined by their respective returns.

The parameters $\phi$ and $\gamma$ in equation (5) determine the effect of utilization and maintenance on the rate of depreciation of capital, respectively. When $\phi > 0$, $\frac{\partial \delta}{\partial U} > 0$, whereas when $\phi = 0$, capital utilization does not affect the rate at which capital depreciates. Similarly, if $\gamma > 0$, $\frac{\partial \delta}{\partial M} < 0$ and $\frac{\partial^2 \delta}{\partial M^2} > 0$, while $\gamma = 0$ implies that maintenance expenditures do not affect the capital depreciation rate. Moreover, when the capital stock is not utilized and maintenance expenditures are very high, there is no depreciation, i.e. $\delta(0, \infty) = 0$. Notice that specification (5) nests the one used by McGrattan and Schmitz (1999) for $\psi = 0$ and the one in Burnside and Eichenbaum (1996) for $\psi = 1$. When maintenance is assumed to be constant in the benchmark model, the depreciation function takes the form $\delta(U_t) = \delta U_t^{\phi}$, in line with Greenwood et al. (1988) and Burnside and Eichenbaum (1996).

As described in Boucekkine and Ruiz-Tamarit (2003) and Boucekkine et al. (2010), the sign of the cross derivative $\frac{\partial^2 \delta}{\partial M \partial U}$ is crucial to determine the degree of complementarity or substitutability between investment and maintenance. The sign of this derivative is determined by $\theta$: when $\theta > 1$ ($\theta < 1$) the cross derivative is negative (positive). In the steady state the value of $\theta$ depends on the values of parameters $\gamma$, $\phi$ and $\alpha$. In our exercise we let $\theta$ to take values larger or smaller than one.

### 2.3 Production side and market clearing

Firms use capital services and labor hours to produce a final good, $Y_t$, that can be used for consumption, investment and maintenance activities. The representative firm then chooses its factor inputs, hours worked, $h_t$, and capital services, $U_tK_t$, to produce a given level of $Y_t$ in order to minimize the production costs:

$$w_t h_t + r_t U_t K_t$$

(6)
subject to the technological constraint:

\[ Y_t = (U_t K_t)^{1-\alpha} (X_t h_t)^\alpha \]  \hspace{1cm} (7)

where \( X_t \) represents a neutral labor-augmenting technology TFP shock: \( \log(X_t/X) = \rho_x \log(X_{t-1}/X) + \epsilon_t^x \), where \( \epsilon_t^x \) is an i.i.d. Normal error.

In equilibrium the goods market clears and we have:

\[ Y_t = C_t + I_t + M_t + G_t \]  \hspace{1cm} (8)

where \( G_t \) is a public spending shock: \( \log(G_t/G) = \rho_g \log(G_{t-1}/G) + \epsilon_t^g \), where \( \epsilon_t^g \) is an i.i.d. Normal error.

### 2.4 Model solution

The representative agent chooses sequences of \( C_t, h_t, U_t, I_t, \) and \( M_t \), to maximize (1) subject to (2) and (3). The first-order conditions are given by:

\[ \eta^b_t \lambda^h_th^n_t = \alpha C_t^{-\gamma} Y_t \]  \hspace{1cm} (9)

\[ (1 - \alpha) \frac{Y_t}{U_t} = \xi \theta_\psi \left[ \psi U_t^\phi + (1 - \psi) e^{-\gamma \frac{M_t}{K_t}} \right]^{\theta-1} \frac{K_t}{Z_t} U_t^{\phi-1} \]  \hspace{1cm} (10)

\[ \xi \theta_\gamma (1 - \psi) \left[ \psi U_t^\phi + (1 - \psi) e^{-\gamma \frac{M_t}{K_t}} \right]^{\theta-1} e^{-\gamma \frac{M_t}{K_t}} = Z_t \]  \hspace{1cm} (11)
\[ \beta E_t \left\{ \eta^{u}_{t+1} C_{t+1}^{-\sigma} \left[ r_{t+1} U_{t+1} - \frac{M_{t+1}}{K_{t+1}} \right] + \frac{1 - \delta}{Z_{t+1}} \left( U_{t+1} \frac{M_{t+1}}{K_{t+1}} \right) + \frac{b}{2} \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \left( \frac{K_{t+2}}{K_{t+1}} - 1 \right) \right\} \]

Equation (9) gives the first-order condition for hours worked and equation (10) sets the marginal return of a rise in the capital utilization rate equal to its opportunity cost measured by the increased capital depreciation rate. Equation (11) is the optimality condition with respect to maintenance and sets the marginal benefit of maintenance equal to its cost. Finally, equation (12) modifies the usual optimality condition that equates the marginal productivity with the user cost of capital, since a marginal increase in the capital stock implies a rise in its required maintenance cost. Firms set the marginal products of effective capital and hours worked equal to the return of capital and the wage rate, respectively.

3 Estimation and dynamics

3.1 Data and priors

In order to investigate the dynamics of the model, we log-linearize the equilibrium conditions around the steady state.\(^4\) The log-linearized model is estimated with Bayesian techniques. We estimate the mode of the posterior distribution by maximizing the log posterior function, which combines the log of prior with the log likelihood of the data, using the numerical method by Sims (1999). The Metropolis-Hastings algorithm is then used to get an estimate of the joint posterior distribution of the parameter vector. A sample with 250000 draws is used (the first 20\% draws are discarded) and the scale for the jumping distribution is set to 0.4 (0.5 for the US).

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\(^4\)The steady-state conditions, the log-linear equations, and a full description of the data sources are presented in the online Appendix.
For Canada we use as observables output, capital utilization, total investment, consumption, and hours worked for the period 1956-2005. Maintenance expenditures are not included in the set of observable variables, but are used ex-post to evaluate the model’s performance. Due to data availability and in order to maintain consistency, all series refer to the manufacturing sector. Output, total investment and consumption are deflated with the Industrial Selling Price index and divided by total working population. Hours are adjusted for total working population. The discount factor is fixed at 0.98, implying a steady-state real interest rate of 2%. The steady-state depreciation rate, which corresponds to the steady-state ‘new’ investment to capital ratio, is set to 0.0882, as in Hwang (2002/3), and the steady-state $MK$ ratio is set at 0.0494. Both figures equal the corresponding averages of the series from the Canadian Survey on Capital and Repair Expenditures. The ratio of public spending to GDP is fixed to 17%. We also use steady-state relationships to determine the values of parameters $\theta$, $\xi$ and $\psi$, and estimate the values for $\phi$ and $\gamma$. Table 1 displays the parameters which are fixed in the estimation and their steady-state values.

Data on US manufacturing output, employment, hours worked, capital expenditures, and capital are obtained from the NBER-CES Manufacturing Industry Database, provided by Becker and Gray (2009), which covers the period 1958-2005. The series are extended to 2009 using the corresponding entries as reported in the U.S. Census Annual Survey of Manufacturers. A series on capital utilization in U.S. manufacturing is compiled using data from the Board of Governors of the Federal Reserve System. The steady-state depreciation rate, which equals the steady-state investment to capital ratio, is derived as the average of the series for manufacturing machinery and equipment over the sample and set to 0.117. This figure is in line with existing estimates of depreciation rates in US manufacturing. For example, Hulten and Wykoff (1981) estimate a depreciation rate of 13.3% for equipment; Nadiri and Prucha (1996a) an average of 5.9% between 1960-1988; Jorgenson (1996) and Fraumeni (1997) report average depreciation rates of 15% for durable equipment in US manufacturing; Kollintzas and Choi (1985) and Bischoff and Kokkelenberg (1987) report values of 12.5% of 10.6%, respectively; Epstein and Denny (1980) account for endogenous utilization and find an average depreciation rate of 12.6%
for the period 1947-1971. The steady-state $MK$ ratio is obtained by multiplying this value by the average maintenance to investment ratio for the available US data for total manufacturing, thus, obtaining 0.0309 for the machinery and equipment sector. The rest of the parameters are determined as with Canadian data.

All other structural and auxiliary parameters are estimated. Prior shapes, prior means and standard deviations are collected in Table 2. The priors are independent of the data used. We set most priors so that parameters vary in a range compatible with estimates obtained in the literature and describe most parameters with truncated Normal or Gamma distributions. We allow the intertemporal elasticity of substitution, $\sigma$, to vary in the (0.01,6) interval. The inverse of the Frisch elasticity of labor supply, $\theta_n$, varies in the (0.01,10) interval. The existence of endogenous maintenance might affect the low of motion for capital since it provides an additional investment smoothing mechanism relative to the standard model where depreciation is fixed. For that reason we pose a prior that reflects this idea assuming a priori that capital adjustment costs should be close to zero and let $b$ vary in the (0,10) interval. The interval for the share of labor, $\alpha$, in the production function is centered about the standard estimated value for this parameter and is represented by a Normal distribution. The parameters of the depreciation function $\gamma$ and $\phi$, which determine the elasticity of depreciation to changes in maintenance expenditures and capital utilization respectively, are described by Gamma distributions. Given the absence of calibrated values for parameter $\gamma$ we assume a relatively diffused prior and we set the prior mean of $\phi$ to 0.9 with a standard deviation of 0.2. Finally, the persistence parameters of the AR(1) processes are Beta distributed and the standard errors of the innovations are assumed to follow an Inverse-gamma distribution.

### 3.2 Posterior estimates

The left panel of Table 3 shows the results of the model for Canada. The first panel displays results for the standard RBC model and the second panel results for the model with endogenous maintenance. We report the posterior mean and 90% credible intervals.\(^5\) Regarding the shocks considered, all exhibit low persistence with the labor supply shock

\(^5\)In the online appendix we present prior and posterior distributions.
displaying the highest persistence and the preference and government spending shocks being the least persistent. Standard deviations of the shocks are estimated to be low in accordance to our priors.

All posterior estimates assume economically plausible values. The posterior mean for $\theta_n$ equals 2.05, which implies a Frisch elasticity of 0.49. This number is in the interval (0.01,0.85) of the values estimated in microeconomic studies for Canada.\(^6\) The posterior value for the labor share is somewhat higher than the initial value of 0.7, but remains within reasonable bounds. The relative risk aversion parameter, $\sigma$, is usually estimated in the $[0.5,6]$ range, with lower values typically estimated from microeconometric data. The credible intervals for $\sigma$ are within this range. In calibration exercises of DSGE models the parameter determining capital adjustment costs, $b$, varies from values around 3 (Woodford, 2003) to 19 (Casares and McCallum, 2006). Our estimates for $b$ vary between (7,10) with Canadian data and (5.3, 9) with US data.

Turning to the parameters of the depreciation function, the posterior value of parameter $\phi$ is slightly higher than values assumed for this parameter in the literature and the posterior estimate for parameter $\gamma$ is positive, confirming that as maintenance expenditures increase the depreciation rate decreases. Using the posterior mean values of $\gamma$, $\phi$, and $\alpha$, we find that the implied value for parameter $\theta$ is 2.25, whereas the implied mean values for $\psi$ and $\xi$ are, respectively, 0.52 and 0.19. These estimates imply that when capital is fully utilized and there are no expenditures in maintenance, the capital stock depreciates at the rate $\xi = 19\%$ per year. When capital is not utilized and there are no maintenance expenditures, the estimated ”natural” depreciation rate equals $\xi(1-\psi)^\theta = 3.7\%$ per year. Instead, when the capital stock is fully utilized and maintenance expenditures are high the depreciation rate of the capital stock equals $\xi\psi^\theta = 4.4\%$ per year.

The right panel of Table 3 presents the corresponding estimates for the US. In contrast with the results for Canadian data, the variances of the preference and the labor supply shocks assume significantly lower values, while there are no significant differences in the estimated persistence of the different shocks. Technology shocks seem to be more volatile in the US. The estimated mean values of $\sigma$ and $\theta_n$ in the US are substantially lower than

\(^6\)See Evers et al. (2008) for a summary of such estimates.
their estimated values for Canada. The estimates of risk aversion for the US are within reasonable ranges, although the mean estimate for $\theta_n$ implies a labor supply elasticity of 2.9. Adjustment costs are also estimated to be smaller in the US and the mean estimate for the elasticity of depreciation to changes in utilization and maintenance is also smaller. A higher labor supply elasticity implies that the economy can adjust to shocks relying more on the labor margin and less on adjustments in utilization, which come at a cost of higher depreciation. As a result, the high labor supply elasticity, due to complementarities in the production function, busts utilization movements. Given the low estimates of $\phi$ in the US, depreciation is not affected substantially by utilization. At the same time, $\gamma$ is relatively smaller and maintenance needs to increase relatively more to mitigate the effects of utilization on depreciation. The movements in utilization imply that capital will move less in equilibrium in response to shocks reducing the need of high capital adjustment costs in the capital low of motion. Finally, given the US estimates for $\gamma$ and $\phi$, the implied values for the parameters of the depreciation function are: $\theta = 2.73$, $\psi = 0.58$ and $\xi = 0.16$. Such values imply that the “natural” depreciation rate of US manufacturing capital equals $\xi(1 - \psi)^\theta = 1.5\%$, which is lower than the corresponding Canadian rate.

The estimates of the maintenance model are comparable with estimates of a standard RBC model without capital maintenance keeping the same set of observables, calibrated parameters and priors. When we contrast the log data densities in the two models for Canada and the US, the differences do not appear to be substantial for Canada, but they are significant for the US. The model with endogenous maintenance for Canada attains a log data density of 462.7, whereas the value for the standard RBC model is 462.5. The corresponding figures for the US are 477.0 and 473.8. Posterior estimates for the elasticity of depreciation to changes in utilization, $\phi$, turn out to be significantly different in both estimations. Table 3 indicates that $\phi$ will assume lower values when one considers the positive effects of maintenance in the depreciation of the capital stock. There is little knowledge about how utilization affects the depreciation of capital. Basu and Kimball (1997) estimate a log-linear production function incorporating variations in both capital utilization and effort for a panel of US firms from 21 manufacturing industries for the period 1949–1985 and estimate $\phi$ to be approximately unity. Burnside and Eichenbaum
(1996) calibrate $\phi = 1.56$, while Neiss and Pappa (2005) calibrate a slightly higher value for this parameter. Our estimates of $\phi$ are not comparable though with the existing ones and the ones of the standard RBC model since the cross derivative $\frac{\partial^2 \delta}{\partial M \partial U}$ is non zero. That is, unlike the standard model, in our specification endogenous maintenance movements affect the sensitivity of depreciation to changes in utilization.

### 3.3 Model dynamics

To gain some intuition on the mechanism present in the model we compare the impulse response functions with those generated by a standard RBC model with variable utilization. In the exercise we use posterior mean estimates obtained for Canada. In general, responses are similar. The presence of endogenous maintenance induces a relatively higher volatility of utilization and the depreciation rate.

The first row of Figure 2 plots the estimated IRFs of the variables of interest to a one-standard-deviation shock to TFP. The productivity shock raises the marginal product of the production factors and, as a result, capital utilization, investment and capital increase in response to the shock, generating a surge in output and consumption. Given the increase in utilization, the depreciation rate increases. Maintenance also increases to balance the detrimental effects of the surge in capital utilization on depreciation.

The investment-specific shock affects the production of investment goods. The second row of Figure 2 presents IRFs of the two models in response to shocks in the price of investment. The fall in the price of investment does surge investment in the impact period, increasing capital and, due to complementarities in production, hours and capital utilization. In the model with maintenance, the fall in the price of investment increases also the relative price of maintenance. Agents find it optimal to decrease maintenance expenditures on impact, which further increases the depreciation rate. Notice that the response of maintenance to investment-specific shocks is crucial to identify this type of shocks in the short run. After an investment-specific shock maintenance expenditures are reduced on impact while output increases, while for the rest of the disturbances considered the two variables always comove.

The third row of Figure 2 plots the IRFs to a negative labor supply shock. The shock
reduces hours on impact and, due to factor complementarity in the production function, it also reduces capital utilization and investment. The fall in capital utilization reduces maintenance expenditures and the induced movements in utilization and maintenance reduce the depreciation rate in equilibrium.

The next row of Figure 2 shows that a positive preference shock crowds out investment and, as a result, reduces hours, capital, and capital utilization. Consequently output also falls in equilibrium. The fall in utilization decreases capital depreciation and the need for capital maintenance and maintenance falls also in equilibrium.

Finally, the last row of Figure 2 presents the IRFs to a government spending shock. The increase in government spending crowds out investment, but, due to the negative wealth effect, labor supply and capital utilization increase in equilibrium. The rise in capital utilization raises the depreciation of capital and maintenance expenditures increase as well. In the model with endogenous maintenance utilization increases much more leading to a significant increase in the volatility of depreciation relative to the standard RBC model.

4 The time profile of capital depreciation

Given the estimates we obtained in the previous section, we next analyze the inferred time profile of variable capital depreciation in Canadian and US manufacturing resulting from our model. Although several studies have attempted to estimate the depreciation rate (especially in US manufacturing) using various single or multi-equation econometric approaches (see Epstein and Denny, 1980; Hulten and Wykoff, 1981a, 1981b; Nadiri and Prucha, 1996a, 1996b; Jorgenson, 1996; Huang and Diewert, 2011), there is no study that has provided estimates for depreciation series that are generated within a general equilibrium framework. An exception that uses time-varying depreciation within a general equilibrium setup is Chen et al. (2006), who calibrate the Japanese economy in order to investigate the driving forces of the saving rate. The time profile of their reported (exogenous) depreciation rates indicates that they were exceptionally high in the 50s and 60s, but declined substantially over the following decades. Recently, Liu et al. (2011) and Furlanetto and Seneca (2013) show that a reduced form depreciation shock is extremely
important in fitting the business cycle.

4.1 Canada

Prior to our calculations, it is useful to assess the fit of our model by comparing model estimates for capital maintenance and the actual series from the Canadian Survey. Since actual series for maintenance are not used in the estimation of our model, this exercise can serve as an additional test of our specification. The Bayesian estimation uses the Kalman filter to derive the log-likelihood, conditional on the set of observables. The same recursive algorithm enables us to sequentially update a linear projection for the system and as a by-product to generate smoothed estimates for the endogenous variables. Figure 3 displays the estimated trend deviations of the series for maintenance to capital ratio versus the actual trend deviations of the series from the Canadian Survey on Capital and Repair Expenditures. The model fits fairly well the pattern for the $MK$ ratio for most of the period covered with most of the peaks captured well by the estimated series, which are less volatile in general. The contemporaneous correlation between the actual and the estimated series amounts to 0.50. In line with their data counterpart, the estimated series are highly procyclical with the contemporaneous correlation of actual output and estimated maintenance equal to 0.66. Moreover, the cross-correlations remain high for lags (-1) to (-3) and for lead (+1) of output, similarly to the actual series.7 To further assess the fit of our model we also calculate the ratio of maintenance to ‘new’ investment series for Canada, which are two key variables in our setup. The estimated series are depicted in Figure 4. Again, our estimates track well the actual series: the correlation between actual and simulated series is 0.72.8

In Figure 5 we depict the estimated depreciation rate of equipment capital in the Canadian manufacturing sector over the period 1956-2005 (centered at 8.82%) along with actual output trend deviations. Table 4 contains the detailed figures for the depreciation

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7Detailed cross-correlograms are available upon request.
8An implication derived from Figures 3 and 4 is that the endogenously determined capital stock in our model is too volatile and does not replicate the second moment of the official capital stock series. This is due to the fact that endogenous maintenance dampens the responses of investment and does not operate directly through the accumulation of capital. Instead, our model performs much better in estimating the second moment of investment series.
rates of machinery-equipment capital in Canadian manufacturing. The depreciation rate of equipment capital is found to have a standard deviation of 1.2% and ranges between 5.4% and 11.3% over the period with a strongly procyclical profile: the correlation coefficient with output trend deviations amounts to 0.56. The correlation is higher (0.71) in the 1956-83 period of the sample, when output and the MK ratio exhibit a high correlation, and drops substantially (0.36) in the 1984-2005 period. This picture indicates that the long-run depreciation rate of equipment capital in Canadian manufacturing has exhibited substantial swings reflecting periods of fast and slow growth in the manufacturing sector and the associated pattern of capital maintenance.9

4.2 US

Given the success of the model in replicating the main features of the actual series of capital maintenance in Canadian manufacturing, we use our approach to estimate series for capital maintenance in the US manufacturing sector, where there has been no systematic collection of data on capital maintenance until 2007. Our estimates, thus, provide an assessment of the behavior of capital maintenance in the US over the last 50 years using the average value of years 2007-9 as a proxy of the steady-state maintenance to ‘new’ investment ratio. Figure 6 plots the estimated series for maintenance to capital ratio and output (in trend deviations) and Table 5 contains the estimated series of maintenance expenditures for machinery-equipment capital in US manufacturing for the period 1958-2009 expressed in current thousands USDs. As in the case of Canada, maintenance is found to be highly procyclical with a correlation coefficient of 0.85 with significant positive correlations also for the first lag and the first lead of output. The main picks and troughs of the business cycle are well captured by movements in maintenance. A similar picture emerges for the maintenance to ‘new’ investment ratio (Figure 7), which is also procyclical with a correlation coefficient of 0.51.

Similarly to Canada, we use our estimated series for capital maintenance in the US to

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9We note that a straightforward extension would be to generate alternative capital stock series that can be contrasted with official capital stock estimates. However, the comparison would be internally inconsistent as the official capital stock series are created under a different set of assumptions than those maintained here.
obtain estimated series for the depreciation rate in machinery-equipment capital over the period 1958-2009. Figure 8 plots the estimated depreciation rate and the output trend deviations for US manufacturing capital and Table 5 contains the detailed figures for the depreciation rates of machinery-equipment capital in US manufacturing. As in Canada, the depreciation rate in US manufacturing has been quite volatile and procyclical. In particular, the estimates indicate that the estimated range of the depreciation rate of equipment capital in US manufacturing varies between 9.3% and 13.7% over the period 1958-2009. In accordance with the Canadian estimates, the correlation with output trend is positive and equals 0.56.

These results shed some further light on the variability of capital depreciation, as few studies have focused on its behavior over time. Epstein and Denny (1980) report that the average depreciation rate in total US manufacturing over the period 1947-1971 ranged between 10.8% and 14.5%. Kollintzas and Choi (1985) report a similar range of 10.7%-14.1% over the same time period, whereas Bischoff and Kokkelenberg report a range of 9.6-11.8% over a period extended to 1978. Tevlin and Whelan (2003) report that the depreciation rate of non-computing equipment capital in the private business sector rose from 11% in 1965 to 14% in 1997, a rise that is largely attributed to the pattern of capital depreciation in the computing sector, which rose from 8% in 1965 to 16% in 1997. Notably, Nadiri and Prucha (1996a) report that the constant depreciation rate assumption cannot be rejected for the US electrical machinery industry. Our evidence, based on machinery-equipment capital, generates a somewhat wider spread for capital depreciation, which is not unreasonable given the 50-year time span of our study. Importantly, our implied depreciation rate follows a highly procyclical pattern, a feature that has only been indirectly captured by Epstein and Denny (1980) for some cycles.

5 Conclusions

This paper formulated and estimated a DSGE business-cycle model in which the depreciation rate is endogenously determined by expenditures on capital maintenance. An important feature of our approach, apart from its general-equilibrium character, is that we were able to derive the cyclical movements of capital depreciation, in the absence of
time-series data on capital maintenance that are largely unavailable. Our evidence on the time profile of the capital depreciation rate, which has been found to be procyclical and quite volatile, is contrasted to the standard assumption of constant capital depreciation, adopted routinely in most studies of macroeconomic fluctuations, and can provide significant insights in their sources and propagation mechanisms.

Our evidence may provide important potential insights for the tax treatment of capital assets and their depreciation. Given the procyclicality of depreciation, the state of the economy should be taken into account in the formation of the tax code and the calculation of variables affecting the values of assets, like interest rates. Nevertheless we emphasize that our implied estimates are in no way intended to provide definitive estimates of depreciation or their cyclical pattern. There is a great deal of room for further research, particularly in the areas of using more disaggregated data for the assessment of depreciation rates related to sectoral capital stocks within the context of a general equilibrium approach. Our findings should, thus, be viewed as an example of what can be achieved with a DSGE approach that accounts for capital maintenance. In this vein, the model can also be used to estimate unmeasured capital expenditures, like spending on capital maintenance, in other countries, as they form an important part of economic activity in order to estimate cross-country depreciation rates stemming from a general-equilibrium setup.
References


[38] Parks R., 1979, ‘Durability, maintenance and the price of used assets’, *Economic Inquiry*, 17(2), 197-217.


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<td>depreciation</td>
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<td>$\frac{M}{I}/\frac{I}{K}$</td>
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<td>$r^*$</td>
<td>net real interest rate</td>
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<td>output to capital ratio</td>
<td>$r^*/(1 - \alpha)$</td>
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Table 2: Prior distribution of structural parameters and shock processes

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Table 3: Posterior distributions of structural parameters and shock processes of the models for Canada and US

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<td>90% credible set</td>
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<td>90% credible set</td>
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<td>90% credible set</td>
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Table 4: Estimated depreciation rate of equipment capital in Canadian manufacturing (1956-2005)

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<td>1980</td>
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Table 5: Estimated capital maintenance (in current million USD) and depreciation rates of equipment capital in US manufacturing (1958-2009)

<table>
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<tr>
<th>year</th>
<th>maintenance</th>
<th>depreciation rate</th>
<th>year</th>
<th>maintenance</th>
<th>depreciation rate</th>
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</table>
Figure 1: Maintenance, capital and output: Canada, 1956-2005.

(a) Maintenance vs output

(b) Maintenance to capital ratio vs output
Figure 2: Impulse responses of variables to all shocks (in rows).

Note: solid black line for endogenous maintenance model, dashed grey line for standard RBC model.
Figure 3: Estimated MK ratio: Canada, 1956-2005.

![Graph showing the estimated MK ratio for Canada from 1956 to 2005. The graph includes two lines, one representing the actual values and the other the estimated values. The x-axis represents the years from 1960 to 2005, and the y-axis represents the ratio values from -2 to 0.2.](image)

Figure 4: Actual and estimated maintenance to 'new' investment ratio: Canada, 1956-2005.

![Graph showing the actual and estimated maintenance to new investment ratio for Canada from 1956 to 2005. The graph includes two lines, one representing the actual values and the other the estimated values. The x-axis represents the years from 1960 to 2005, and the y-axis represents the ratio values from 0.3 to 0.8.](image)
Figure 5: Output (trend deviations) and estimated depreciation rate (equipment capital): Canadian manufacturing, 1956-2005.

![Figure 5](image)

Figure 6: Estimated capital maintenance and output (trend deviations) in US manufacturing.

![Figure 6](image)
Figure 7: Estimated maintenance to ‘new’ investment ratio and output (trend deviations) in US manufacturing

Figure 8: Output (trend deviations) and depreciation rate: US manufacturing capital (equipment and structures)
Appendix of
‘Capital Maintenance and Depreciation
over the Business Cycle’

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October 2013

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A Technical Appendix

A.1 Steady state and determination of $\theta$, $\phi$ and $\gamma$

Using equations (11) and (10) in the paper we get that in the steady state it must hold:

$$\psi \phi U^\phi = (1 - \alpha) \gamma (1 - \psi) \frac{Y}{K} e^{-\gamma \frac{M}{K}}$$

From the capital law of motion it also holds:

$$\frac{I}{K} = \xi \left[ \psi U^\phi + (1 - \psi) e^{-\gamma \frac{M}{K}} \right]^\theta$$

Equation (12) implies that:

$$\frac{1}{\beta} = \left[ r^*U + \left( 1 - \frac{I}{K} \right) - \frac{M}{K} \right]$$

Moreover, equation (10) can be rewritten as:

$$\xi \theta \phi \psi \left[ \psi U^\phi + (1 - \psi) e^{-\gamma \frac{M}{K}} \right]^{\theta-1} U^{\phi-1} = r^*$$

Using the above equations, and imposing $U = 1$ in the steady state, we can derive the following conditions for the values of $\theta$, $\psi$ and $\xi$:

$$\theta = \frac{r^* \gamma + \phi}{\phi \gamma \frac{1}{K}}$$

$$\psi = \frac{r^* \gamma e^{-\gamma \frac{M}{K}}}{r^* \gamma e^{-\gamma \frac{M}{K}} + \phi}$$

$$\xi = \left( \frac{I}{K} \right)^{1-\theta} \left( \frac{r^*}{\theta \psi \phi} \right)^\theta$$
A.2 Log-linear conditions

The log-linear first-order conditions are given by the following set of equations (hatted variables denote log-deviations from steady-state values):

\[ \theta_n \hat{h}_t + \hat{\eta}_t^h = -\sigma \hat{C}_t + \hat{Y}_t - \hat{h}_t \]

\[ \left[ \gamma \frac{M}{K} + \frac{\theta - 1}{\theta} \frac{M}{I} \right] (\hat{K}_t - \hat{M}_t) + \frac{\theta - 1}{\theta} (1 - \alpha) \frac{Y}{I} \hat{U}_t - \hat{Z}_t = 0 \]

\[ \frac{(\theta - 1)}{\theta} \frac{M}{I} (\hat{K}_t - \hat{M}_t) + \left[ \phi + \frac{\theta - 1}{\theta} (1 - \alpha) \frac{Y}{I} \right] \hat{U}_t - \hat{Z}_t - \hat{Y}_t + \hat{K}_t = 0 \]

\[-\sigma \hat{C}_t - \hat{Z}_t - b \hat{K}_t + \hat{\eta}_t^u = -\sigma \hat{C}_{t+1} + \beta b \hat{K}_{t+2} + \beta (1 - \alpha) \frac{Y}{K} \hat{Y}_{t+1} \]

\[-\beta (1 - \alpha) \frac{Y}{K} \hat{U}_{t+1} - \beta (1 - \delta) \hat{Z}_{t+1} - \left[ b (1 + \beta) + \beta (1 - \alpha) \frac{Y}{K} \right] \hat{K}_{t+1} + \hat{\eta}^u_{t+1} \]

\[ \frac{I}{K} \hat{I}_t + \frac{I}{K} \hat{Z}_t = \hat{K}_{t+1} - \left[ (1 - \delta) - \frac{M}{K} \right] \hat{K}_t + (1 - \alpha) \frac{Y}{K} \hat{U}_t - \frac{M}{K} \hat{M}_t \]

\[ \hat{Y}_t = (1 - \alpha) (\hat{K}_t + \hat{U}_t) + \alpha (\hat{X}_t + \hat{h}_t) \]

\[ \hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{I}{Y} \hat{I}_t + \frac{M}{Y} \hat{M}_t + \frac{G}{Y} \hat{G}_t \]
The above seven equations describe the path of the seven endogenous variables of the model: output $\hat{Y}_t$, utilization $\hat{U}_t$, capital $\hat{K}_t$, hours $\hat{h}_t$, consumption $\hat{C}_t$, investment $\hat{I}_t$, and maintenance $\hat{M}_t$.

B Data Appendix

B.1 Canada

The first part of the Data Appendix describes briefly first the dataset from the Canadian survey on 'Capital and Repair Expenditures'. Private firms, households and government organizations in Canada were asked in an annual survey over the period 1956-1993 about their capital and repair expenditures on equipment and structures. The survey (conducted after 1993 in an updated form) is a census with a cross-sectional design and a sample size of 27,000 units; the target population is all Canadian businesses and governments from all the provinces and territories in Canada and the response rate is roughly 85%. Prior to the selection of a random sample, establishments are classified into homogeneous groups (i.e. groups with the same NAICS codes, same province/territory etc).

Capital expenditures are gross expenditures on fixed assets, which are assumed to cover spending devoted to ‘new’ investment, in accordance to the broad definition given earlier. These include expenditures on (i) fixed assets (such as new buildings, engineering, machinery, and equipment) which normally have a life of more than 1 year, (ii) modifications, additions, major renovations, and additions to work in progress (iii) capital costs such as feasibility studies and general (architectural, legal, installation and engineering) fees, (iv) capitalized interest charges on loans with which capital projects are financed, (v) work by own labor force. On the other hand, repair expenditures cover spending devoted to ‘maintenance’ cost, again in accordance to the broad definition given earlier. These expenditures cover (i) maintenance and repair of nonresidential buildings, other structures, and on vehicles and other machinery, (ii) building maintenance (janitorial services, snow removal, sanding), (iii) equipment maintenance (such as oil changes and lubrication of vehicles and machinery), (iv) repair work by own and outside labor force machinery and
equipment.

The following variables from the Canadian Survey on Capital and Repair Expenditures of Canada Statistics were used for capital and repair expenditures. Backward values for the manufacturing sector up to 1956 were obtained by using the growth rates for capital expenditures (the growth rates for 1992 and 1993 are common for the two surveys) and then by extrapolating the series for repair expenditures through their share in total capital and repair expenditures over 1956 to 1993.

- Repair expenditures in manufacturing, machinery and equipment variable v754445 [D878256], 1994 to 2005, and variable v62550 [D843232], 1956 to 1993.

The rest of the Canadian variables used in the paper and their sources are as follows.

- Manufacturing capital stock in machinery and equipment: manufacturing sector end-year capital stock, variable v1071437 [D819523], 1955 to 2007 (Canada Statistics, Table 031-0002, current prices).
- Manufacturing employment: index 2000=100 (source: International Financial Statistics, variable 15667EY.ZF...).
- Hours worked: Annual average number of hours worked for all jobs; Non-durable manufacturing, index 1992=100 (source: Canada Statistics). The number of hours worked in all jobs is the annual average for all jobs times the annual average hours worked in all jobs. According to the retained definition, hours worked means the
total number of hours that a person spends working, whether paid or not. In general, this includes regular and overtime hours, breaks, travel time, training in the workplace and time lost in brief work stoppages where workers remain at their posts. Time lost due to strikes, lockouts, annual vacation, public holidays, sick leave, maternity leave or leave for personal needs are not included in total hours worked.

- **Consumption**: Annual nominal consumption in million Canadian dollars (source: Penn World Tables). It is obtained by multiplying consumption share of GDP per capita (variable: cc) times GDP per capita (variable: cgdp) and total population (variable: POP).

- **Capital utilization**: Industrial (total non-farm goods producing industries) capacity utilization rate (source: Canada Statistics, variables v142812, Table 028-0001, percent), averaged from quarterly data available from 1962 onwards. Backward values were extrapolated by fitting a linear regression on total fixed non-residential capital stock for all industries (source: Canada Statistics, variable: D99027311000) divided by Canada Gross National Product (source: International Financial Statistics, variable 15699A.CZF).

- **Industrial selling price index**: Industrial selling price index (source: International Financial Statistics, variable: 15663...ZF...)

- **Population**: Population 15-64 (source: OECD, ALFS Summary Tables).

Maintenance, total investment, capital stock, consumption and output were deflated with the industrial selling price index and divided by total working population 15-64. Hours per worker were obtained by multiplying manufacturing employment and hours worked and dividing by total working population 15-64, as in Smets and Wouters (2007).

### B.2 United States

The following variables for U.S. manufacturing data (NAICS classification) were used.
• **Manufacturing output**: Total value added in million US dollars from Becker and Gray (2009), variable vadd, period 1958-2005. The series was extrapolated for years 2006-9 using the growth rate of value added from the U.S. Census Annual Survey of Manufacturers (Table: Statistics for Industry Groups and Industries). The item is derived by subtracting the cost of materials, supplies, containers, fuel, purchased electricity, and contract work from the value of shipments (products manufactured plus receipts for services rendered), adjusted by the addition of value added by merchandizing operations plus the net change in finished goods and work-in-process between the beginning and end of year inventories.

• **Hours worked**: Production worker hours in millions from Becker and Gray (2009), variable prodh, period 1958-2005. The series was extrapolated for periods 2006-9 using the growth rate of production workers hours from the U.S. Census Annual Survey of Manufacturers (Table: Statistics for Industry Groups and Industries). The item covers all hours worked or paid for at the manufacturing plant, including actual overtime hours (not straight-time equivalent hours). It excludes hours paid for vacations, holidays, or sick leave when the employee is not at the establishment.

• **Capital expenditures**: Total capital expenditures in million US dollars from Becker and Gray (2009), variable invest, period 1958-2005. The series was extrapolated for years 2006-9 using the growth rate of total capital expenditures from the U.S. Census Annual Survey of Manufacturers (Table: Statistics for Industry Groups and Industries). The item represents the total new and used capital expenditures reported by establishments in operation and any known plants under construction. These data include expenditures for permanent additions and major alterations to manufacturing and mining establishments, new and used machinery and equipment used for replacement and additions to plant capacity, including work done by contract, as well as by the establishment’s own workforce. Capital expenditures for machinery-equipment and buildings & other structures were obtained by using the average shares of these expenditures over the period 2002-2009, reported in the U.S.
Census Annual Survey of Manufacturers (Table: Statistics for Industry Groups and Industries).

- **Capital stocks**: Real capital stocks (total, equipment, structures) in million US dollars from Becker and Gray (2009), variables cap, equip, plant, period 1958-2005. The series were converted in current U.S. dollars using the investment deflator from Becker and Gray (2009), variable piinv and extrapolated for years 2006-9 assuming a constant capital-output ratio and constant shares for equipment and structures.

- **Capital utilization**: Capacity utilization in manufacturing for years 1958-2009 was obtained using the historical series from the Board of Governors of the Federal Reserve System (Table G.17).

- **Consumption**: Annual nominal consumption in million US dollars (source: Penn World Tables). It is obtained by multiplying consumption share of GDP per capita (variable: cc) times GDP per capita (variable: cgdp) and total population (variable: POP).

- **Population**: Population 15-64 (source: OECD, ALFS Summary Tables).


Capital expenditures, capital stock, consumption and output were deflated with the Producer Price Index and divided by total working population 15-64. Hours per worker were obtained by dividing Hours worked by total working population 15-64, as in Smets and Wouters (2007).