Another look at the linear $q$ model: 
an empirical analysis of aggregate business capital spending with maintenance expenditures

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Abstract. The paper revisits the empirical investment literature, which has established that aggregate business fixed investment is not found to be related linearly to marginal or average Tobin’s $q$. The theoretical background is extended here by developing a supply-side model where the depreciation rate of private capital is determined endogenously. The firm can either invest in ‘new’ capital, which adds directly to the existing capital stock at the presence of convex adjustment costs, or extend the durability of installed capital through maintenance expenditure, which affects its depreciation rate. The model shows that Tobin’s $q$ is then a positively related sufficient statistic for both components of aggregate capital expenditures. This central implication is tested empirically using aggregate time-series survey data from Canada on ‘new’ investment and maintenance expenditures covering the period 1956–93. The estimated relationships produce significant and plausible parameter estimates for the structural parameters of the $q$ model. JEL classification: D92, E22

Un autre coup d’œil au modèle linéaire fondé sur le coefficient $q$ de Tobin: une analyse empirique des dépenses agrégées d’investissement en capital des entreprises quand il y a des dépenses de maintenance. Le texte examine la littérature empirique sur l’investissement qui a établi que le niveau d’investissement agrégé des entreprises en capital fixe n’est pas relié de façon linéaire au coefficient moyen ou marginal $q$ de Tobin. On enrichit l’arrière plan théorique en développant un modèle d’offre dans lequel le taux de dépréciation du capital privé est déterminé de façon endogène. L’entreprise peut soit investir dans du capital « nouveau » (ce qui ajoute directement au stock de capital existant) soit allonger la vie du capital en place par des dépenses de maintenance qui modifient le taux de dépréciation. Le modèle montre que le coefficient $q$ de Tobin est alors positivement relié aux deux composantes des dépenses agrégées en capital. Cette implication importante est mise au

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test empiriquement en utilisant les séries chronologiques en provenance des enquêtes de Statistiques Canada sur les nouveaux investissements et les dépenses de maintenance pour la période 1956–93. Les relations estimées produisent des paramètres plausibles et significatifs pour les paramètres structurels du modèle \( q \).

1. Introduction

A persistent puzzle in the empirical investment literature is that various measures of Tobin’s \( q \) do not appear to have a notable impact on investment as originally hypothesized by Tobin and Brainard (1968) and Tobin (1969) following a concept put forward by Keynes back in 1936. The current paper aims at revisiting the empirical literature on this relationship by differentiating between ‘new’ investment, which adds directly to the capital stock, and maintenance, which affects the depreciation rate of private capital. The paper then uses a largely unexplored data set, the Statistics Canada Survey of Capital and Repair Expenditures, to test the relationship of these expenditures with \( q \) and asserts that the linear \( q \) model contains substantial information for the determination of both components of capital expenditures.

In the theoretical field the idea of \( q \) was encapsulated in the neoclassical model with adjustment costs for investment by Eisner and Strotz (1963), Lucas (1967), Gould (1968), and was subsequently further reinforced by the finding of Hayashi (1982) that under certain assumptions marginal \( q \) can be replaced by average \( q \), which is observable via the stock market valuation of firms. Albeit these theories have provided solid foundations on the link between investment and \( q \), the empirical counterpart of the literature is characterized by the failure of reduced-form relationships in accommodating a robust relationship between investment and various measures of \( q \). Abel and Blanchard (1986) stress that ‘measures of \( q \) […] constructed in very different manners, leave large serially correlated residuals in investment,’ whereas Cummins, Hassett, and Hubbard (1994) state that ‘\( q \) models have […] explained investment very poorly using aggregate time-series data or firm-level data [producing] very small estimated effects of \( q \) on investment [and] implying implausibly high […] cost of adjusting the capital stock.’ The general picture is surveyed by Chirinko (1993) who claims that ‘the usefulness of \( q \) theory is called into question by its generally disappointing empirical performance.’ Recent attempts to rehabilitate the connection between investment and \( q \) have relied on introducing more complex processes arising from regimes switches, non-convex adjustment costs, investment irreversibility and other factors,1 whereas Bond and Cummins (2001) seek to redefine the concept of \( q \) by using earnings forecasts in order to gain a better approximation of average \( q \).

However, little or no work has been done so far in addressing the left-hand side of this relationship, namely, investment. Typically, there are two broad classifications of business or private investment data. The first category

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1 For explorations of similar effects on the relationship between firm investment and \( q \) see Barnett and Sakellaris (1998, 1999) and Abel and Eberly (2002).
includes data on fixed non-residential private investment from standard macroeconomic time-series data sources, such as national accounts, the OECD database (Sensenbrenner 1991), or the Penn World Tables. The second category of investment data is based on panel data for 2-digit or plant-level manufacturing firms (usually obtained from the U.S. Compustat industrial database) and refers to investment as capital expenditures on property, plant, and equipment.\(^2\)

These definitions for investment aim at capturing the concept of gross capital expenditures with no distinction made regarding the targeted capital stock. As first pointed out by Feldstein and Foot (1971), though, according to a survey on planned investment in the United States for the period 1949–68 only one-half of ‘gross’ investment involved new capital expenditures (‘expansion’ investment).\(^3\) The other half of planned investment concerned funds for ‘replacement and modernization,’ defined by Feldstein and Rothschild (1974, 394) as ‘the actual purchase of equipment to maintain the output capacity that is lost through output decay’ brought about by capital ageing. In almost all studies ‘replacement’ investment is assumed to be a constant fraction of the capital stock, thus implying that ‘gross’ investment ‘can be explained and forecast by a simple mechanical “technological” rule’ (Feldstein and Foot 1971, 49).

Yet, as emphasized by Feldstein and Rothschild (1974, 394) a constant ‘replacement’ investment to capital stock ratio implies a constant exponential rate of output decay and a constant composition of the capital stock by deterioration pattern, both of which are inconsistent with standard stylized facts from U.S. data. ‘Replacement’ investment appears to be closer to capital ‘maintenance,’ defined as the deliberate utilization of all resources, which preserves the operative state of capital goods (Bitros 1976). In contrast to ‘expansion’ investment, ‘maintenance’ expenditures are directly related to the capital depreciation rate. Along these lines, in a series of papers during the 1970s several authors investigated the firm’s problem between the optimal maintenance level and the maintenance dependent depreciation rate.\(^4\) Subsequent empirical studies have found that, although decaying constant rates of depreciation often provide a reasonable approximation at a given point in time, there is mounting evidence that capital deterioration is endogenous and, in particular, associated with maintenance expenditure.\(^5\) Generally, these findings point out that expenditures for capital maintenance affect the operating life of the capital stock and thus are likely to provide significant empirical insights for investment models.

Although theoretically sound, the empirical testing of these ideas confronts the lack of consistent data on capital maintenance expenditures. Globally, there is

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\(^2\) Blundell et al. (1992) use direct purchasing of new fixed assets from company accounting data reported in the Datastream International database.

\(^3\) The evidence came from the McGraw-Hill Survey of Business’ Plans for New Plants and Equipment.


\(^5\) For a brief survey on related empirical findings, see Nelson and Caputo (1997) and the references cited therein.
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only one source of long-run data on capital expenditures in newly purchased assets and maintenance, namely, the Statistics Canada Survey of Capital and Repair Expenditures, which contains evidence obtained from private firms, households, and government organizations. The figures from this survey show that total (private and public) maintenance and repair expenditures in Canada amounted on average to around 6.3% of GDP for the period 1956–93. This number was roughly equal to one-third of spending on ‘new’ investments and, when compared with other so-called engines of growth, was somewhat lower than education spending (6.8% of GDP), but far above the average spending on R&D (1.4% of GDP). Capital expenditures by business enterprises constituted the largest portion of these expenditures, amounting to 14.1% of GDP, of which more than one-fourth was oriented to maintenance and repair expenditures.

Through use of this largely unexplored data set the paper aims at revisiting the empirical literature on the relationship between aggregate capital expenditures and \( q \). To this end, a supply-side model is outlined in which maintenance expenditures are embedded in the economic choice of the firm and are not considered a technological necessity. In this context the depreciation rate is endogenous and depends upon maintenance expenditures: the firm can spend on capital either by directly adding ‘new’ investment to the capital stock or by extending the durability of the existing capital stock through maintenance expenditures. Within this framework both choice variables of the firm (‘new’ investment and maintenance expenditures) are shown to be functions of \( q \).

The paper then uses the Canadian survey data on business capital expenditures and tests their relationship with \( q \). The main finding of the paper is that the empirical reduced-form relationships utilizing the survey data on capital expenditures reveal that \( q \) contains substantial information for the determination of business’s ‘new’ investment and maintenance expenditures. The results produce reasonable estimates of the adjustment costs for ‘new’ investment and of the relationship between the depreciation rate and maintenance expenditures. All findings are found to be robust to alternative specifications. Thus, investment equations that ignore maintenance expenditures are likely to omit something important. The picture in favour of the model with maintenance expenditures remains when a decomposition of each type of capital expenditures on construction and machinery-equipment spending is considered. The evidence from the disaggregated data shows that the impact of \( q \) is different for these two components of capital spending. Consequently, capital heterogeneity is likely to play an important role in the \( q \) model of business expenditures and should be taken into account in the empirical modelling of investment spending.

Recently, Mullen and Williams (2004) developed a model of the behaviour of the firm to explore the linkages between maintenance-repair expenditures and various potential determinants, such as the user cost of capital, capacity utilization, and scrappage, as well as ‘new’ investment expenditures. The authors then adopt a generalized dynamic empirical specification and use disaggregated
survey data from Canada covering the period 1991–2000 to investigate the impact of the aforementioned variables on maintenance and repair expenditures. They find that these expenditures are positively associated in the Canadian economy with the user cost of capital, possibly reflecting the tradeoff between investing in 'new' versus existing capital goods.

In contrast to Mullen and Williams (2004), the current study attempts to improve the empirical performance of $q$-type aggregate investment models by extending the concept of capital expenditures to include maintenance and, subsequently, by focusing on the links between 'new' investment and maintenance-repair expenditures with $q$. Although the empirical specification adopted here is more restricted in terms of the potential determinants included, it permits a structural interpretation of the estimated parameters in terms of the $q$ model with convex adjustment costs. Thus, Mullen and Williams (2004) offer a rich, flexible, and more data-disaggregated framework, whereas the present paper aims at improving the class of aggregate investment models broadly termed as $q$ models and allows for a structural interpretation of the estimated parameters.

A by-product of the empirical analysis is the apparent resolution of a 'puzzling' result on Canadian investment pointed out by Barro (1990). Using the growth rate of Canadian real fixed non-residential private investment and Canadian real stock price changes as a measure of changes in average $q$, Barro (1990) found that Canadian investment reacts to the U.S. stock market rather than to the domestic market. The evidence presented here suggests that this 'puzzle' on the impact of U.S. stock prices on Canadian investment evaporates when the survey data on capital expenditures are used in empirical specifications.

The findings of the paper do not imply, however, that newer theories about the effect of $q$ on investment, such as non-linearities due to threshold effects, non-convex adjustment costs, and fixed costs, do not add substantial insights to the exploration of the determinants of investment. These models appear to be extremely useful for analysing data at the plant level, which can directly capture discrete investment movements. Rather, the present model aims at highlighting at the aggregate level a more or less neglected factor of capital accumulation, namely, maintenance expenditures, which intuitively affects the accumulation of capital, but its impact is rarely examined in empirical investment relationships. To this extent, endogenizing the depreciation rate in order to analyse aggregate investment decisions in a richer context can complement the newer theories on $q$.

The rest of the paper is organized as follows. Section 2 outlines a supply-side model with endogenous capital depreciation rate and derives the optimality conditions for capital expenditures by firms (the steady-state properties of the model are sketched in appendix A). Section 3 describes briefly the Canadian Survey of Capital and Repair Expenditures. Section 4 presents the empirical specifications and results. Section 5 gives some sensitivity tests by examining extensions of the basic empirical model and provides some estimates using disaggregated data. Section 6 concludes the paper.
2. A $q$ model of business investment with capital maintenance expenditures

This section develops a supply-side model for the behaviour of firms. The environment is a variant of the standard neoclassical growth model; see, for instance, Barro and Sala-I-Martin (1995, chap. 3). For simplicity we assume zero population growth and no technological progress. The model is then augmented to include endogenous depreciation of the capital stock, which will appear as a choice variable for firms dependent upon spending for capital maintenance expenditures in addition to the standard expenditures for ‘new’ investment.

Consider an economy where production is carried out by many identical competitive firms. The production function of firm $i$ is given by

$$Y_i = f(K_i, L_i),$$

where $Y$ denotes output, $K$ the capital stock, $L$ the amount of labour, and the production function $f(\cdot)$ is assumed to be constant returns to scale and also satisfies the neoclassical properties (time subscripts are omitted).

Now, following among others Nickell (1978), Schworm (1979), and, more recently, McGrattan and Schmitz (1999) and Boucekkine and Ruiz-Tamarit (2003), it is assumed that the capital accumulation process of the firm depends upon two decision variables: ‘new’ investment, $I_i$, and the endogenous depreciation rate of the existing capital stock, $\delta(\cdot)$, which is a function of maintenance expenditures by the firm, $M_i$. More specifically, the capital stock evolves over time according to the following law of motion:

$$\dot{K}_i = I_i - \delta \left( \frac{M_i}{K_i} \right) \cdot K_i,$$

where the depreciation function satisfies the properties $\delta'(M/K) < 0$ and $\delta''(M/K) > 0$. By choosing maintenance expenditures the firm determines the depreciation rate of its capital stock.

Turning to ‘new’ investment, the standard adjustment costs specification is adopted. More specifically, the cost of ‘new’ investment is assumed to involve convex unit installation costs given by $\varphi(I_i/K_i)$, with $\varphi'(\cdot) > 0$ for $I_i/K_i > 0$ and $\varphi'(\cdot) \geq 0$. The total cost of ‘new’ investment faced by the firm is thus given by $[1 + \varphi(I_i/K_i)] \cdot I_i$.\(^6\)

To ease exposition, it will be further assumed that the unit price of ‘new’ investment equals the unit price of maintenance expenditure. Thus, a unit of ‘new’ investment can be replaced with a unit of maintenance and vice versa without

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\(^6\) The unit cost of adjustment specification may come from a strictly convex total adjustment cost function; that is, $C(I, K) = \varphi(I/K) \cdot I$ where $C'(I, K) > 0$ and $C''(I, K) > 0$ with respect to $I$. This standard homogeneous specification will allow below the substitution of marginal $q$ with the average (Tobin’s) $q$, which is directly observable. This will prove useful in the empirical counterpart put forward in section 4.
any additional complications. Also, it will be assumed throughout that the firm's optimal 'new' investment policy always results in positive investment, which is a reasonable assumption at the aggregate level, empirically investigated below.

The decision problem of the representative firm is to choose \( L_i, I_i, M_i \), which maximize its discounted net cash flows subject to the capital accumulation constraint and an initial value \( K(0) \):

\[
\max_{L_i, I_i, M_i} \int_0^\infty e^{-rt} \left\{ f(K_i, L_i) - wL_i - \left[ 1 + \varphi \left( \frac{I_i}{K_i} \right) \right] \cdot I_i - M_i \right\} dt
\]

s.t. (2), \( K(0) = K_0 \),

where it is assumed that the representative firm faces an economy-wide constant real interest rate \( r \). The optimality conditions are then given after aggregation across firms by the following equations:

\[ f(k) - k \cdot f'(k) = w \quad (3a) \]

\[ q = 1 + \varphi \left( \frac{I}{K} \right) + \left( \frac{I}{K} \right) \cdot \varphi' \left( \frac{I}{K} \right) \quad (3b) \]

\[ \frac{1}{q} = -\delta' \left( \frac{M}{K} \right) \quad (3c) \]

\[ \frac{1}{q} \left[ f_k(K, L) + \left( \frac{I}{K} \right)^2 \varphi' \left( \frac{I}{K} \right) \right] - \left[ \delta \left( \frac{M}{K} \right) - \left( \frac{M}{K} \right) \delta' \left( \frac{M}{K} \right) \right] + \frac{\dot{q}}{q} = r \quad (3d) \]

where \( k \) denotes the capital-labour ratio, and \( q \) is the shadow price related with the capital accumulation constraint (2) representing the current-value shadow price of installed capital in units of contemporaneous output. For given \( w, r, q, \) and \( \dot{q} \), equations (3a)–(3d) ensure that all firms have the same ratios of 'new' investment and maintenance to the capital stock.

Equations (3a) and (3b) are standard optimization conditions for this type of problem and state that the marginal product of labour equals the real wage rate \( w \) and that the shadow price of capital \( q \) exceeds unity for positive investment, owing to adjustment costs. The relationship between \( q \) and \( (I/K) \) is monotonically increasing as a result of the properties of the cost of adjustment function \( \varphi(I/K) \). Similarly, equation (3b) can be inverted to yield the familiar investment function:

\[ \frac{I}{K} = \psi(q) \quad \text{(3b)}' \]

\footnote{Boucekkine and Ruiz-Tamarit (2003) investigate in detail the implications of differential prices between 'new' capital goods and maintenance.}
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with $\psi'(q) > 0$. Equation (3c) then emerges as an extra efficiency condition that equates the benefit from the reduction in the depreciation of capital by $-δ'(M/K)$ as a result of an extra unit of maintenance expenditures evaluated at the shadow price of capital to the price of maintenance services (assuming a unitary price of maintenance services).\(^8\)

Equation (3d) modifies the standard condition, which states that the market rate of return $r$ is equal to the sum of the total marginal product of capital (consisting of its marginal contribution in output plus the reduction in the opportunity cost of installation made possible by an additional unit of capital) deflated by the cost of capital $q$, the capital gain, less the maintenance dependent depreciation rate. The extra term $(M/K) \cdot δ'(M/K)$ measures the rise required in the market rate of return as a result of the marginal increase in the level of maintenance expenditures due to an increase in the capital stock by one unit scaled by the marginal reduction in the depreciation rate (the curvature of the depreciation function). Therefore, a rise in maintenance expenditures enters in a twofold manner in the determination of the market rate of return: positively by reducing the depreciation rate and negatively by raising expenditures and reducing profits. Finally, the standard transversality condition related to this problem requires that the interest rate exceed the zero steady-state growth rate.

The similarities and dissimilarities of the no-maintenance model can be more easily shown if the differential equation (3d) is solved forward, subject to the transversality condition, to yield (with time subscripts included):

$$q_t = \int_t^\infty e^{-s} \left[ f_K(K_s, L_s) + \left( \frac{I_s}{K_s} \right)^2 \cdot \psi' \left( \frac{I_s}{K_s} \right) - \left( \frac{M_s}{K_s} \right) \right] ds. \quad (4)$$

The shadow price $q$ is the present value of total future marginal products minus the maintenance expenditures discounted by the real interest rate and the maintenance dependent depreciation rate.

The steady-state properties of the model are outlined in appendix A. An interesting feature of the model is that the firm’s problem involves one extra choice variable, namely, maintenance expenditures, which given an appropriate parametric specification for the depreciation function summarizes via (3c) all the information needed to obtain a well-defined demand function for maintenance expenditures linked to average $q$, as in the case of ‘new’ investment. In other words, the two choice variables of the firm regarding capital accumulation, namely, $I$ and $M$, will be represented by monotonic relationships through $q$, which will be a sufficient statistic for both types of capital spending.

In particular, for higher marginal or average $q$ (exceeding unity) ‘new’ investment increases, as investment in the form of ‘new’ capital is more profitable.

\(^8\) For a similar derivation see also McGrattan and Schmitz (1999) and Boucekkine and Ruiz-Tamarit (2003).
relative to existing capital. In addition, by (3c) a higher value of $q$ implies that the slope of $\delta(M/K)$ is less negative at the optimum. Thus, it is more profitable for firms to increase maintenance expenditures as well, as the yield of existing capital in terms of output is now larger. Changes in the marginal or average $q$ are therefore expected to mirror changes in both ‘new’ investment and maintenance expenditures in the same direction.

Finally, it should be noticed that the homogeneity properties of the production and the total adjustment cost functions ensure the equality between marginal and average $q$. The shadow price of capital times the cost of capital equals, then, the present value of all future dividends:

$$q_0K_0 = \int_0^\infty e^{-rt} \left[ f(K, L) - wL - M - I(1 + \cdot \varphi(I/K)) \right] dt.$$  

In an efficient stock market, this present value will be exactly equal to the stock market price of a firm, and marginal $q$ will be equal to the ratio of the stock market value of a firm to the replacement cost of its capital (Tobin’s $q$).

3. Data on ‘new’ business investment and capital maintenance expenditures: the Canadian

3.1. Survey of Capital and Repair Expenditures

This section describes the Canadian survey data set on ‘new’ investment and maintenance expenditures, which will be utilized to test empirically the relationships derived by the $q$ model for aggregate business investment with capital maintenance. As mentioned in the introduction, the only available data set worldwide on ‘new’ investment and maintenance expenditures is the Canadian Survey of Capital and Repair Expenditures. This section describes briefly this data set, which will be utilized to test empirically the relationships derived by the $q$ model for business investment with capital maintenance. For more details the reader is also referred to McGrattan and Schmitz (1999).

Private firms, households and government organizations in Canada were asked in an annual survey over the period 1956–93 about their capital and repair expenditures on equipment and structures. The survey (conducted after 1993 in an updated form) is a census with a cross-sectional design and a sample size of 27,000 units; the target population is all Canadian businesses and governments from all the provinces and territories in Canada and the response rate is roughly 85%. Prior to the selection of a random sample, establishments are classified into homogeneous groups (i.e. groups with the same NAICS codes, same province/territory etc).

9 This can be easily confirmed by taking the time derivative of $(qK)$, substituting (3a) into (3d) and using Euler’s Theorem for the production function. Forward integration of the resulting relationship then yields equation (5); see, for instance, Sala-i-Martin (2001).
Here, we focus on *capital* and *repair* expenditures by business enterprises where the government controls less than 50% of the voting rights. In particular, *capital* expenditures are gross expenditures on fixed assets, which are assumed to cover spending devoted to ‘new’ investment, in accordance to the broad definition given earlier. These include expenditures on (i) fixed assets (such as new buildings, engineering, machinery, and equipment), which normally have a life of more than one year; (ii) modifications, additions, major renovations, and additions to work in progress; (iii) capital costs such as feasibility studies and general (architectural, legal, installation, and engineering) fees; (iv) capitalized interest charges on loans with which capital projects are financed; (v) work by own labour force. On the other hand, *repair* expenditures cover spending devoted to ‘maintenance’ cost, again in accordance with the broad definition given earlier. These expenditures cover (i) maintenance and repair of non-residential buildings, other structures, and on vehicles and other machinery; (ii) building maintenance (janitorial services, snow removal, sanding), (iii) equipment maintenance (such as oil changes and lubrication of); (iv) repair work by own and outside labour force machinery and equipment.

These expenditures are allocated, in turn, to *non-residential construction* and *machinery and equipment* expenditures. In particular, non-residential building and engineering construction expenditures include spending on (i) manufacturing plants, warehouses, office buildings, shopping centres, and so on; (ii) roads, bridges, sewers, electric power lines, underground cables, and so on; (iii) demolition cost of buildings, land servicing, and site preparation; (iv) leasehold and land improvements. Spending on machinery and equipment covers (i) automobiles, trucks, professional and scientific equipment, office and store furniture, appliances; (ii) motors, generators, transformers; (iii) capitalized tooling expenses; (iv) prepaid progress payments.

Table 1 gives a synoptic presentation of aggregated and disaggregated data for the business enterprises sector. For comparison purposes the corresponding figures for the total economy (which also includes expenditures by the private institutions-housing and the public sectors) are also reported. Business expenditures constitute by far the largest component of total capital and repair expenditures (which are tabulated in the last part of table 1). The general picture shows that total business expenditures in ‘new’ investment and maintenance amounted to 14.1% of GDP for the period under consideration, with the average maintenance share covering 27% (3.8% of GDP); see also figure 1a. However, there are substantial disparities when the allocation of these expenditures on construction and machinery and equipment is considered. In particular, the bulk of maintenance expenditures by business enterprises was concentrated in machinery and equipment (76% of total business maintenance expenditures).

10 The remainder of the private sector consists of private institutions and housing.
11 As shown by McGrattan and Schmitz (1999), ‘new’ investment expenditures exhibited larger variability than maintenance expenditures. However, issues like the relationship between maintenance variability, depreciation, and business-cycle fluctuations will not be addressed here; for related papers, see Collard and Kollintzas (2000) and Licandro and Puch (2000).
TABLE 1

<table>
<thead>
<tr>
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<th>Mean</th>
<th>Std. Dev.</th>
<th>Maximum</th>
<th>Minimum</th>
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<td><strong>BUSINESS ENTERPRISES</strong></td>
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<tr>
<td>Total ‘new’ capital and maintenance expenditures in business enterprises</td>
<td>14.1</td>
<td>1.4</td>
<td>17.7</td>
<td>10.8</td>
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<tr>
<td>Disaggregation in ‘new’ capital and maintenance expenditure</td>
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<tr>
<td>‘New’ capital expenditures</td>
<td>10.2</td>
<td>1.2</td>
<td>13.3</td>
<td>7.8</td>
</tr>
<tr>
<td>Maintenance expenditures</td>
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<td>0.4</td>
<td>4.5</td>
<td>3.1</td>
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<tr>
<td>Disaggregation of ‘new’ capital and maintenance expenditure in construction and machinery-equipment expenditure</td>
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<td></td>
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<tr>
<td>‘New’ capital expenditures in construction in business enterprises</td>
<td>4.3</td>
<td>0.7</td>
<td>6.2</td>
<td>2.7</td>
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<td>5.9</td>
<td>0.6</td>
<td>7.3</td>
<td>4.8</td>
</tr>
<tr>
<td>Maintenance expenditures in construction in business enterprises</td>
<td>0.9</td>
<td>0.1</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Maintenance expenditures in machinery-equipment in business enterprises</td>
<td>2.9</td>
<td>0.2</td>
<td>3.4</td>
<td>2.4</td>
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<tr>
<td><strong>TOTAL ECONOMY</strong></td>
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<tr>
<td>Total expenditures in ‘new’ capital and maintenance</td>
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<td>2.4</td>
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<td>22.5</td>
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<tr>
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<td>1.9</td>
<td>25.7</td>
<td>16.9</td>
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<tr>
<td>Total maintenance expenditures</td>
<td>6.3</td>
<td>0.7</td>
<td>7.6</td>
<td>5.2</td>
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</tbody>
</table>

SOURCE: CANSIM database, Statistics Canada and author’s calculations.

whereas the corresponding figure for ‘new’ investment expenditures was 58% of the total. Figures 1b and 1c depict ‘new’ investment and maintenance in construction and equipment as shares of the corresponding aggregates, respectively, for the period under consideration.

It should be noticed that total (private and public) expenditures in ‘new’ investment and maintenance amounted to 27.2% of GDP on average for the period under consideration, of which 76% was allocated to ‘new’ investment and 24% to maintenance. These figures imply that, according to the Canadian Survey of Capital and Repair Expenditures, the figures for total capital expenditures by business enterprises are quite different from those reported in national accounts or other official and widely used statistical sources. For instance, the Gross Domestic Private Investment as a percentage of real GDP, as reported in Penn World Tables version 6.1, is 21.1% of GDP for the period under consideration, of which private investment is estimated to be 18.8% of GDP. Moreover, according to the National Accounts classification system, business investment in fixed non-residential capital as a percentage of GDP amounted to 17.7% during the same period. These discrepancies suggest that widely used national or international sources of aggregate investment are likely to underestimate expenditures on capital, which in turn may affect substantially the estimates of empirical investment equations.
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4. Empirical specifications and results

The introductory section underlined the robust conclusion reached by several past studies on the broad failure of estimating a robust relationship between investment and marginal or average $q$. This section moves on to provide empirical estimates of investment equations using data on 'new' investment and maintenance from the Statistics Canada Survey on Capital and Repair Expenditures; the relationship between 'new' investment, maintenance, and $q$ are prescribed by the first-order conditions generated by the solution of the model. Hence, after stressing theoretically the role of $q$ for the determination of maintenance
FIGURE 1c ‘New’ investment and maintenance in machinery-equipment

expenditures, the goal of the empirical methodology is to provide an econometric environment, aiming at capturing the link of both types of capital expenditures with $q$.

4.1. Empirical specifications

Given an appropriate parametric specification, equation (3b) has provided a standard benchmark in the empirical investment literature. As mentioned earlier, this is reinforced by Hayashi’s (1982) well-known result, which states that as long as the production function and the adjustment cost function are homogeneous of degree one and the firm is a price taker, the shadow (marginal) price of capital equals the average price of capital or the firm’s market value (Tobin’s $q$). In particular, a convenient parametric specification for the unit installation cost function, which is often adopted in the relevant empirical literature, is given by

$$\phi \left( \frac{I}{K} \right) = \frac{\phi}{2} \left( \frac{I}{K} \right).$$

Consequently, after using the approximation $\ln(q) \approx (q - 1)$, which holds for values of $q$ close to unity, and, taking first differences, the first-order condition (3b)’ can be parameterized to the following empirical specification:

$$\Delta \left( \frac{I_t}{K_{t-1}} \right) = \left( \frac{1}{\varphi} \right) \Delta \ln(q_t) + \epsilon_t,$$

where $E[\epsilon_t] = 0$, $E[\epsilon_t \epsilon'_t] = \Sigma$, and $\Sigma$ is unrestricted; that is, the disturbances may be autocorrelated and/or heteroscedastic (time subscripts for the remaining variables are omitted for simplicity). This empirical specification for ‘new’ capital is in accordance with the majority of related empirical relationships, which have proved unsuccessful in identifying a robust association between investment and $q$. 
Another look at the linear \( q \) model

Regarding the empirical specification for maintenance, a functional form for the depreciation equation in terms of maintenance expenditures is required. To this end, an exponential specification is postulated of the form \( \delta (M/K) = \exp [-\gamma (M/K)] \), where \( \gamma > 0 \). Accordingly, the first-order condition (3c) can be parameterized to the empirical specification:

\[
\Delta \left( \frac{M_t}{K_{t-1}} \right) = \left( \frac{1}{\gamma} \right) \Delta \ln(q_t) + u_t, \tag{7}
\]

where \( E[u_t] = 0 \), \( E[u_t, u_t'] = \Sigma \), and \( \Sigma \) is unrestricted. Following this approach, the reduced-form investment-\( q \) equations based on (6) and (7) will now include separately the two components of capital expenditures as independent variables.\(^{12}\)

Regarding the determination of \( q \) in equations (6) and (7), the value of marginal \( q \) for Canadian firms was proxied under the assumptions of the model by two measures to ensure the robustness of the results to alternative definitions of \( q \). The first measure is average \( q \) for private non-financial corporations calculated from balance-sheet data (an analytical description of the calculation can be found in appendix B). As shown earlier, this approximation is justified theoretically by the set-up developed in section 2 that meets the requirements of Hayashi (1982) on the equality of marginal and average \( q \). The second proxy for \( q \) is given by real stock prices. Several studies have stressed that real stock prices closely follow estimates of marginal \( q \) and yield similar estimates in empirical investment equations.\(^{13}\) In addition, the choice of real stock price changes, as a proxy for \( q \) is dictated by the study of Barro (1990) for Canadian investment, where both Canadian and U.S. real stock prices are used as proxies for the corresponding \( q \) measures. Therefore, the results with Canadian real stock prices presented here are comparable to Barro’s (1990) evidence.\(^{14}\)

A final remark at this stage regards the estimation method. Consistent identification of the structural parameters in models (6) and (7) clearly depends on whether the errors \( e_t \) and \( u_t \) are uncorrelated with the measure of \( q \) utilized. To circumvent related econometric pitfalls we utilize a number of observable variables as instruments, such as lagged values of \( q \), Canadian and U.S. business profits, U.S. real stock prices, and U.S. investment. These instrument sets are employed in a data-oriented second-order GMM estimation framework.\(^{15}\)

---

12 Given that these two equations correspond to the first-order conditions of the firm’s problem, they can be viewed as long-run specifications for determining capital expenditures with \( q \) incorporating all future information. In contrast, models with, say, cash flows could be treated as short-run specifications.

13 Notice that, as pointed out by a referee, when deviations between the stock market and fundamental values are both highly persistent and correlated with the true value of the firm, the semi-log transformation will be accurate in identifying the structural parameters of the investment equation only at low levels of adjustment costs.

14 In fact, Barro (1990) also shows that the rate of the return of the U.S. stock market explains U.S. investment better than the rate of change in \( q \). This finding is also confirmed by Blanchard, Rhee, and Summers (1993).

15 Some preliminary tests indicated that the residuals have an autocorrelated structure of order one or two in some regressions; this should be expected, given the time series nature of the data at
absence of any formal test on weak identification in non-linear GMM estimation, the insensitivity of the results to a combination of instrument sets should be regarded as an indication of inference robustness. In turn, the correlation of the instruments with the error term is investigated with the standard $J$-test of overidentifying restrictions.

4.2. Estimates with ‘new’ capital and repair expenditures
The upper part of table 2 contains the estimation results for equation (6). As can be readily seen, the coefficient estimates for the two measures of $q$ (average $q$ and real share prices) are very close across the various instrument sets, indicating that the choice of instruments does not play a substantial role in the robustness of the empirical findings. The point estimates for the log changes in average $q$ range from 0.042 to 0.048 and are on average smaller than those derived using real stock price changes as the dependent variable (which range from 0.042 to 0.069). All estimates are significant at the 1% significance level, but those with larger values have relatively larger standard errors (e.g., those from models I, IV, and V with real stock price changes), which leaves some room in favour of the relatively smaller estimates. To sum up, these estimates imply that a rise in $q$ by 10% would raise ‘new’ investment as percentage of the capital stock by 0.4 to 0.7 percentage points.

An interesting comparison with earlier findings can be made in terms of the structural parameters of the model and, in particular, in terms of the magnitude of adjustment costs implied by the estimates of table 2. Previous studies have often been criticized for yielding implausibly high adjustment costs; for instance, Abel and Eberly (2002) found that in the linear model adjustment costs vary between 59% and 545% for the U.S. manufacturing sector, and they stress that this wide range is a typical finding in the relevant literature. Here, some indirect evidence on the magnitude of these costs could be obtained by the coefficient on the growth rate of $q$, which gives the inverse of the parameter in the adjustment cost function. The adjustment cost for ‘new’ investment as a share of total ‘new’ investment found from the various regressions ranges between 44.1% and 71.9%. These figures are far lower and intuitively more plausible than those reported in previous empirical studies of the linear investment model.

Estimating equations (6) and (7) jointly did not substantially affect the coefficient estimates and yielded – as expected – slightly smaller standard errors. These results are available upon request.

Adjustment costs as a ratio to ‘new’ capital expenditures are then $(\phi/2)(I/K)^2$. These values are obtained by noting that the total amount of adjustment costs is $(\phi/2)(I/K)^2$. Adjustment costs as a ratio to ‘new’ capital expenditures are then $(\phi/2)(I/K)$, where the ‘new’ capital expenditures to the capital stock ratio is evaluated at its mean, which was 6.1% for the period 1956–93.
TABLE 2
GMM results for business expenditures in ‘new’ capital

<table>
<thead>
<tr>
<th>Model</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dependent variable: differenced ‘new’ capital expenditures in business enterprises as percentage of previous period-end business capital stock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C$</td>
<td>$-0.024$</td>
<td>$0.012$</td>
<td>$-0.156$</td>
<td>$0.009$</td>
<td>$-0.172$</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.143)</td>
<td>(0.078)</td>
<td>(0.136)</td>
<td>(0.074)</td>
<td>(0.133)</td>
</tr>
<tr>
<td>$\Delta \log [q_t(-1)]^a$</td>
<td>$0.046^\ast\ast$</td>
<td>$0.047^\ast\ast$</td>
<td>$0.048^\ast\ast$</td>
<td>$0.042^\ast\ast$</td>
<td>$0.044^\ast\ast$</td>
<td>$0.044^\ast\ast$</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.013)</td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$GRSH(-1)^b$</td>
<td>$-0.069^\ast\ast$</td>
<td>$0.060^\ast\ast$</td>
<td>$0.064^\ast\ast$</td>
<td>$0.057^\ast\ast$</td>
<td>$0.042^\ast\ast$</td>
<td>$0.058^\ast\ast$</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.017)</td>
<td>(0.018)</td>
<td>(0.020)</td>
<td>(0.014)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Specification tests</td>
<td>$J$ test$^a$</td>
<td>$0.127$</td>
<td>$0.243$</td>
<td>$0.089$</td>
<td>$0.583$</td>
<td>$0.189$</td>
</tr>
<tr>
<td></td>
<td>$-0.001$</td>
<td>$-0.030$</td>
<td>$-0.009$</td>
<td>$-0.027$</td>
<td>$-0.002$</td>
<td>$-0.023$</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.020)</td>
<td>(0.014)</td>
<td>(0.021)</td>
<td>(0.012)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Specification tests</td>
<td>$J$ test$^a$</td>
<td>$0.175$</td>
<td>$0.153$</td>
<td>$0.519$</td>
<td>$0.889$</td>
<td>$0.321$</td>
</tr>
</tbody>
</table>

NOTES: $^a$$\Delta \log [q_t(-1)]$ denotes the log difference of average $q$ (described in appendix B), and $GRSH$ denotes the growth rate of real stock price changes. $^b$The values reported in the $J$-test are the probability values of the corresponding test of over-identifying restrictions. $^c$Canada business profits are expressed as percentage of the business capital stock. US investment and US business profits are expressed as a percentage of U.S. Gross National Income.
Turning to the estimates corresponding to equation (7), we see that the lower part of table 2 contains the empirical results for the same instrument sets as in the upper part of the table (with the exception of the change in the lagged dependent variable). All estimates on both measures on $q$ are statistically significant at the 1% level, but the estimates derived from average $q$ are somewhat lower than those derived from real stock returns. The most interesting feature of these results is the interpretation of the coefficient on $q$ in terms of the structural parameter $\gamma$ that appears in the depreciation function and measures the sensitivity of the depreciation rate with respect to changes in the rate of maintenance. According to the results, a plausible value for $\gamma$ is in the vicinity of 1.5 (when the depreciation and the maintenance rate are expressed in percentage terms). This implies that a marginal rise in aggregate maintenance expenditures by one-tenth of a percentage point at its mean level (from 2.25% to 2.35%) will lower the depreciation rate of the aggregate capital stock in the business sector from 3.42% to 2.95%. Stated otherwise, a 10% increase of maintenance expenditures evaluated around its mean level would reduce the rate of depreciation by 0.98%. This figure is remarkably close to that obtained by Nelson and Câputo (1997) for the aircraft industry and, in general, supports the hypothesis that the depreciation rate is an endogenous variable with a strongly time-varying profile.

In fact, the structural estimates for $\gamma$ allow a quantification of the unobservable depreciation function. To illustrate the implications of aggregate maintenance expenditure for the depreciation rate with respect to $\gamma$ and its standard error, the estimates of table 2 were simulated to yield an approximation of the depreciation function and a one standard error confidence interval. Figure 2 plots the relevant curves and it is evident that the depreciation function depends strongly upon the ratio of maintenance expenditures to the capital stock. Taking into account that over the period under consideration maintenance as a percentage of the capital stock has varied between 0.8% and 2.8%, we see that the depreciation rate is estimated to have moved in a range of 20 percentage points. Hence, albeit the analysis developed here is purely indicative, the general image supports the view that treating depreciation as an exogenous and technically determined variable may have led to substantial mismeasurements of the capital stock in the context of empirical treatments regarding its durability and evolution.

5. Robustness and extensions

The previous section has established that $q$ is a significant determinant of ‘new’ capital and maintenance business expenditures in Canada. However, it is known...
Another look at the linear $q$ model

from section 2 that strong restrictions on adjustment costs and stock market efficiency are required for average $q$ to be a sufficient statistic for marginal $q$ and investment. Therefore, the current section empirically extends the core model tested in section 4 to allow for some sensitivity tests by including additional explanatory variables in the basic equation. We stress that these tests serve as robustness tests of the parameters on $q$, rather than as general-to-specific methodologies that nest the linear $q$-model. Furthermore, ‘new’ capital and repair expenditures are disaggregated into construction and machinery-equipment expenditures in order to offer a broader picture of the quantitative impact of $q$ on investment under capital heterogeneity.

5.1. Sensitivity tests
To investigate the robustness of the above results to modifications of the model, table 3 presents some additional sensitivity tests of the $q$ model. In particular, the model is first extended to include U.S. real stock price changes as an explanatory variable. This approach is close to the spirit of Barro’s (1990) study on the determination of Canadian investment, which found that U.S. real stock price changes are, on the whole, a better predictor of Canada’s real fixed non-residential private investment than Canadian real stock prices. Moreover, both
### TABLE 3
Sensitivity tests

<table>
<thead>
<tr>
<th>Model</th>
<th>With U.S. stock prices</th>
<th>With quadratic q</th>
<th>With stock market uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dependent variable: differenced ‘new’ capital expenditures in business enterprises as a percentage of previous period-end business capital stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.093 (0.109)</td>
<td>0.018 (0.077)</td>
<td>0.773 (0.154)</td>
</tr>
<tr>
<td>(\Delta \log {q(-1)}_a)</td>
<td>0.067** (0.017)</td>
<td>0.072* (0.031)</td>
<td>0.029* (0.013)</td>
</tr>
<tr>
<td>(\text{GRSH}(-1)_b)</td>
<td>–0.031** (0.006)</td>
<td>–0.011 (0.012)</td>
<td>–0.299 (2.320)</td>
</tr>
<tr>
<td>(\text{USGRSH}(-1)_b)</td>
<td>–0.004 (0.002)</td>
<td>–0.000 (0.002)</td>
<td>–0.015 (0.293)</td>
</tr>
<tr>
<td>(\text{UNCERT}_b)</td>
<td>–0.004 (0.002)</td>
<td>–0.000 (0.002)</td>
<td>–0.015 (0.293)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specification tests</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(J)-test(^b)</td>
<td>0.543 (0.018)</td>
<td>0.918 (0.002)</td>
<td>0.000 (0.002)</td>
</tr>
<tr>
<td>(\text{GRSH}(-1)_b)</td>
<td>0.014** (0.005)</td>
<td>0.005* (0.002)</td>
<td>0.005* (0.002)</td>
</tr>
<tr>
<td>(\text{USGRSH}(-1)_b)</td>
<td>–0.004 (0.002)</td>
<td>–0.000 (0.002)</td>
<td>–0.015 (0.293)</td>
</tr>
<tr>
<td>(\text{UNCERT}_b)</td>
<td>–0.004 (0.002)</td>
<td>–0.000 (0.002)</td>
<td>–0.015 (0.293)</td>
</tr>
</tbody>
</table>

(continued)
Another look at the linear q model

<table>
<thead>
<tr>
<th>Model</th>
<th>With U.S. stock prices</th>
<th>With quadratic q</th>
<th>With stock market uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification tests</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J-test*0.521</td>
<td>0.492</td>
<td>0.637</td>
<td>0.393</td>
</tr>
<tr>
<td>List of instruments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q measure (−3),</td>
<td>q measure (−3),</td>
<td>q measure (−3),</td>
<td>q measure (−3),</td>
</tr>
<tr>
<td>US real stock</td>
<td>US real stock</td>
<td>q measure (−3),</td>
<td>q measure (−3),</td>
</tr>
<tr>
<td>prices (−2),</td>
<td>prices (−2), depend.</td>
<td>squared,</td>
<td>squared,</td>
</tr>
<tr>
<td>Can. bus. profits (−2)</td>
<td>variable (−2), Can.</td>
<td>dependent variable (−2)</td>
<td>conditional standard deviation of stock returns (−1), dependent variable (−2)</td>
</tr>
<tr>
<td>(−3),</td>
<td>bus. profits (−2) and (−3),</td>
<td>US investment (−2)</td>
<td>US investment (−2)</td>
</tr>
<tr>
<td>US investment (−2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTES: *See table 2 for definitions of variables and tests.

USGRSH denotes the growth rate of U.S. stock returns. UNCERT denotes the conditional standard deviation from a GARCH (1,1) model for Canadian stock returns.
markets were found to be only weakly related to Canadian investment over the post-war period. Barro (1990) attributed this finding to a manifestation of the classic example of misalignment between marginal and average \( q \) when variations in the relative price of energy drive the values of an additional capital unit and the existing capital stock in opposite directions. According to the author, given that the Canadian economy is strongly dependent on natural resources and energy, a rising wedge between marginal and average \( q \) for Canadian firms in the presence of such variations might thus yield a more significant coefficient for U.S. real stock prices.

The left part of table 3 contains the estimates for ‘new’ investment (upper part) and maintenance expenditures (lower part) with the addition of U.S. real stock prices. The coefficients on both measures of \( q \) remain significant for two alternative sets of instruments chosen. The coefficients for ‘new’ capital expenditures (left part) with the measure of average \( q \) are again somewhat lower than those obtained with Canadian real stock price changes. On the other hand, the coefficient on U.S. real stock price changes is significant in two cases, but enters with the wrong sign. Likewise, the coefficients for repair expenditures (left side of lower part) are again significant and with values close to those obtained in table 2, whereas the coefficients on U.S. real stock prices are insignificant. In general, the estimates of table 2 are found to be robust to the inclusion of U.S. real stock prices as an independent variable and thus offer an answer to Barro’s (1990) ‘puzzling’ result on the behaviour of Canadian investment with respect to domestic and U.S. developments.

Another strand of the literature on \( q \) models emphasizes the impact of non-linearities in investment patterns. This point is put forward in a theoretical context by Abel and Eberly (1994), who show that under investment irreversibility and the presence of convex and fixed costs there are regions where investment is insensitive and regions where it is responsive (positively or negatively) to the level of \( q \). In an empirical context, Eberly (1997) examines investment data from 11 countries and finds that non-linearities are important in explaining and predicting firm and aggregate investment rates. Barnett and Sakellaris (1998) find that U.S. investment is estimated to be convex for low values of \( q \) and concave for high values of \( q \). In a similar vein, the evidence of Abel and Eberly (2002) is supportive of non-linearities, which arise from concave adjustment costs and disinvestment at the plant level for low values of \( q \), and leads the authors to underline that these findings have implications for cyclical movements in aggregate investment.

To assess the effects of non-linearities in \( q \) in the present context, a log quadratic term is introduced in the basic specification. The general structure

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21 The results presented here correspond to the two instruments sets that include U.S. variables. Additional tests with alternative instrument sets (available upon request) did not yield substantial differences in the estimates.

22 A similar experiment was performed by including log changes of \( q \) for the United States, as calculated by Blanchard, Rhee, and Summers (1993) for 1900–90. The results were very similar to those reported in the text.
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of the instrument sets remains then identical to those used to generate the results of section 4 with the inclusion of lagged log $q$. In this framework, the coefficient on the squared term measures the extent to which investment responds non-linearly to $q$ and should be zero if adjustment costs do not deviate from the standard symmetric quadratic form. The results are tabulated in the middle part of table 3. The coefficient on the quadratic term is almost always statistically insignificant (with one exception). On the other hand, the coefficients for ‘new’ investment and maintenance expenditures on both measures of $q$ retain their statistical significance and do not deviate from the point estimates reported in section 4.

Finally, another sensitivity test involves the impact of uncertainty on capital expenditures. A number of studies have examined theoretically the effect of uncertain macroeconomic conditions, such as technology, demand, and price uncertainty, on investment; see, among others, Caballero (1991). Empirical studies that have attempted to assess the macroeconomic impact of uncertainty on investment have identified uncertainty, for instance, by excess volatility in GDP (Asteriou and Price 2005), by the risk premium in the term structure of interest rates (Federer 1993), by the response in survey questionnaires regarding expected future demand conditions (Temple, Urge, and Driver 2001), or by the forecasts of analysts regarding future profits (Bond and Cummins 2004).

To explore the impact of uncertainty in the present context, we use excess volatility in Canadian stock returns. To this end, we obtain the conditional standard deviation of stock returns from a GARCH(1,1) model and we use it as an additional variable in the estimated equation. The results are shown in the right side of table 3 and the overall picture does not alter the previous findings. The coefficients on excess volatility are, in general, negative and insignificant (with one exception), whereas the coefficients on ‘new’ investment and maintenance retain their magnitude and statistical significance.

Thus, the overall picture from table 3 corroborates the central findings of the previous section. The addition of U.S. stock market developments, non-linearities, or uncertainty is almost always rejected, whereas the qualitative results for ‘new’ capital and maintenance expenditures are robust across specifications. This evidence suggests that the standard $q$ model with ‘new’ capital and maintenance expenditures under convex adjustments costs for Canadian business firms is insensitive to well-known digressions and provides a compelling strategy for modelling aggregate business capital expenditures.

5.2. Disaggregated estimates for construction and machinery-equipment

A key assumption underlying the estimation of empirical investment functions is that capital can be treated as a homogeneous good. Some authors who have

23 We thank a referee for suggesting this robustness exercise and the use of excess volatility in stock returns as a measure of uncertainty.
24 We also experimented with two additional variables that might provide useful information in the present context: real interest rates and business profits. Including these variables in the linear $q$ model did not substantially affect the magnitude or the significance of the estimated coefficients on $q$, whereas the estimates on the two variables were found to be insignificant.
relaxed this assumption (e.g., Abel and Eberly 2002) claim that capital heterogeneity may lead to a mismeasurement of the relationship between the various forms of capital and $q$. In an empirical context, Oliner, Rudebusch, and Sichel (1995) have found that investment models for structures perform worse than the corresponding ones for equipment, whereas Bontempi et al. (2004) show that the standard convex costs model performs well for equipment, but not for structures where evidence of non-convex adjustment costs is found.

An interesting extension of the relationship between ‘new’ investment, maintenance, and $q$, therefore, might involve the discrimination between the various forms of these expenditures by type of asset. Fortunately, the Canadian Survey also provides data on capital spending in ‘new’ investment and maintenance disaggregated to expenditure in construction and machinery-equipment (see section 3 for a brief description of the data at the disaggregated level). Here, since shocks in ‘new’ investment and maintenance are likely to affect both components (construction and machinery-equipment), system equations are estimated using average $q$ as the dependent variable to gauge any differential relationship of $q$ with the components of capital spending.

The point estimates and the standard errors are reported in table 4. The upper part of table 4 contains the estimates for ‘new’ investment, where the various models correspond to alternative instrument sets following the reasoning of section 3. The constant term is almost everywhere statistically insignificant, but, more important, the coefficients measuring the impact of $q$ on ‘new’ investment in construction and machinery-equipment differ largely. In particular, the coefficients for ‘new’ investment in construction are smaller and statistically insignificant in four out of six specifications, whereas ‘new’ investment in machinery and equipment is more sensitive to movements in $q$. The point estimate of the coefficient is always statistically significant at the 1% level and is three to six times larger than the coefficient for construction. A similar picture emerges from the estimates on disaggregated maintenance expenditures. The coefficients of $q$ in the equations with machinery-equipment are five to ten times larger than those for construction and are significant at the 1% level.

So, it seems that the impact of $q$ operates on investment mainly through spending in machinery and equipment and to a much lesser extent through construction. It should be noticed that in the absence of a formal theoretical context, like that of section 2, for the aggregate capital stock, the coefficients on $q$ in the disaggregated capital spending equations do not bear the previous interpretation in terms of the structural parameters for the adjustment cost and the depreciation functions, but reflect simple reduced-form empirical relationships aiming at investigating if (and by how much) the two components of capital spending respond differently to variations in $q$.\footnote{Wildasin (1984) shows that in the many-capital-goods case, the investment equations cannot, in general, be inverted to obtain monotonic relations with respect to $q$ and analyses the strict conditions that should hold for adjustment cost functions in order for the investment equations to be uniquely determined by a $q$-type variable.} If we keep this in mind, two potential explanations that are related to the current set-up could be in line here. First,
Another look at the linear q model

### TABLE 4
System GMM results for business expenditures in construction and machinery-equipment

<table>
<thead>
<tr>
<th>System</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Investment type</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>-0.064</td>
<td>0.100</td>
<td>-0.012</td>
<td>0.225*</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.050)</td>
<td>(0.118)</td>
<td>(0.058)</td>
<td>(0.109)</td>
<td>(0.044)</td>
</tr>
<tr>
<td></td>
<td>Δlog[q,(-1)]</td>
<td>0.024*</td>
<td>0.081**</td>
<td>0.018</td>
<td>0.073**</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.010)</td>
<td>(0.016)</td>
<td>(0.009)</td>
<td>(0.017)</td>
<td>(0.007)</td>
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<td>Specification tests</td>
<td>J-test</td>
<td>0.116</td>
<td>0.221</td>
<td>0.360</td>
<td>0.237</td>
</tr>
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<tr>
<td></td>
<td>Specification tests</td>
<td>J-test</td>
<td>0.433</td>
<td>0.427</td>
<td>0.583</td>
<td>0.632</td>
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<td>List of instruments</td>
<td>q,(-3), Can. bus. profits (-2) and (-3)</td>
<td>q,(-3), dependent variable (-2), Can. bus. profits (-2)</td>
<td>q,(-3), dependent variable (-2), Can. bus. profits (-2) and (-3)</td>
<td>q,(-3), Can. bus. profits (-2) and (-3), US real stock prices (-2)</td>
<td>q,(-3), dependent variable (-2), Can. bus. profits (-2) and (-3), US real stock prices (-2)</td>
</tr>
<tr>
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<td></td>
<td>q,(-3), Can. bus. profits (-2)</td>
<td>q,(-3), dependent variable (-2), Can. bus. profits (-2) and (-3)</td>
<td>q,(-3), Can. bus. profits (-2) and (-3), US real stock prices (-2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTES: 1) See table 2 for definitions of variables and tests. 2) Column A denotes investment in equipment, and column B denotes investment in structures.
adjustment costs for construction are different (and probably much larger) than those for machinery-equipment and may have affected the estimated coefficients in equations for ‘new’ investment. Cummins, Hassett, and Hubbard (1994) report marginal adjustment costs of 30% for an extra dollar of equipment investment. Even though no estimates for construction investment are provided, it is likely that these adjustment costs are generally larger for construction, owing to the nature of these works, which mainly involve installation costs of new plants, relocation costs of existing ones, and so on and may have led to a wedge in the estimated impact of $q$. Second, the structure of the depreciation function may be a potential cause for the differential estimates in maintenance expenditures. As expected, the various categories of business assets in machinery and equipment are found to exhibit much larger depreciation rates than those in non-residential construction. Here, it is hard to reconcile the finding of larger coefficients in machinery-equipment with the larger depreciation rates in absolute terms generally observed in the sector. A potential explanation is that maintenance expenditures are much lower as a share of the capital stock in construction, which may mitigate or fully outweigh the impact through the depreciation function and thus justify both the differential estimates and the lower (higher) depreciation rates in construction (machinery-equipment).

In the absence of a theoretical framework that captures the firm’s optimal decision in spending in the various types of capital (either in the form of ‘new’ investment or of maintenance) it is hard to interpret these findings in a more structural manner. Still, a clear conclusion that emerges from these results is that many previous empirical studies on aggregate investment that investigated the linear $q$ model have been carried out at an inappropriate level of aggregation. There is no a priori reason to expect marginal or average $q$ to carry the same information for ‘new’ investment and maintenance expenditures in construction and machinery-equipment, respectively. Addressing the heterogeneous effect of $q$ on investment components therefore seems an interesting route for further research.

6. Conclusions and policy implications

This paper has developed an optimizing model of business investment with convex adjustment costs where maintenance expenditures affect the depreciation rate of capital. The model led to equations for ‘new’ capital and maintenance expenditures containing $q$ as the single dependent variable. When the central predictions of the model were tested on survey data from the Canadian business sector, the $q$ variable was found to be statistically significant in explaining both types of capital expenditures. The results yielded plausible estimates of the adjustment

26 See, for example, Jorgenson (1996). This general finding is confirmed by Koumanakos and Hwang (1998, table A9) for the Canadian industry sectors.
Another look at the linear $q$ model

costs for ‘new’ investment expenditures and of the depreciation function with respect to maintenance expenditures. The empirical findings were robust to modifications suggested in the existing literature of business investment. When a less theory-based approach was implemented by considering the decomposition of aggregate ‘new’ investment and maintenance expenditures into construction and machinery-equipment, $q$ was found to impact significantly only on machinery and equipment capital expenditures.

The message of the paper is that the $q$ theory may be able to deliver significant results if aggregate spending on capital is measured more accurately, so that it tends to reflect all expenditures leading to an increase in the capital stock either through the formation of ‘new’ capital or through the preservation of the existing capital stock. In addition, a distinction should be made between expenditures targeting construction and machinery-equipment, as these two forms seem to embed different information related to $q$.

Some last remarks regard the limitations of the current approach. First, given the nature of spending in ‘new’ capital and maintenance, a more direct comparison (like forecasting investment spending) between the approach adopted here and other investment models is not feasible, as there is only partial overlapping of investment as recorded in national accounting or at the firm level and the types of expenditures covered in the Canadian Survey of Capital and Repair Expenditures. For instance, some forms of maintenance traditionally appear under various categories in national accounts expenditure systems, such as repairs or renovations, whereas ‘new’ investment does not correspond to aggregate investment at the national level. Second, at the present stage of data availability the broader cross-country empirical testing of these relationships does not seem possible. With the exception of Canada, there has been no particular interest in systematically collecting data on maintenance expenditures, albeit scarce existing evidence reveals that maintenance in developed economies covers a substantial fraction of total capital expenditures. The lack of data is again largely due to the nature of maintenance expenditures: as there is no market and recorded transactions for maintenance, data collection requires the planning of surveys to obtain accurate measures of maintenance expenditures. To this extent, data collection on this type of expenditure either via a unified questionnaire or by the inclusion of related questions in existing surveys would yield valuable information for the understanding of investment behaviour.

Another important determinant of the depreciation rate that merits further investigation involves the utilization rate of capital. Although casual empiricism suggests that utilization varies along with depreciation, endogenous utilization is not often explored in the context of endogenous determination of depreciation.  

The empirical investigation of the joint determination of depreciation, maintenance expenditures, and capital utilization becomes more complicated in the absence of wide-ranging data on capital utilization.\textsuperscript{28} Hence, another promising route for further research might deal with the empirical in-depth analysis of firm activities involving maintenance expenditures and capital utilization and their implications for the depreciation rate and, consequently, the user cost and measurement of capital.

Finally, we emphasize that the present study has not addressed the important issue of the impact of taxation on ‘new’ investment and maintenance expenditures.\textsuperscript{29} Typically, maintenance expenditures are treated as current operating expenses and can therefore be fully deducted from pre-tax revenues; on the other hand, ‘new’ investment expenditures are only deducted through depreciation allowances (see Mullen and Williams 2004). Incorporating differential forms of capital expenditures’ taxation in the theoretical analysis within a general equilibrium framework could offer new insights for theoretical research on the determination of investment and further improve the empirical performance of the $q$ model.

Appendix A: Solving the $q$ model with maintenance expenditures

Appendix A solves the model developed in section 2. To this end, a parametric specification for the cost of adjustment function is adopted. A standard specification is given by the linear relationship $\varphi(I/K) = \varphi/2 \cdot (I/K)$. Consequently, equation (6) is parameterized to

$$\frac{I}{K} = \frac{(q - 1)}{\varphi}. \quad (A1)$$

Now, adopting the parametric specification $\delta(M/K) = \exp[-\gamma (M/K)]$ for the depreciation function and substituting the latter along with (A1) and the parametric version of (3c) into (3d), one gets after some manipulation

$$\frac{\dot{q}}{q} = \left[ r + \left( \frac{1}{\gamma q} \right) \right] - \left( \frac{1}{\gamma q} \right) \ln \left( \frac{1}{\gamma q} \right) - \left( \frac{f_K(K, L)}{q} \right) - \frac{(q - 1)^2}{2\varphi q}. \quad (A2)$$

\textsuperscript{28} Shapiro (1986) emphasizes the spurious correlation between capacity utilization and capital utilization and the difficulties associated with the measurement or construction of the latter.

\textsuperscript{29} We thank a referee for pointing out the taxation effect on the various forms of capital expenditures.
In a similar vein, the capital accumulation equation is modified to

\[ \dot{K} = \left[ \frac{q - 1}{\varphi} - \frac{1}{\gamma q} \right] K. \] (A3)

Equations (A2) and (A3) give a system of two differential equations that can be solved for the steady-state values of \( q \) and \( K \). More specifically, the \( q = 0 \) locus is downward sloping at the steady-state value of \( q \); its slope equals \( dq/dK|_{q=0} = f_{xK}(K, L)/[r + (1/\gamma q) - (q - 1/\varphi)] < 0 \), as the denominator is positive at the steady-state value implied by (A3). The steady-state value of \( q \) is then given by the positive root of the quadratic equation associated with (A3) at \( K = 0 \):

\[ q|_{K=0} = \frac{1 + \sqrt{1 + 4\varphi/\gamma}}{2}. \] (A4)

As displayed in figure A1, the \( K = 0 \) locus is a horizontal line in the phase plane. The arrows of motion show the steady-state properties of the system, and it can be easily established that the linearization around equilibrium involves a saddle path. In the original Solow model without maintenance expenditures, for low values of the capital stock \( K \) the shadow price of capital needs to be higher than its steady-state value, in order to boost capital formation and bring the economy to equilibrium. Here, the steady-state value of \( q \) is attained by a simultaneous rise in ‘new’ investment and a fall in the depreciation rate as a result of increased maintenance expenditures, both of which are triggered by the rise in \( q \). The equilibrium value of \( q \) is higher than the one implied by the Solow model without maintenance expenditure, because under the impact of maintenance expenditures as a ratio of the capital stock on the depreciation rate a higher capital stock implies a higher depreciation rate, and consequently the marginal value of capital should exceed its replacement cost as ‘new’ investment involving extra adjustment costs replaces worn-out capital.

Appendix B: Calculation of average \( q \) for Canadian private firms

Appendix B presents the calculation of approximating aggregate \( q \) for Canadian private non-financial corporations. The standard definition of average \( q \) is given by the following formula (see, e.g., Lindenberg and Ross 1981):

\[ q = \frac{\text{replacement value of the firm}}{\text{replacement value of assets}} = \frac{\text{market value (equity + debt + preferred stock)}}{\text{replacement value (plant + equipment + inventories)}} \]
Owing to data availability, the adopted definition of average $q$ in the current paper follows the approximation given by equation (4) in Perfect and Wiles (1994):

$$q_s = \frac{(MVE + DEBT)}{TVA},$$

where $MVE$ denotes the sum of the market value of outstanding common shares and the liquidating value of outstanding preferred stock, $DEBT$ denotes the year-end book value of short-term and long-term debt, and $TVA$ denotes the book value of total assets. Chung and Pruitt (1994) show that equation (B1) provides a satisfying proxy for use in empirical applications of the theoretically correct formula by Lindenberg and Ross (1981).

Here, $q_s$ is calculated for private non-financial corporations. In particular, the calculation of $MVE$ is obtained by first estimating the market value of corporations’ equity by dividing dividend payments by the dividend yield: the sum of dividends paid to Canadian residents [D16461] and to non-residents [D16458] is divided by the average annual composite stock dividend yield of the Toronto stock exchange [B4245]. The obtained equity value of corporations is then multiplied by the market value of non-financial corporations divided by the total market value. These data were obtained by Datastream codes TOTLICN and TOTMKCN, respectively, and are available from 1973 onward; values for the missing years 1956–72 were then generated by backward extrapolation. As a final step, these figures were multiplied by the current value of non-financial government enterprises (Datastream code CN160144) divided by the current value of non-financial corporations to yield an estimate of the equity value of private non-financial corporations.

Balance sheet data for the calculation of $DEBT$ and $TVA$ are obtained through the balance-sheet series for private non-financial corporations available from Canadian Statistics. Regarding the calculation of $DEBT$, following Blanchard, Rhee, and Summers (1993) we first net out short-term credit from non-interest-bearing assets to obtain the book value of short-term debt. Short-term credit is given by the sum of trade accounts payable [D163116] and other short-term paper [D163120], and non-interest-bearing assets are given by currency and bank deposits [D163093], Deposits in other institutions [D163094], Foreign currency deposits [D163095] and trade accounts receivable [D163098]. Long-term debt is given by the sum of bank loans [D163118], other loans [D163119], mortgages [D163121], other Canadian bonds [D163123], and corporate claims [D163125]. $TVA$ is given by total assets [D163082]. All the aforementioned series are available for the post-1960 period for which balance sheets of private non-financial corporations are available; values for the missing years 1956–60 were then generated by backward extrapolation.
Another look at the linear $q$ model

\[ q = \bullet q \]

\[ K = \bullet K^* \]

FIGURE A1 Dynamics of $q$ and $K$ with maintenance expenditures

Data appendix

The first part of the data appendix gives the sources for capital and repair expenditures from the Canadian Survey on Capital and Repair Expenditures of Statistics Canada (for a general description of these variables, see section 3).

1) Capital and repair expenditures by business enterprises: variable D843800.
2) Capital expenditures by business enterprises: variable D842986.
3) Capital expenditures by business enterprises in construction: variable D842987.
4) Capital expenditures by business enterprises in machinery and equipment: variable D842988.
5) Repair expenditures by business enterprises: variable D843801.
6) Repair expenditures by business enterprises in construction: variable D843802.
7) Repair expenditures by business enterprises in machinery and equipment: variable D843803.
8) Total capital and repair expenditures: variables (D843800 + D843804 + D843829).
9) Total capital expenditures: variables (D842986 + D842989 + D844014).
10) Total repair expenditures: variables (D843801 + D843805 + D843830).

The second part of the data appendix describes the rest of the Canadian variables.
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2) *Business end capital stock in construction*: Business sector end-year gross fixed non-residential capital stock in building and engineering construction (source: Statistics Canada, variables v1408306 and v1408307, table 031-0002, current prices).

3) *Business end capital stock in machinery and equipment*: Business sector end-year gross fixed non-residential capital stock in machinery and equipment (source: Statistics Canada, variables v1408308, table 031-0002, current prices).


The third part of the data appendix describes the U.S. variables.


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