Auctions

Costas Courcoubetis
George D. Stamoulis
{courcou,gstamoul}@aueb.gr

Auctions

1. Introduction

What is an auction ?

- A method for allocating scarce resources based on competition

- Bidding mechanism:
  - The seller (auctioneer) defines the auction rules:
    - how the winner is determined
    - how much he must pay
  - Each buyer chooses a bidding strategy

- Basic distinction: single-item vs. multi-item auctions

Examples

- Ancient cases:
  - 500BC: Herodotus mentions about auctions in Babylon
  - Ancient Rome: commercial trading, selling war booty
    - 193 A.D.: auction for the entire empire
- More recent cases: auctions
  - for rare collective items
  - in wholesale markets of fish, flowers, etc.
  - for public contracts
  - in stock market
- Very recent cases: auctions
  - over Internet (E-bay, ONSALE, etc.)
  - for bandwidth (Interxion, Band-X, Bandwidth Market Ltd.), spectrum

An auction is a game

- The auction rules define a game among bidders

- Should use the basic game-theoretic concepts to analyze auctions
  - Define economic objectives of bidders
    - Maximize Net benefit → Maximize (Value – Charge)
  - Try to predict bidding behavior → Look for
    - Dominant strategies
    - Nash equilibria

Dominant strategy

- Best strategy for one player, regardless of how opponents play
- Example: Pay-off matrix of 2-player game
  - Up is the dominant strategy for Row player
  - Knowing this, Column player, will play Left

Column Player

Row Player

	Left	Right
Up	2,3	4,1
Down	1,2	3,5

Nash equilibrium

- Set of strategies from which no player has incentive to deviate
  → Best strategy for individual players,
  if others also maintain their strategies
- Example: The battle of sexes
  - Two pure Nash equilibria
  - No dominant strategies exist

Alice

Bob

	Box	Ballet
Box	2,1	0,0
Ballet	0,0	1,2

An important observation

- The best outcomes are not always Nash equilibria
- Example: Prisoner's dilemma
  - The co-operative outcome
    - Is optimal, but …
    - is not an equilibrium

Prisoner 2

Prisoner 1

	Coop.	Def.
Cooperate	3,3	0,4
Defect	4,0	1,1

Alternatives to auctions

- Negotiate, perform beauty contests or hearings
- Set fixed prices (take it or leave it)

Assume independent buyer valuations, unknown to seller $V = (V_1, \ldots, V_n)$

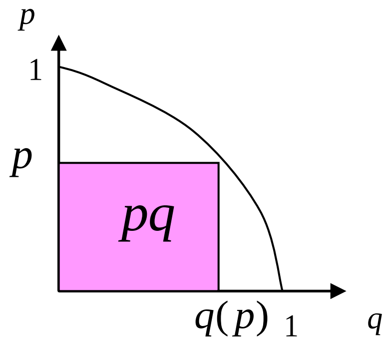
- distributions iid, uniform on [0,1], **known to seller**
- problem: find price that maximizes seller's net profit

$$\Pr\{\max V_i \geq p\} = 1 - \Pr\{V_1 < p, \ldots, V_n < p\} = 1 - p^n$$

Expected quantity sold at price *p* : $q(p) = 1 - p^n$ = Demand function

Expected revenue: $R(p) = q(p)p = p(1 - p^n)$

$R(q) = qp(q)$



Optimal price: $p^*(n) = \sqrt[n]{\dfrac{1}{n+1}}$ $\quad R = p^*(n)\dfrac{n}{n+1}$

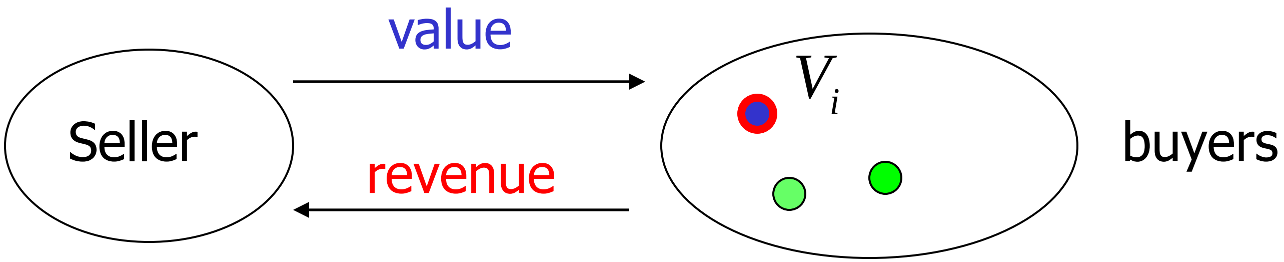
Why use auctions ?

- Useful in the context of incomplete information
  - when selling a commodity of undetermined value
    - value depends on buyer, or actual value found ex-post
  - when no information is available about value distribution (i.e., demand) of buyers

- Auctions offer simplicity, speed of sale, transparency,
- … and reveal information about buyer’s valuation and demand
  - Allocation of good(s) and price(s) are decided by the market
  - … thus preventing dishonest dealing between seller-buyer

Auctions and resource allocation

- An auction is a market mechanism that
  - allocates resources (goods) to buyers
    - generates value for the consumers
  - generates revenue for the seller
    - generates revenue for the producer
- Is used where traditional market mechanisms (e.g. fixed price) can not be used
  - can serve as an internal mechanism



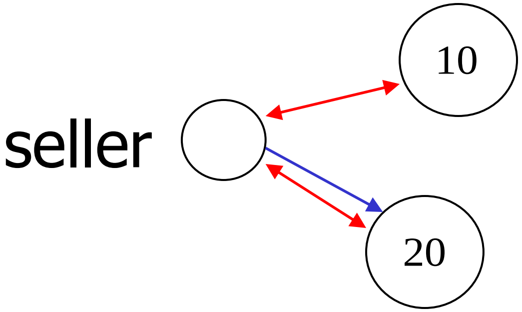
Performance measures

- Wide variety of performance assessment criteria and measures may be used in auction design:
  - social efficiency → maximize the total value to buyers
  - revenue (seller profit)
  - bidder profit
  - time, complexity, susceptibility to collusion
  - promotion of competition
  - In multi-item auctions
    - flexibility: different outcomes for different conditions
    - fairness: target small variation of prices

Why are auctions hard to design?

1. Because the main performance objectives are conflicting with each other

2. Due to lack of information!



- Auction: incentive mechanism
  - buyer: maximizes expected profit
  - seller: maximizes performance measure

Auctions

2. Single-item auctions

Types of single-item auctions

- Also called simple auctions
- Basic types:
  - Open auctions
    - English, with several variations
    - Dutch
  - Sealed-bid auctions
    - First price
    - Second price (Vickrey)
- Rare types:
  - *k*-th price, all-pay,...

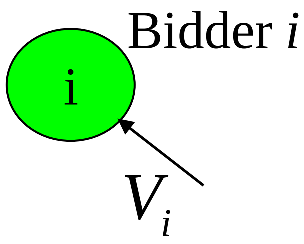
Bidder and seller characteristics

- Valuation
  - private values
  - common values
  - correlation

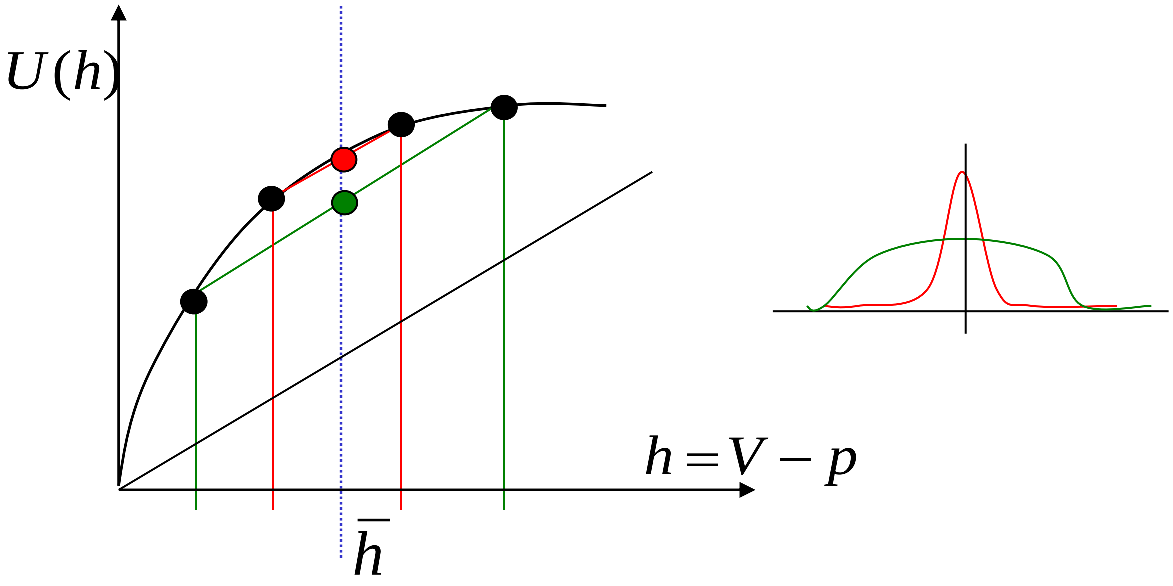
$$U_i = V_i$$

$$U_i = V, V_i = V + x_i$$

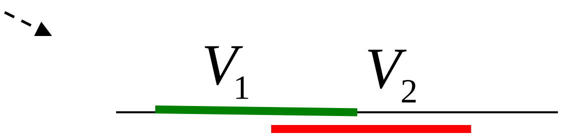
$$U_i = aV_i + b\sum_{j \neq i} V_j$$



- Risk assessment
  - risk neutral
  - risk averse

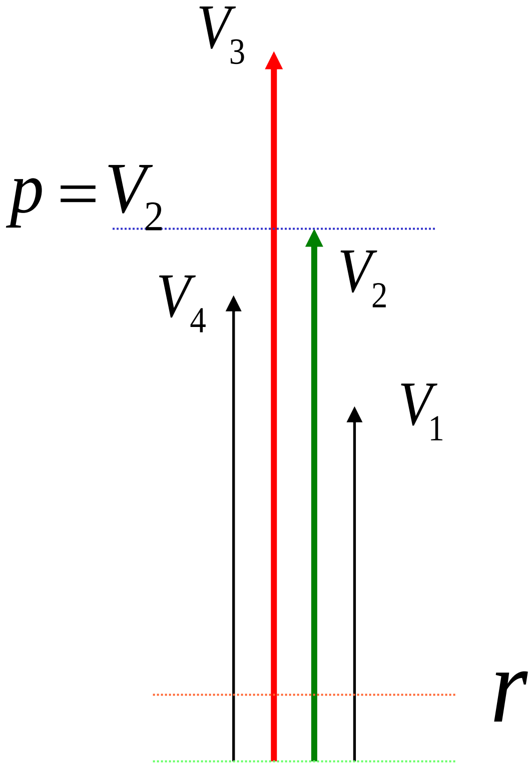


- Symmetry
  - symmetric
  - asymmetric



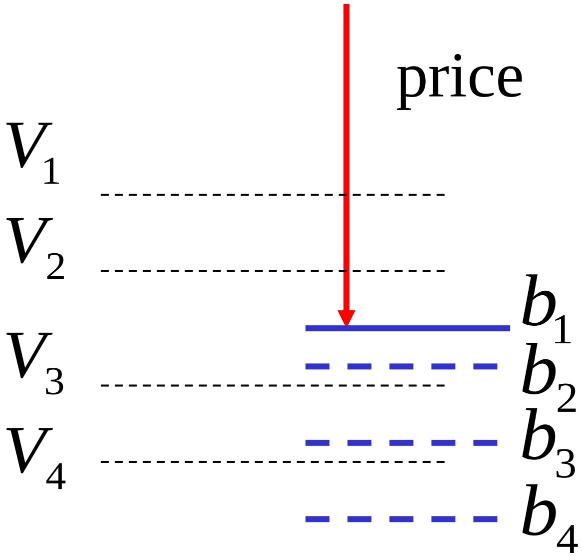
English auction

- Ascending bid, open-outcry
  - other formats: Japanese, ...
- Item is sold at least at the reserve price
- Dominant strategy for bidder
  - bid a small amount more than i the previous high bid until bidder’s valuation is reached, then stop
- For non-private values, auctioneer has great influence
- Most emotional and competitive of auctions
- Much information regarding demand is revealed



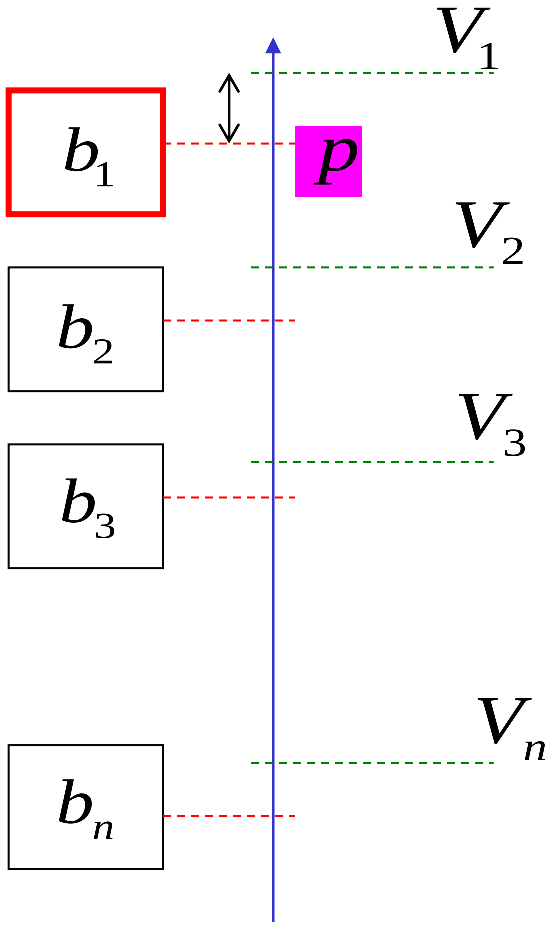
Dutch auction

- Descending price, often by "dutch clock"
- Open-outcry, first price wins
  - Winner → bidder that first shouts "mine"
- Auctioneer usually has no influence
- Little information on demand is revealed



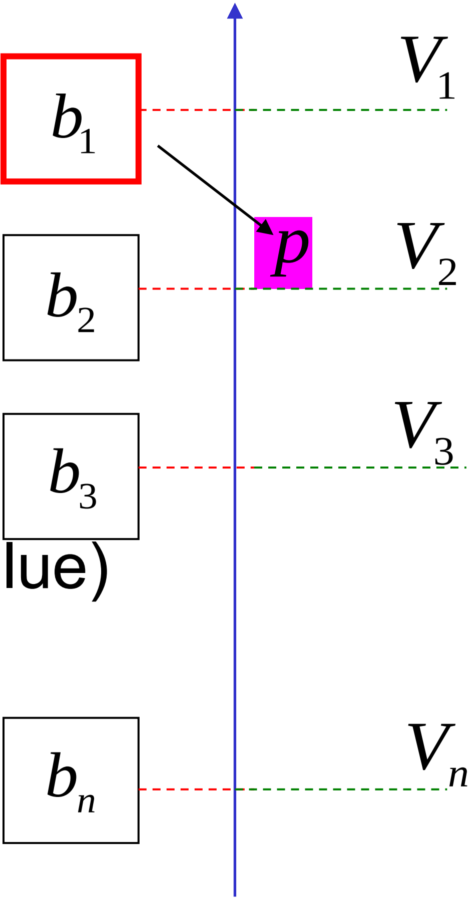
First price sealed-bid auction

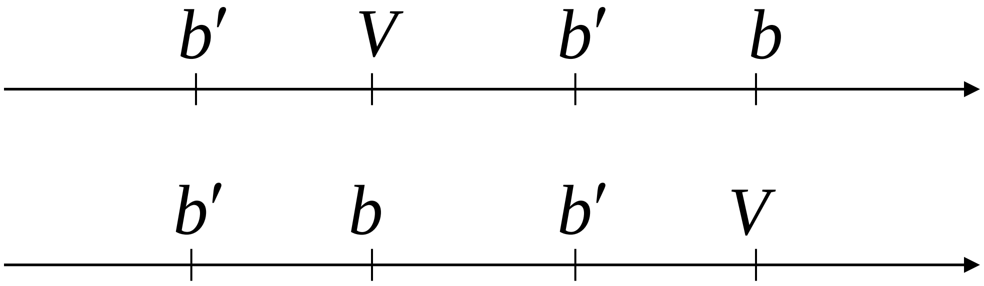
- First price (top bid) wins
- Sealed: each bidder is ignorant of other bids
- Performed in two phases
  1. Bidding, with usually one bid per bidder
  2. Resolution, i.e. winner determination
- Bidder's strategy: shade bids
  - to generate a profit due to pay-your-bid
  - to avoid winner's curse (for common value)
- Little information revealed about demand
- Potential for dishonest bidding



Vickrey (second-price sealed-bid) auction

- Sealed
- Item is awarded to the highest bidder
  - at a price equal to the second highest bid
- Dominant strategy: submit a bid equal to true valuation
  → incentive compatibility;
  guarantees efficiency
  - less fear of winner's curse (for common value)





Basic equivalence results

- Dutch and First price sealed bid auctions:
  - strategically equivalent, i.e. one-to-one mapping of strategies → equivalent payoffs
- English and Vickrey auctions:
  - equivalent in outcome, under private values,
  - … but not strategically equivalent
    - unlike in a sealed bid, in an English auction bidders can respond to rivals' bids
- If Symmetric Independent Private Value (SIPV) model applies, all four basic auctions:
  - achieve efficiency
  - are equivalent in terms of expected revenue

The SIPV model

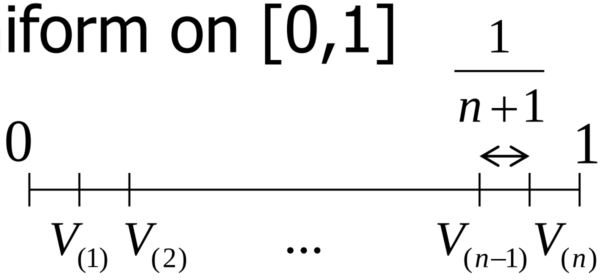
- Assume:
  - a single indivisible object
  - private values
  - all bidders are indistinguishable (symmetry)
  - valuations are independent and identically distributed continuous random variables
  - bidders and seller are risk neutral

- Revenue Equivalence Theorem:
  - *all auctions that: award the item to the highest bidder and lead to the same bidder participation are payoff equivalent*

Applying the RE Theorem (I)

Assume private value case, values iid, uniform on [0,1]

$$V_{(r)} = \max{}^{(r)}\{V_1, \dots, V_n\} \qquad E[V_{(r)}] = \frac{r}{n+1}$$



English (or Vickrey) auction: $b^*(u) = u,\ \bar{p} = \frac{n-1}{n+1}$

=

Dutch (or sealed bid first-price) auction:

$$b^*(u) = au,\ \bar{p} = a\frac{n}{n+1} \Rightarrow a = \frac{n-1}{n}$$

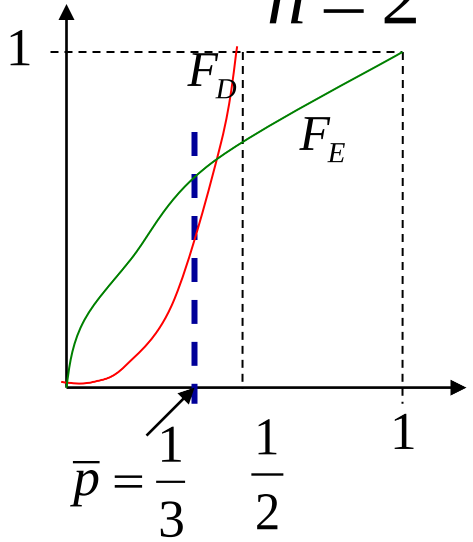
→ Seller's expected gain = $\bar{p}$

Bidder's expected gain = $u^{n-1} \times \frac{u}{n} = \frac{u^n}{n}$

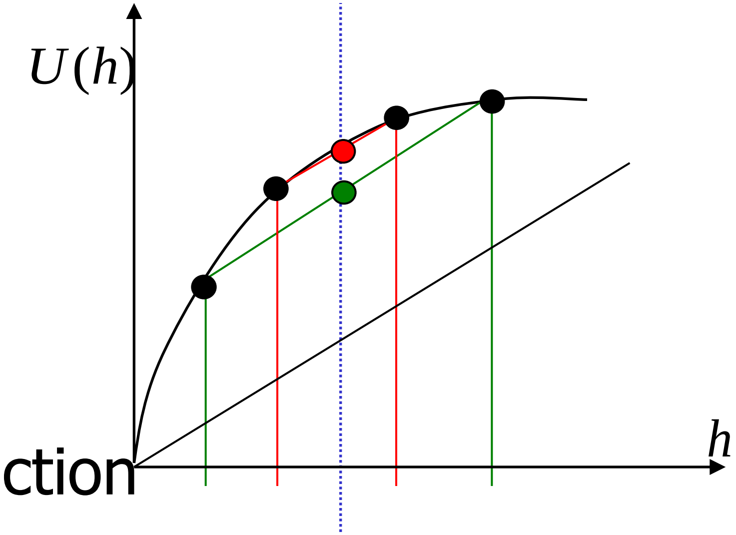
Applying the RE Theorem (II)

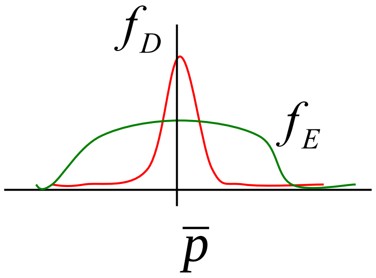
The RE theorem does not imply same price dispersion

$$n = 2$$



$$b_E^*(u) = u,\ \ b_D^*(u) = \frac{n-1}{n}u,\ \ \bar{p} = \frac{n-1}{n+1}$$





- Risk-averse sellers prefer the Duch auction

- Expected revenue (1/3) is lower than under optimal fixed price (0.385) which is based on more information

Procurement auctions

- So far, overviewed auctions for selling a good
- Similar auction types for buying a good, yet the direction of price progress is reversed
  - E.g., in the English auction, a new valid bid should be below the standing bid
- Similar results apply on strategies, equivalence properties etc.

Roadmap

- So far, for single-unit auctions:
  - Defined the basic types
  - Stated and applied Revenue Equivalence Theorem

- Issues to follow:
  - Variations: risk averse bidders, common value auctions, asymmetric cases
  - Improving seller's revenue
  - Collusion

Risk averse bidders

- In a first-price auction, risk averse bidders shade less their bids, thus resulting in
  - higher average prices and seller revenue
  - lower expected bidder profit

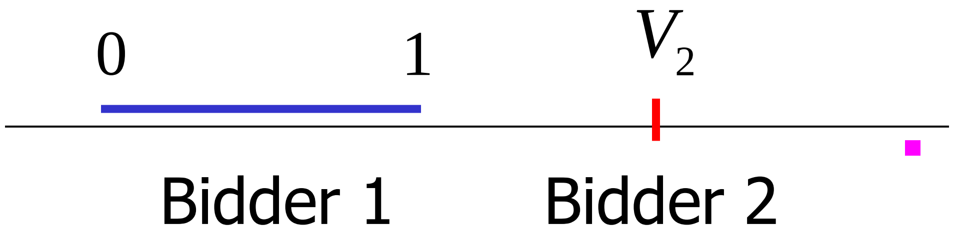
- With risk averse bidders, the seller prefers the Dutch to the English auction
- Revenue equivalence no longer applies

Common value auctions

- Value of bidder is not fixed before the auction
  - True value of item is not known ex-ante, although defined
  - Value is (almost) the same for all bidders
  - Examples: sealed box with coins, oil-lease
- Complex strategies, no general results
- Winner’s curse: the winner discovers that he overestimated the value of the item
- Strategic approach: shade the bid to account for the adverse selection bias

Asymmetric cases

- Different distributions for bidders' valuations
- Revenue equivalence does not apply
- First price auctions are not socially optimal
- Public authorities should use second price auctions for efficiency purposes
  - otherwise, possibility for inefficiency



$$\Pr\{b_1 > b_2\} > 0$$

- e.g., if bidder 1 is truthful, then $b_2 = \min\{1, 1/2 * V_2\}$

Participation fees

- Seller imposes a participation fee *c*

- Buyer participates only if his expected payoff is larger than *c*

- Fiercer competition → higher expected seller revenue

- Positive probability of non-participation
→ not socially optimal

Optimal auctions

- Seller seeks to maximize expected revenue
- Design methodology:
  - If SIPV applies: use standard auctions with a reserve price p_0
    - optimize revenue w.r.t. p_0
  - In the general case: construct incentive compatible mechanisms
    - Make bidders reveal their valuations
    - Item is allocated to bidder with highest marginal revenue
    - Highest bid does not always win → not socially efficient

Comparing auctions

	Risk neutral	**Risk averse**
Private value	equivalent	1. Dutch-FP 2. English-Vickrey
Common value	1. English 2. Vickrey 3. Dutch-FP	?

English and Vickrey auctions:

- best in terms of strategic simplicity
- phantom bid problem

Sealed versus Open format

- Open format:
  - Revelation of information promotes competition
  - More transparent in market determination
- Sealed auctions:
  - Often preferable for private values,
    - collusion members may defect,

  … especially under low competition or asymmetries
  - Vickrey suffers from phantom bids’ problem

Collusion (I)

- Bidders form collusion rings, within which they make agreements to get the item at a lower price:
  1. They select their designated winner
     - the one with the highest valuation
  2. Others promise to follow a specific strategy
     - abstain from bidding

- Which auctions are more vulnerable to collusion than others ?
  - Enforcement issue: incentives for non-winners to keep their promise

Collusion (II)

- First price sealed bid and Dutch auctions: not self-enforcing! no possibility for punishment
  - In FP: agreed winner places bid = $p_{\min} + \varepsilon$, other bidders may abstain or break the ring by bidding slightly higher
  - In Dutch: one of the others may shout “mine” and win!

- English and Second price auctions: self-enforcing!
  - In English: if one of the others bids higher than promised, then the winner may overbid again
  - In SP: winner’s bid = valuation, others’ bid =0

Example of collusion

- A ring of 5 people bid on a necklace.

1. Frank is appointed the designated bidder
   - wins the necklace for $50.
2. The ring members meet then & hold another auction
   - the necklace is sold now to Blanche for $100.
3. Now, from the $100, Blanche:
   - Reimburses Frank for the original $50 he paid, and
   - pays each other person (including Frank and herself) $10.

- So, Blanche ends up paying $90 for a necklace, and all others make a $10 profit.
- The auctioneer loses $50.

Auctions

3. Multi-object auctions

Types of multi-unit auctions

- Multiple items are for sale
- Classification of multi-unit auctions:
  - Are all items the same ?
    - Yes: homogeneous, No: heterogeneous
  - Are all items put for auction at the same time ?
    - Yes: simultaneous, No: sequential
  - Are bids for combinations allowed ?
    - Yes: combinatorial, No: individual
  - Do prices evolve before being finalized ?
    - Yes: progressive, No: one-time sealed bids
  - In a homogeneous auction, are all units charged at the same price ?
    - Yes: uniform price, No: discriminatory

The simple case

- k units (identical items) are for sale
- Each bidder can only bid for one unit

- First price auction: k highest bids win, each winner pays his bid
- Second price auction: k highest bids win, each winner pays the value of the k+1 highest bid
  - i.e. the highest loosing bid
- Revenue equivalence applies

The general case

- Each bidder allowed to bid for multiple units:
    - Revenue Equivalence does not apply

- Key issue:
    - Complements: high value for combination
        - E.g. adjacent spectrum bands → when awarded to the same bidder, he can also exploit the guard band

  or

    - Substitutes: identical or nearly items, with an individual diminishing with quantity

Sequential auctions

- Items are auctioned sequentially one-by-one
- Simple solution, also applies for heterogeneous items
- Problems:
  - Identical items can be sold at very different prices
    - Declining price anomaly often arises
  - Predatory bidding to drive prices high and squeeze rivals
  - Complicated strategies
  - Inefficient aggregation of complements
- What if all items are auctioned simultaneously yet individually ? ➔

Examples of sequential auctions

1. 3 symmetrical bidders, 2 items, Vickrey pricing
   - 1st round: bid less than ordinarily, benefit from option to get the item in second round
   - 2nd round: reduced competition
2. Example: RCA-satellite auction, 1981
   - 7 licenses for cable television broadcast
   - Highest \$14.4 M (1st), lowest \$10.7 M (6th)
   - FCC nullified the auction as "unjustly discriminatory" → enforced a uniform price

Problems with individual bidding (I)

- In general, there do not exist common prices so that
  - each item is demanded by one bidder only
  - outcome is most efficient, taking valuations of combinations into account
- Does not apply if items are mutual substitutes
- Example:

Bidder	A or B	AB
#1	a	2a+c
#2	a+d	2a+d

synergy between A and B

  - If $c/2<d<c$: most efficient outcome: AB $\Rightarrow$ Bidder#1
    - there do not exist equilibrium prices to attain this !

Problems with individual bidding (II)

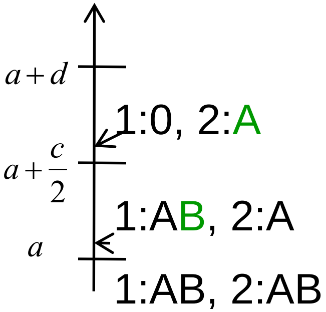
- Exposure Problem: Due to an unsuccessful attempt to win a set (combination) of items, a bidder may pay for a subset of these items more than his respective valuation
  - Arises in cases of complementary items
  - Results in an inefficient outcome and/or lower profit and satisfaction of bidders

Example of exposure

- Common price per item, increased by ε per clock tick, until total demand is for 2 items

Bidder	A or B	AB
#1	a	2a+c
#2	a+d	2a+d

  - Assume c/2< d<c
  - Most efficient outcome:
    AB → Bidder#1
  - When p_A=p_B=a+ε , Bidder#2 prefers to try to win only one item (say A) → he can persist until p_A=a+d
  - Bidder#1 continues trying to win AB after p_A=p_B=a, and he wins B at p_B=a+ε → he can persist for A until p_A=a+c/2+ε
  - Final outcome:
    - A → #2 wins at p_A=a+c/2 +ε,
    - B → #1 wins at p_B=a+ε
    - Both inefficient and wasteful for bidder #1!



Problems with individual bidding (III)

- Demand reduction: large bidders may prefer to withhold part of their demand, in order to depress prices
  - Arises with substitute items, particularly for uniform pricing
  - Results in an inefficient outcome and lower revenues for the seller
  - Can be alleviated by
    - offering discounts to large buyers, or
    - discriminatory pricing, although differential bid shading arises then!

Example of demand reduction

- Uniform price, equal to highest losing bid
- Bidders #2 and #3 demand a single unit
  - they bid truthfully
- Most efficient outcome:
  - AB → Bidder#1, social welfare = 120
  - attained if #1 is truthful
  - price=40, profit of #1 = 40, total revenue = 80
- Most profitable for Bidder#1: bid 70 for one unit and 0 for the other
  - outcome: A → Bidder#1, B → Bidder#2
  - price =20, profit of #1 =50, social welfare = 110, total revenue=40

Bidder	A or B	AB
#1	70	120
#2	40	40
#3	20	20

Allowing combinatorial bids

- Winning subset of bids = subset of non-overlapping bids resulting in maximum total revenue
- Advantages:
  - increased efficiency
  - increased revenue, particularly with reserve prices
  - elimination of exposure problem
- Disadvantages:
  - complexity of determining the winners can be very high
    - exponential number of combinations
    - considerably larger number of possible allocations
    - solution(?): limit permissible combinations
  - threshold and free-rider problems

Free-rider problem

- Small bidders do not manage to displace larger ones
- May lead to an inefficient outcome
- Example:
  - Most efficient outcome:

    A → Bidder#1, B → Bidder#2
  - Bids for AB are placed first: #3 offers 15

Bidder	A	B	AB
#1	40	-	-
#2	-	40	-
#3	0	0	15

  - Bidder #1: “If #2 offers 15 for B, I can take A for 1”
  - Bidder #2: “If #1 offers 15 for A, I can take B for 1”
  - There exist equilibrium randomized strategies for #1 and #2
  - It is possible that none of #1 and #2 decides to offer a high bid, thus resulting in the inefficient outcome AB → Bidder#3

Threshold problem

- Bidders aiming at a single item or a small combination cannot displace bidders of larger combinations by acting unilaterally
  - may lead to an inefficient outcome

Bidder	A	B	AB
#1	10	-	-
#2	-	10	-
#3	0	0	15

- Example: open auction
  - Most efficient outcome:
    A → Bidder#1, B → Bidder#2
  - Initial bids: #3 offers 11 for AB first
  - None of bidders #1 and #2 can overbid #3 alone, and thus does not compete with #3
    - the inefficient outcome prevails, AB → Bidder#3

Roadmap

- So far, for multi-item auctions:
  - Defined the taxonomy
  - Dealt with the simple case of unit demand
  - Stated problems applying to the general case

- Solutions are now to follow:
  - Generalized Vickrey auction
  - FCC auction

Generalized Vickrey Auction

- Sealed multi-item auction

- Winners are charged according to the social opportunity cost their presence entails
  - Extension of the simple Vickrey auction

- Incentive compatibility still holds: truthful bidding is the dominant bidding strategy
  - Unlike "pay-your-bid" auctions, bid shading is not beneficial

Example of a homogeneous GVA

Bids	1st unit	2nd unit	3rd unit
A:	50	40	*37*
B:	*35*	32	30
C:	30	25	10
D:	42	*38*	31

- Three identical units are auctioned to four bidders
  - A wins two units and is charged 38+35=73
  - D wins one unit and is charged 37

Evaluation of the GVA

- Advantage:
  - Efficient outcome, due to incentive compatibility
- Disadvantages:
  - Computationally complex in the non-homogeneous case, because of bids for combinations
  - Possibly low revenue → results in bad publicity
  - Despite incentive compatibility, users consider GVA as complicated
  - Different bidders may pay different charges for the same quantity won
- Mostly of theoretical interest
  - Rarely applied in practice

FCC Simultaneous Ascending Auctions

- Multiple spectrum licenses (=items) put in auction:
  - simultaneously, in discrete rounds
  - the bids:
    - are sealed, then announced at the end of the round
    - are placed per individual license
    - should exceed the corresponding highest previous bid
  - rules for maintaining bidding-activity force bidders to participate, otherwise their eligibility (in terms of their target licenses) is reduced
  - termination rule: one round with no bids at all
  - “pay-your-bid” pricing

Example of an FCC auction

	Round →	1	2	3	4	5	6
Licenses	Bidders and Bids ↓						
A	West	10	**25**			**45**	Termination
	North	**15**		30			
	South	5	20	**35**			
B	West	20	**35**		**50**		
	North	15	30	**45**			
	South	**25**		40			
C	West	15	**25**				
	North	10				**40**	
	South	**20**			**30**		
Future Eligibility	West	3	3	3	2	2	
	North	3	2	2	1	1	
	South	3	3	2	2	2	

- *Activity rule:*
  eligibility(t+1)=activity(t)
  activity(t)≤eligibility(t)

- *Final allocation:* A → West (for 45),
  B → West (for 50),
  C → North (for 40)

Assessment of FCC auctions

- Advantages:
  - Allows the market to decide on allocation
  - Allows for formation of license combinations
- Disadvantages:
  - Possibility for tacit collusion and/or low revenues, particularly for low competition
  - Entry-deterrence: strong rivals discourage competition or even participation
- Considered a simple yet practically successful solution

Auctions

4. Conclusion

Conclusion

- There is no rule prescribing the best auction in each different case!
- Successful auction design requires information on market demand, and experimentation
- Art and science!

Selected References

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