Network Economics

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Network convergence

- In the new public network: from narrowband to a broadband world
- data used to run over a network that was largely built for voice
- from single media to multimedia
- from a fixed environment to a mobile environment
- Convergence: the PSTN, the Internet, wireless, broadcast networks, cable TV, are all coming together to service the same sets of traffic and to deliver the same types of features and services
- Convergence occurs in network services, devices, applications, industries, humans and machines

New economic environment

- Networks not any more state monopolies
- Decisions are profit-oriented
- No central decision making
- Network technologies dictate tussle of network service providers, application service providers, customers
- How will the Internet evolve?
- How will computing evolve?
- What is economics? Why is it relevant?

Questions that need economics

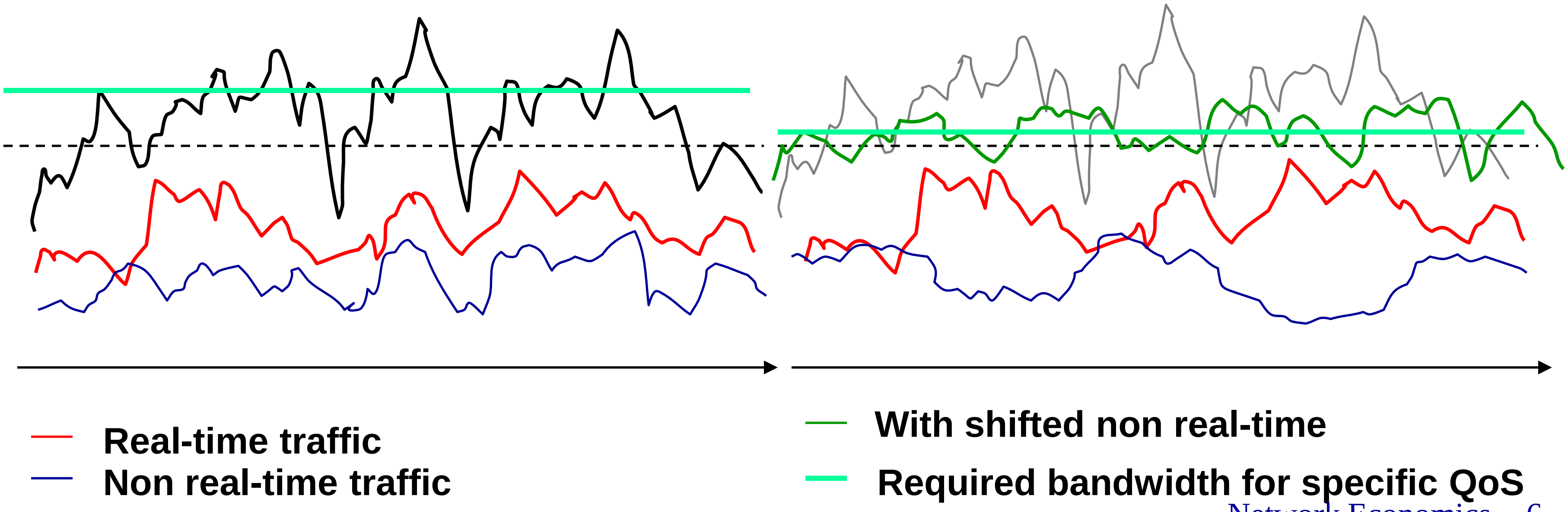
- Simple over-dimensioned networks or more complex control for QoS?
- How should network connectivity be priced?
- How should congestion be handled by new protocols?
- Should eyeball ISPs tax content providers and provide differential treatment to content (Network Neutrality)?
- Interconnection for more than best-effort?
- How will the Internet evolve? Single Internet or multiple specialized networks?
- What is the effect of cheap storage in content distribution?
- What are the correct design principles for network protocols ?
- How can the future Internet be business model neutral?
- Power of position in the value chain? (Google vs T1, T2, T3 networks)
- New business models for network operators?
- What is the impact of cloud computing, p2p,…?
- Business models for Google, Amazon, …?

Economics = incentives

- The taxi tariff $w = a + bT + cX$
- The "all-you can eat" restaurants: flat vs usage-based
- The Internet café tariff: dynamic pricing
- Low Extra Delay Background Transport (ledbat) (BitTorrent clients)

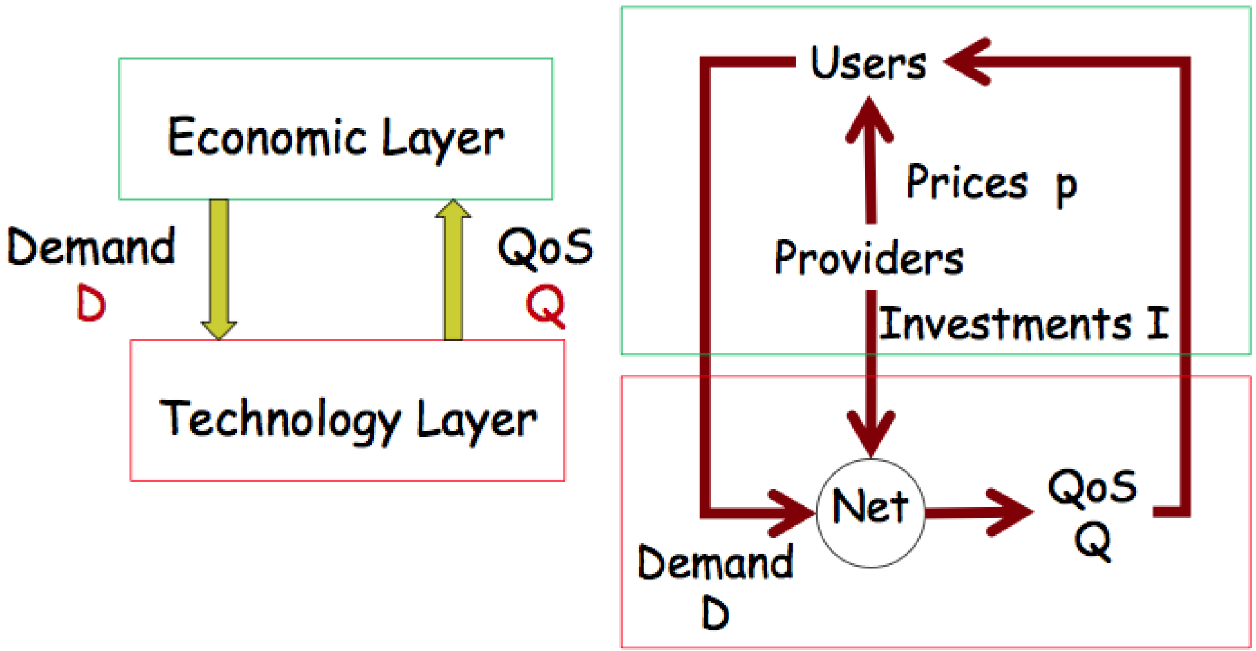
Traffic in the Internet

- Traffic shaping:
  - traffic = real-time + non real-time
  - delay increase => smaller peak rate
  - small delay in non real-time => big difference for the network!
  - Incentives for traffic shaping! But how?
  - The right pricing combined with the appropriate transport protocols



Users and providers

- Ref: J. Walrand, Pricing of Bandwidth and Communication On Demand Services, BoD 2008



$$D = \text{argmax}\ U(D, Q, p)$$

$$(p, I) = \text{argmax}\ \{p.D - \alpha I\}$$

$$Q = Q(D, I)$$

Users and providers are strategic

- Users
  - Exploit the network as much as possible for the given charge (selfish behaviour)
  - Use new applications and protocols if best for them
  - Modify technology is possible (boost TCP, etc.)
  - Over-consume, free-ride
- Providers (network)
  - Price to maximize revenue
  - Invest in capacity only if competition forces them
  - Love to control content and sell their own
  - Decide on deploying new technologies (WiMax, metro WiFi, 4G, QoS, …)
  - Under-invest
- Content
  - Free-ride on networks

Course outline

- Basic economic concepts
- Pricing
- Game theory
- Economics of flow control
- Interconnection economics
- Access network economics
- Network Neutrality
- The telecommunications market and the evolution of the Internet
- New Internet applications

Basic Economics

Basic Economics - Outline

- The consumer
- The producer
- The social planner
- Market mechanisms and competitive equilibria
- Marginal cost pricing and cost recovery
- Externalities and congestion pricing
- Market competition
- Lock-in
- Networks and positive externalities
- Game theory
- The information economy
- Pricing in communication networks

The context

- Communication services are **economic commodities**
- **Demand factors:** amounts of services purchased by users
  - utility of using a service, demand elasticity
- **Supply factors:** amounts of services produced
  - technology of network elements, service control architecture, cost of production
- **Market model:** models interaction and competition
- **Prices:** control mechanism
  - control demand and production, deter new entry
  - provide income to cover costs
  - structure and value depends on underlying model

Economic models and tariffs

- Prices result from the solution of economic models
- Possibly different contexts for deriving optimal prices
  - **surplus maximization**: standard market models with ***actual*** competition: monopoly, oligopoly, perfect competition
  - **stability under competition and fairness**: sustainability against ***potential*** entry, recovering costs, fairness w.r.t. cost causation, no subsidization
  - **asymmetric information models**: principal-agent models, hidden action and hidden information

Terminology

- **Terminology**:
  - **price**: correlated with service unit
  - **tariff:** charge structure
    - more general form of charging (i.e., $a+px$)
    - *control mechanism*
  - **charge**: total amount that must be paid

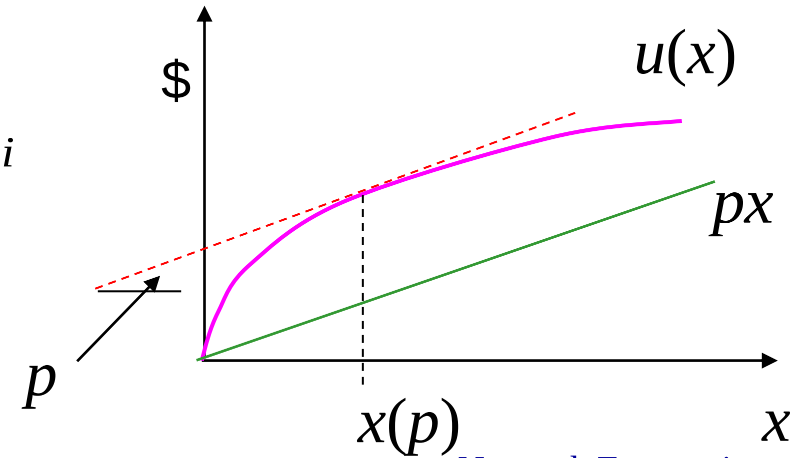
The consumer

The consumer's problem

- **Consumers:**
  - utility function $u(x)$ increasing, concave
  - **consumer surplus** (net benefit): $u(x) -$ charge for x
  - solve optimisation problem (linear prices):

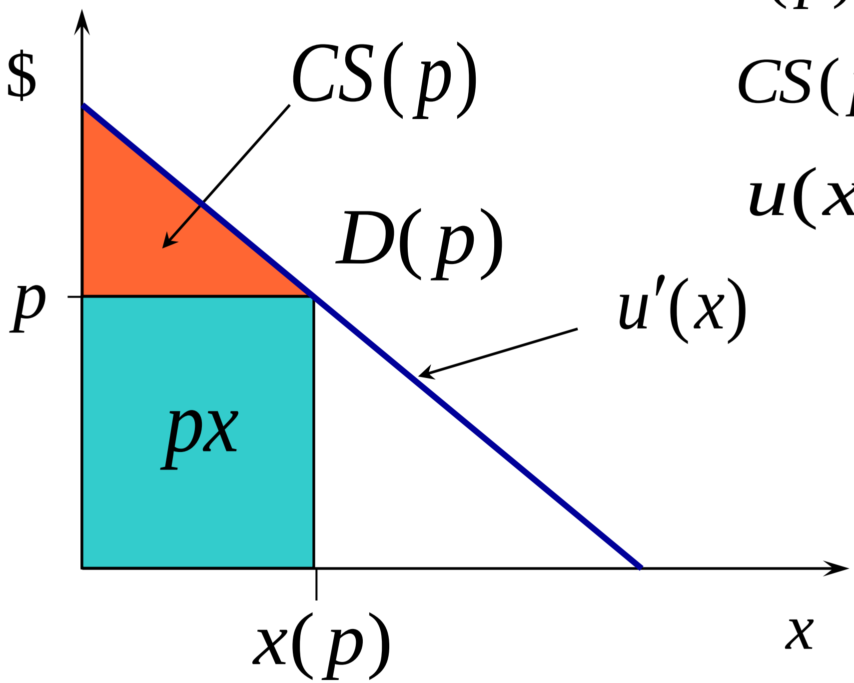
CUM: $x(p) = \arg\max_x [u(x) - p^T x]$

  - at optimum $\dfrac{\partial u(x)}{\partial x_i} = p_i$



The demand curve

The demand curve:



$x(p) =$ quantity demanded at price p

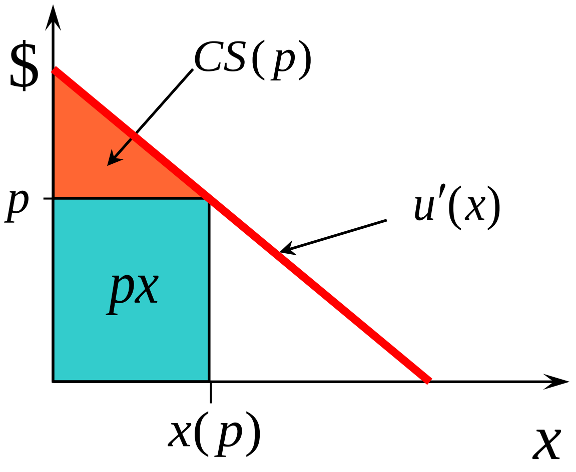
$CS(p) =$ consumer surplus at price p

$$u(x) = CS(p) + px$$

= value of consuming x

$$x(p) := \arg\max\{u(x) - px\}$$

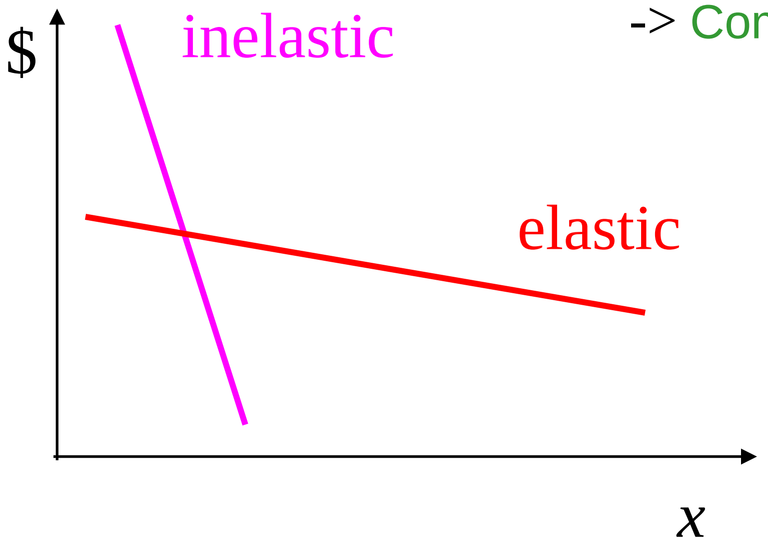
Elasticity of demand



Elasticity of demand: $$\varepsilon_i = \frac{\partial x_i / x_i}{\partial p_i / p_i}$$

Cross-elasticity: $$\varepsilon_{ij} = \frac{\partial x_i / x_i}{\partial p_j / p_j}$$

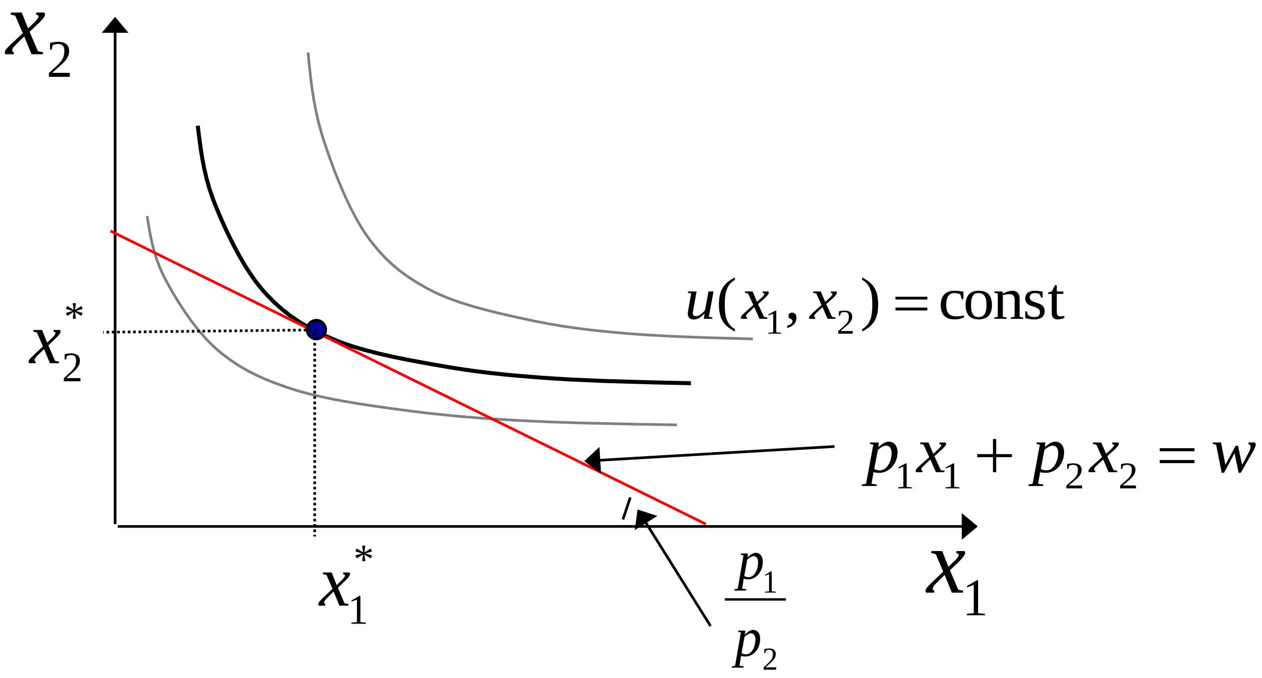
-> Complements, substitutes



Endowment effects

- The consumer has a fixed amount w to spend
- Market prices are given = p

$$\max_{x_1, x_2} u(x_1, x_2) \text{ s.t. } p_1 x_1 + p_2 x_2 \leq w$$



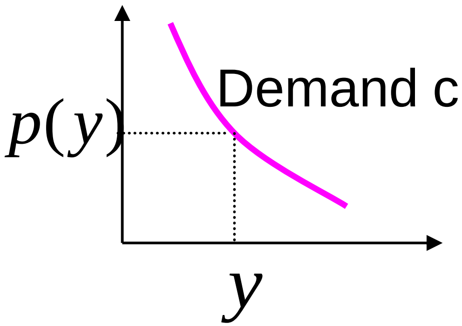
The producer

The producer's problem

- **Producer:** profit function (**producer surplus**):

$$\pi(y) = yp(y) - c(y),\ y \in Y$$

Monopoly: $\max_{y \in Y} [p(y)y - c(y)]$



Perfect competition: $\max_{y \in Y} [py - c(y)]$, for given p

Oligopoly: $\max_{y \in Y} [p(y + z)y - c(y)]$

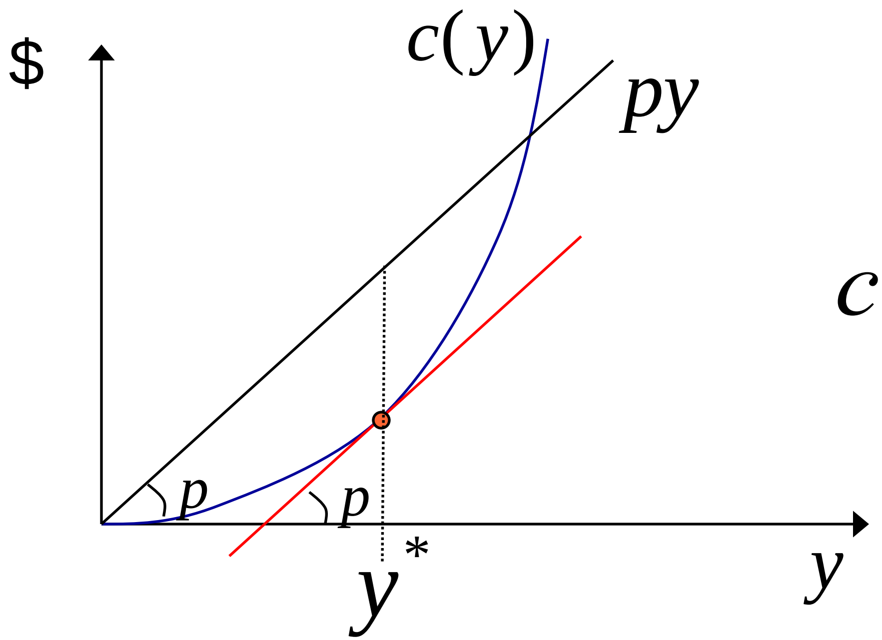
Regulation: fixed p, produce $y=y(p)$

The producer in a competitive market

Competitive market with price $\bar{p}$:

$$D(p) = \begin{cases} 0 \text{ if } & p > \bar{p} \\ \text{any amount produced if} & p = \bar{p} \\ \infty \text{ if } & p < \bar{p} \end{cases}$$

Producer solves: $\max_{y} \; py - c(y) \;\text{ for } p = \bar{p}$

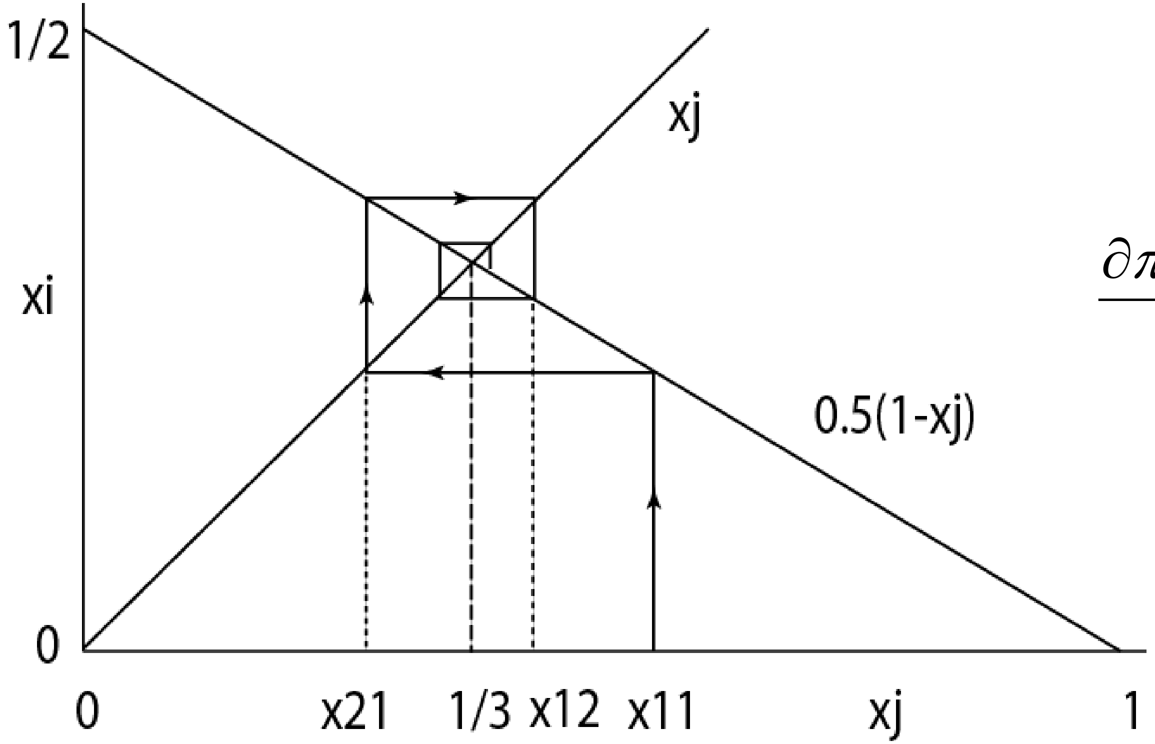


$$c'(y^*) = p$$

The Cournot game

$$\pi_i(x_1, x_2) = p(x_1, x_2)x_i - c_i(x_i),\ i = 1,2$$

$$= (1 - (x_i + x_j))x_i$$



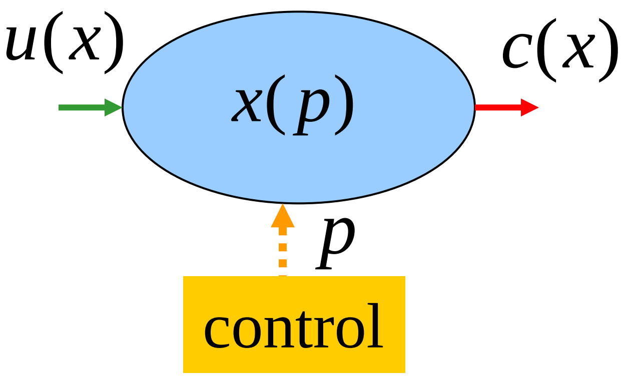
$$p(x) = 1 - x$$

$$c_i(x) = 0$$

$$\frac{\partial \pi_i(x_1, x_2)}{\partial x_i} = 1 - x_j - 2x_i$$

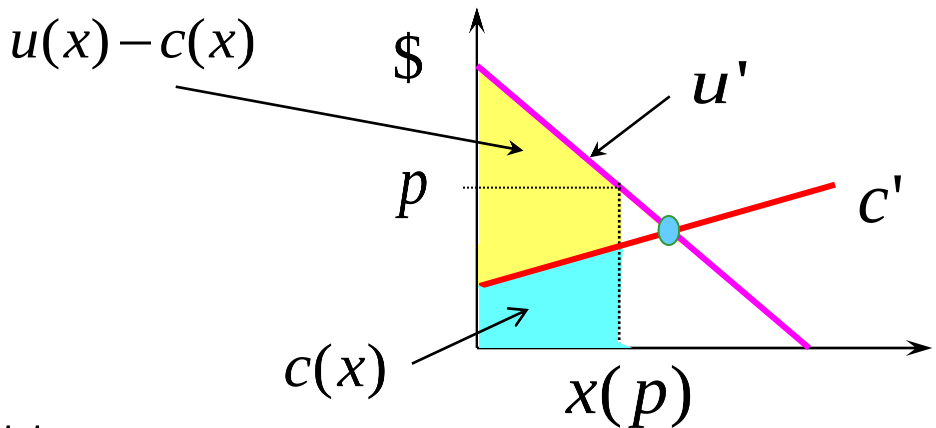
The social planner

The social planner's problem



$$\max_x u(x) - c(x) \Leftrightarrow$$

$$\frac{\partial u(x^*)}{\partial x_i} = \frac{\partial c(x^*)}{\partial x_i} \quad \textbf{= MC}$$

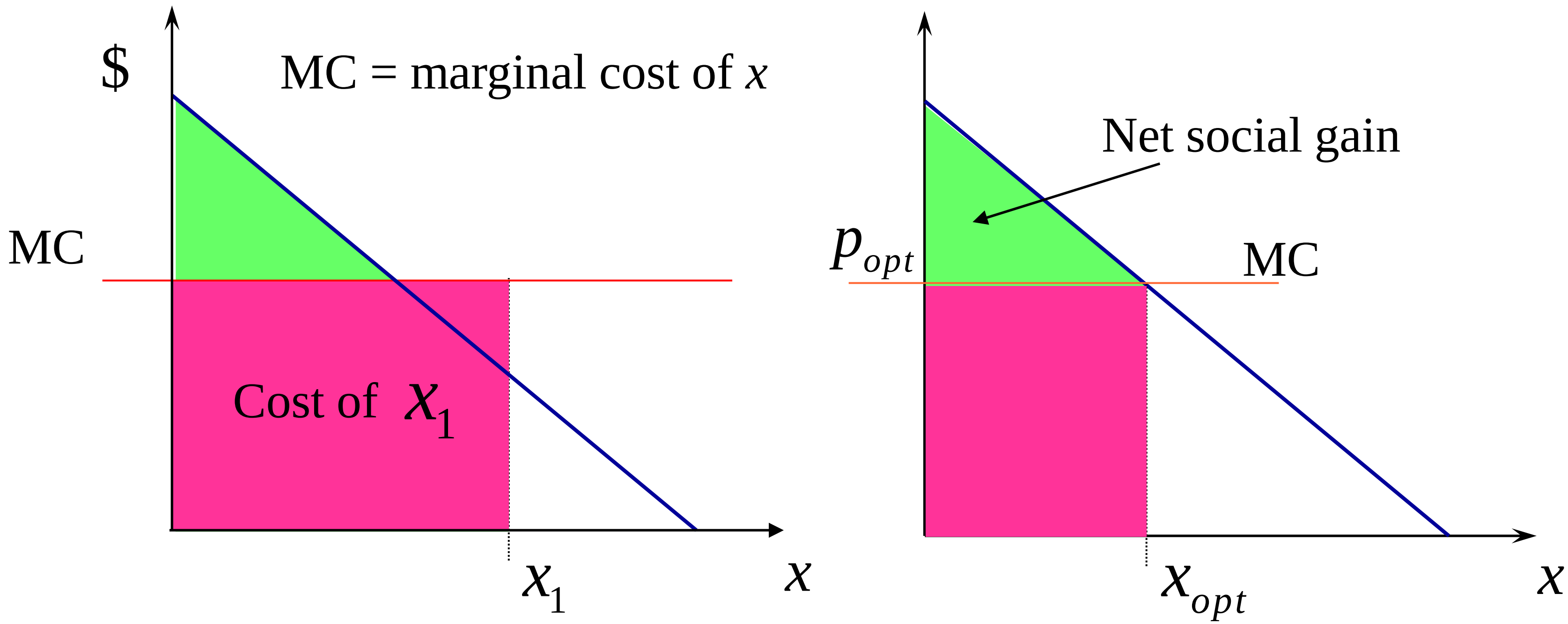


Note that this is equivalent to

$$\max_p [\{u(x(p)) - x(p)p\} + \{x(p)p - c(x(p))\}] = \max[CS + \pi]$$

Constant marginal cost

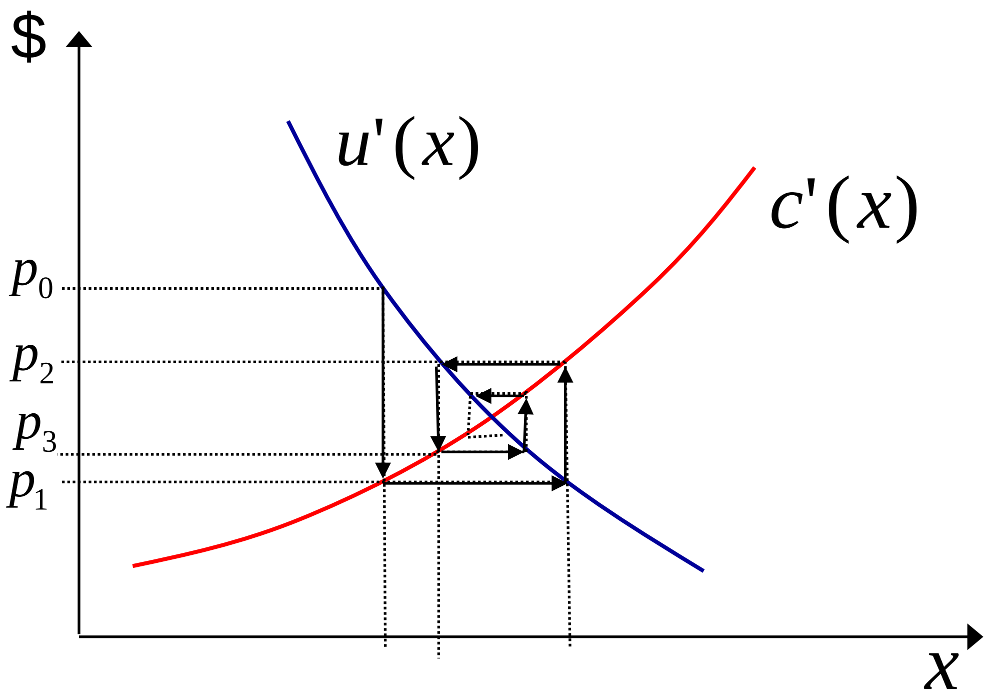
Simple case: constant marginal cost



Set prices = marginal cost

Setting prices equal to marginal cost

- The social planner sets prices equal to marginal cost at the level of production that satisfies demand
- Prices (may) converge to SW optimum



- How does the social planner know the true marginal cost?

SW maximization

- Social planner solves P1:

$$\max_{\{x_i, y_j\}} \sum_i u_i(x_i) - \sum_j c_j(y_j)$$

$$s.t. \quad \sum_i x_i \leq \sum_j y_j$$

- There exist a positive λ for which the solution of P1 is the free maximization of

$$\min_{\lambda} \max_{\{x_i, y_j\}} \sum_i u_i(x_i) - \sum_j c_j(y_j) + \lambda\left(\sum_j y_j - \sum_i x_i\right)$$

Hence at the optimum

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial c_j}{\partial y_j} = \lambda$$

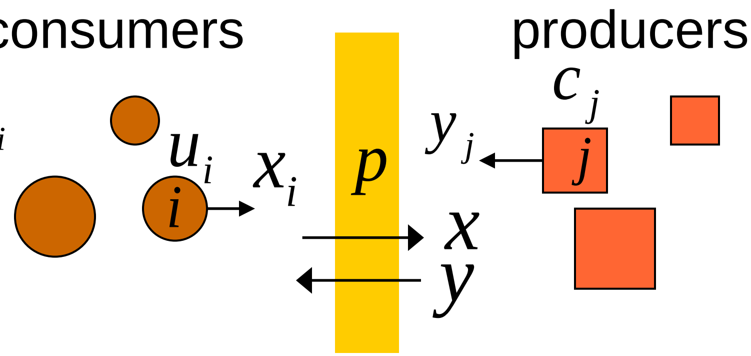
$$\sum_j y_j - \sum_i x_i = 0$$

Market mechanisms and competitive equilibria

Competitive equilibrium

- Market mechanism using prices
- Every participant in the market is small, can not affect prices
- Equilibrium: stable point where production = demand, price *p*

$$\max_{x_i} \; u_i(x_i) - p x_i$$



$$\max_{y_j} \; p y_j - c_j(y_j)$$

Market clearance: $$\sum_i x_i(p) = \sum_j y_j(p)$$

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial c_j}{\partial y_j} = p \Leftrightarrow \max_{\{x_i, y_j\}} \sum_i u_i(x_i) - \sum_j c_j(y_j)$$

$$s.t. \quad \sum_i x_i \leq \sum_j y_j$$

=> **Social welfare optimum!**

=> **Tatonnement**

Single link with capacity constraints

- Total amount of resource available = C , zero cost
- Maximization problem:

$$\max_{\{x_i\}} \sum_i u_i(x_i) \quad s.t. \quad \sum_i x_i \le C \quad (1)$$

- Mathematical solution: minmax the Lagrangian

$$\min_{\lambda} \max_{\{x_i\}} L(\lambda, x_1, \ldots, x_n) = \sum_i u_i(x_i) + \lambda(C - \sum_i x_i) \; (2)$$

The optimal point of (1) is characterized by $\lambda, \{x_i\}$ for which:

$$\frac{\partial u_i}{\partial x_i} = \lambda, \sum_i x_i = C$$

- Problem solution with market mechanism: use price $p = \lambda$
- Each user solves: $\frac{\partial u_i}{\partial x_i} = p$
- λ = **shadow cost** of capacity

Note: although cost = 0,
optimal price is not!

Solving the dual

$$\min_{\lambda} \max_{\{x_i\}} L(\lambda, x_1, \ldots, x_n) = \sum_i u_i(x_i) + \lambda(C - \sum_i x_i)$$

$$\text{For a given } \lambda(t) \text{ solve } \max_{\{x_i\}} \sum_i [u_i(x_i) - \lambda(t) x_i] + \lambda(t) C$$

$$\text{For a given } \lambda(t) \text{ solve } \max_{x_i} [u_i(x_i) - \lambda(t) x_i] \text{ for each } i$$

But this is done by each customer solving CUM using $\lambda(t)$ as a price

$$\text{Then choose } \lambda(t+1) \text{ to reeduce slightly } \lambda(t)(C - \sum_i x_i(t))$$

But this exactly how prices are updated in a market!

Stability

$$\text{Let } V(t) = \sum_i u_i(x_i(t)) + \lambda(t)(C - \sum_i x_i(t))$$

$$\text{where } \dot{\lambda}(t) = k(C - \sum_i x_i(t)) \text{ for a small } k$$

$$\text{and } x_i(t) \text{ maximizes } U_i(x_i) - \lambda(t)x_i \text{ at each } t$$

$$\text{Then } \dot{V}(t) = \sum_i u'_i \, \dot{x}_i(t) + \dot{\lambda}(t)(C - \sum_i x_i(t)) - \sum_i \lambda(t)\dot{x}_i(t)$$

$$= \sum_i (u'_i - \lambda(t))\dot{x}_i(t) + \dot{\lambda}(t)(C - \sum_i x_i(t))$$

$$= \dot{\lambda}(t)(C - \sum_i x_i(t))$$

$$= -k(C - \sum_i x_i(t))^2 < 0$$

But if there are delays??

Market mechanisms

1. Network sets price p^t, users post their demands $x_i^t(p^t)$
2. Network computes excess demand $z^t = \sum_i x_i^t - C$
3. Network updates price: $p^{t+1} = p^t + \alpha z^t, \alpha < 1$

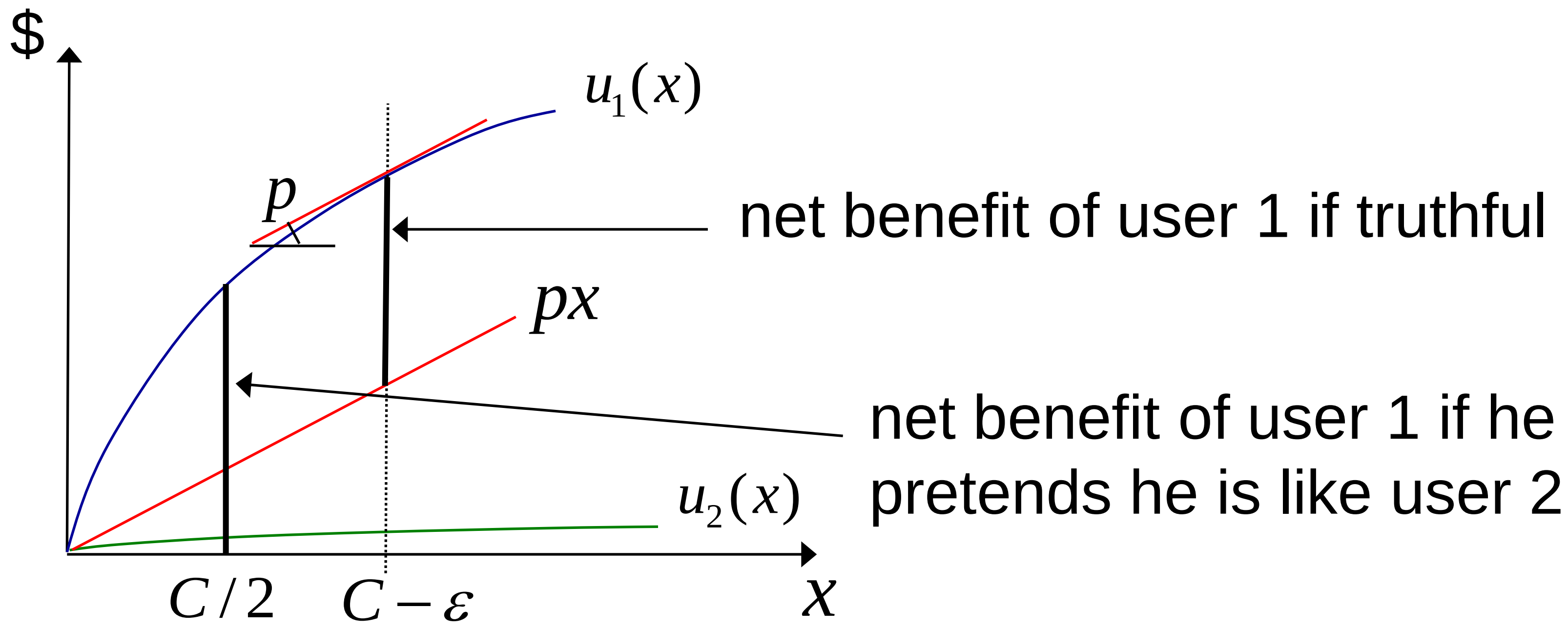
Under general conditions, $p^t \rightarrow \lambda$

where λ is the Lagrange multiplier in (1)

Observe:
- The optimum of (1) is achieved by a decentralized mechanism
- The network does not need to know the utilities of the users

Strategy issues

- Why should users respond truthfully their $x_i(p)$?
- it may be profitable to cheat!
- In a case of 2 unequal users, the large user may pretend he is small



A possible analysis of a user charge

- In general we can analyze the total charge the user is paying as

S = F+U+G+Q, where

F= covers fixed cost,

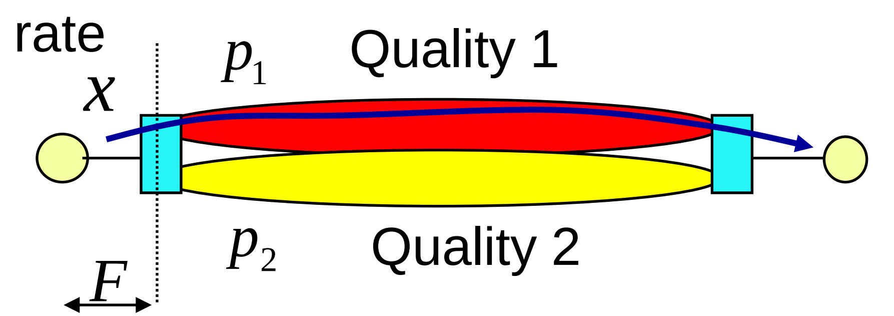
U= covers usage cost,

G= “congestion” part,

Q= quality part

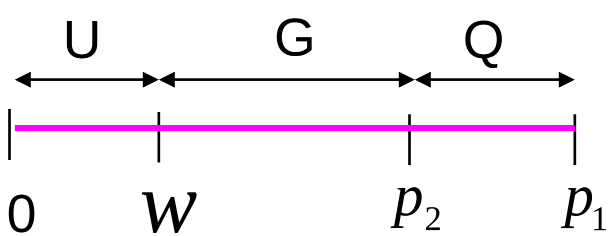
variable part (U, G, Q)

when demand > capacity (G)



volume during T

$$S = F + p_1 x T = F + p_1 q(T)$$



variable part of price

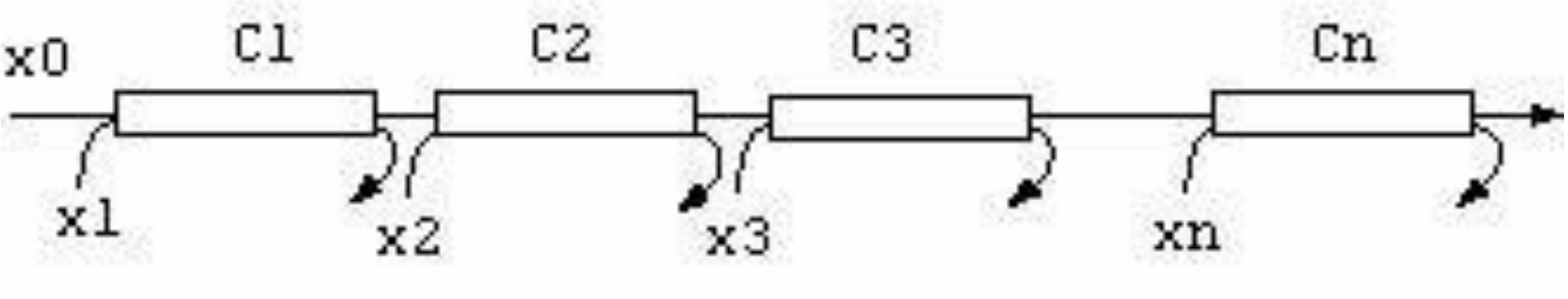
w= real cost/byte (into the network)

F= real connection cost (with the network)

General network flow pricing

Fairness in flow allocations

- Fairness: How should the bandwidth be shared among competing flows?
- Economics: Pareto efficiency, max some form of SW
- Network engineering: Max throughput, max-min, proportional fairness,…
- Can we relate the two?



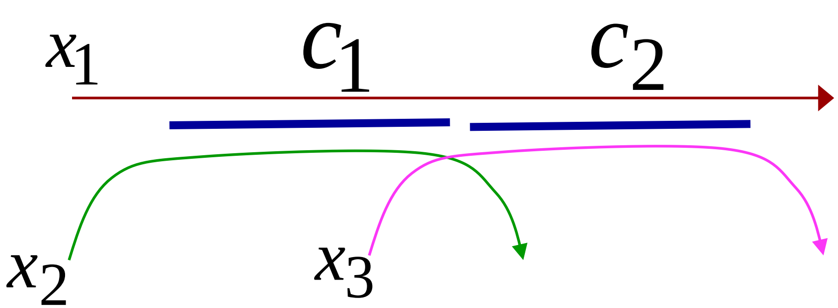
SW max. with capacity constraints

- Generalize the single link case: flow identity defined by a route r (set of links it traverses), x_r = bit rate of flow r
- Assume a utility function for flow x_r, solve

NUM:

$$\max_{\{x_r\}} \sum_r U_r(x_r)$$
$$s.t. \quad Ax \le C, \quad x \ge 0$$

$$A_{jr} = 1 \text{ if } j \in r$$
$$A_{jr} = 0 \text{ if } j \notin r$$



$$\max_{x_1, x_2, x_3} U_1(x_1) + U_2(x_2) + U_3(x_3)$$
$$s.t. \quad x_1 + x_2 \le c_1, \; x_1 + x_3 \le c_2, \; x_1, x_2 \ge 0$$

Example: weighted prop. fairness

- w-Proportional Fairness (WPF): $U_r(x_r) = w_r \log x_r$
- Network problem:

$$\max_{\{x_r\}} \sum_r w_r \log(x_r) \; s.t. \; Ax \le C, x \ge 0$$

$$L(x,\mu) = \sum_r w_r \log(x_r) - \sum_j \mu_j [\sum_{s: j \in s} x_s - C_j]$$

$$(x^*, \mu^*) \begin{cases} \mu_j > 0 \Leftrightarrow \sum_{s: j \in s} x_s - C_j = 0 \\ \frac{\partial L}{\partial x_r} = 0 \Leftrightarrow \frac{w_r}{x_r} = \sum_{j \in r} \mu_j \Leftrightarrow x_r = \frac{w_r}{\sum_{j \in r} \mu_j} = \frac{w_r}{\lambda_r} \end{cases}$$

Nash bargaining solution -> WPF

- Two players bargain to share profit $(u_1, u_2) \in U$
- Alternate in rounds making proposals -counterproposals

$u_1 \quad v_1 \quad u_1 \quad v_1$

$u_2 \quad v_2 \quad u_2 \quad v_2 \quad \ldots.$

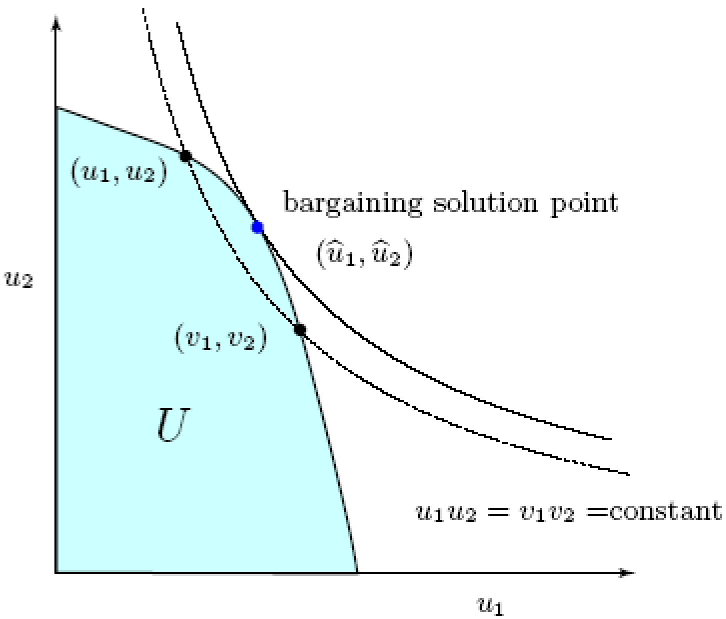
Stationary strategy => $u_2 = v_2 e^{-sn_2}, v_1 = u_1 e^{-sn_1}$

$$u_1^{1/n_1} u_2^{1/n_2} = v_1^{1/n_1} v_2^{1/n_2}$$

As $s \to 0, \; u, v \to \hat{u}$

$$(\hat{u}_1, \hat{u}_2) = \operatorname*{argmax}_{u_1, u_2 \in U} u_1^{w_1} u_2^{w_2}$$

$$(\hat{u}_1, \hat{u}_2) = \operatorname*{argmax}_{u_1, u_2 \in U} w_1 \log u_1 + w_2 \log u_2$$



W-proportional fairness

Marginal cost pricing and cost recovery

Marginal cost prices

- **Strong points:**
  - welfare maximisation under appropriate conditions
  - firmly based on costs
  - easy to understand
- **Weak points:**
  - **do not cover total cost** (need for subsidisation)
  - must be defined w.r.t. time frame of output expansion?
    - **short run marginal cost = 0 or ∞**
    - use long-run marginal cost (planned permanent expansion)
  - difficult to predict demand and to dimension the network
  - difficult to relate cost changes to marginal output changes

Marginal cost pricing problems

- Marginal cost = covers all sacrifices, present or future, external or internal to the company, for which production is at **the margin** **causally** responsible
- Problem1: **specifying the time perspective**
  - should we use long-run MC rather than short-run MC?
  - MC includes present and future causally attributed costs
  - problem: total cost coverage
- Problem2: **specifying the incremental block of output**
  - incremental cost depends on size of increment
  - charge the shortest run MC for the smallest output increment?
- Problem3: **large proportions of common costs**

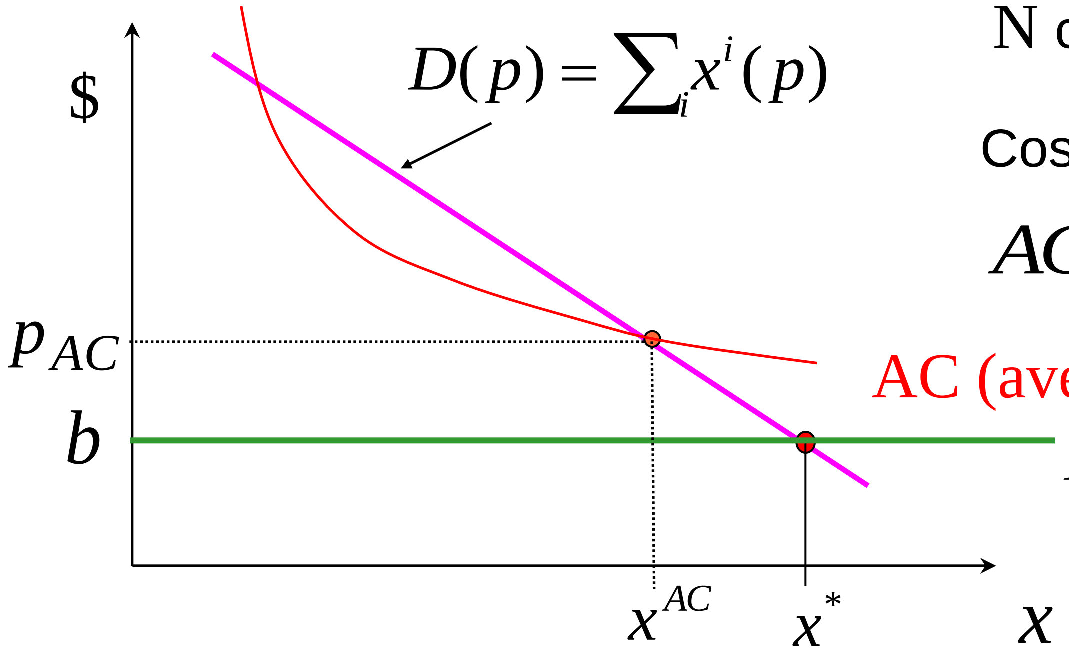
Recovering network cost

- **Pricing at marginal cost maximises efficiency but does not necessarily recover network cost**
  - example: assume $c(x) = \alpha + \beta x$

  Then under marginal cost pricing, $p = \beta$

  and the network revenue is βx, hence we are short of α
- **Ways out:**
  - Ramsey prices (linear prices)
  - add fixed fee (two-part tariffs)
  - general non-linear tariffs
    - $r(x)$
    - $(a_k, p_k), k = 1, \ldots, K$

Two-part tariffs



N customers

Cost = $a + bx$

$AC = a/x + b$

Under *MC* pricing, network needs to recover an additional amount a

Use tariff $a/N + bx$

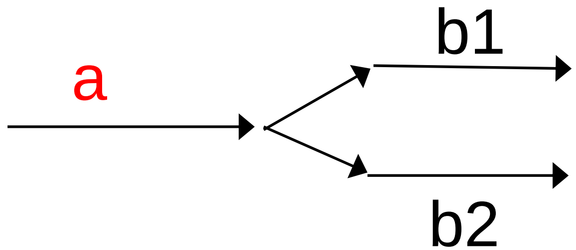
Customer benefit = $u_i(x^i(b)) - a/N - bx^i(b)$ **< 0 ?**

$x(b) =$ user demand at price b

Sharing common marginal costs

The case of common marginal cost

- Consider two products that are *jointly* produced in equal quantities. How to attribute the joint marginal cost a?



cost = $a\max\{x_1,x_2\}+b_1x_1+b_2x_2$

$$\max_{x_1,x_2} \quad u_1(x_1)+u_2(x_2)-a\max\{x_1,x_2\}-b_1x_1-b_2x_2$$

$$\max_{x_1,x_2,z} \quad u_1(x_1)+u_2(x_2)-az-b_1x_1-b_2x_2$$

$$s.t. \quad x_1\le z, x_2\le z$$

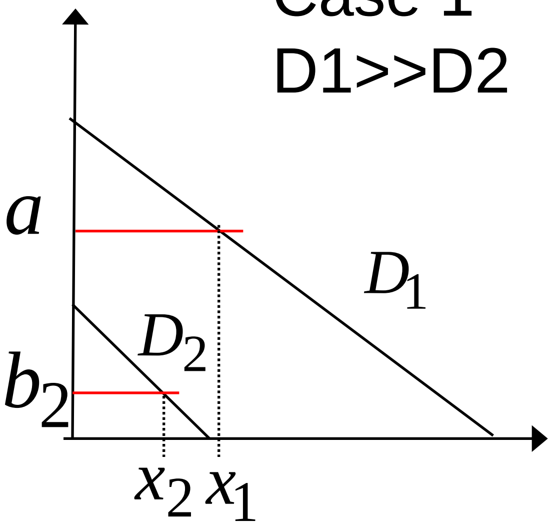
$$L=u_1(x_1)+u_2(x_2)-az-b_1x_1-b_2x_2-\lambda_1(x_1-z)-\lambda_2(x_2-z)$$

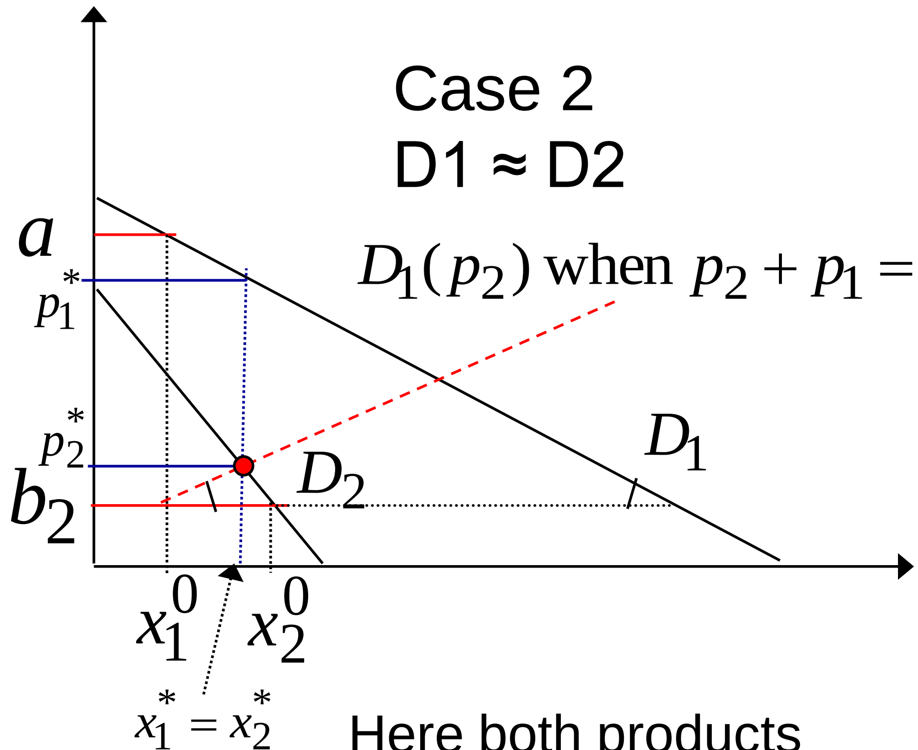
Hence $\quad p_1=b_1+\lambda_1,\ \ p_2=b_2+\lambda_2,\ \ \lambda_1+\lambda_2=a$

and $\quad x_i<z\Rightarrow\lambda_i=0$

A graphical calculation of prices

- Assume $b_1 = 0$





The joint MC cost a
is provided only by
the high demand product

Here both products
share the joint MC cost a

Peak load pricing

$$T \text{ periods, } x_t = x_t(p_1, \ldots, p_T)$$

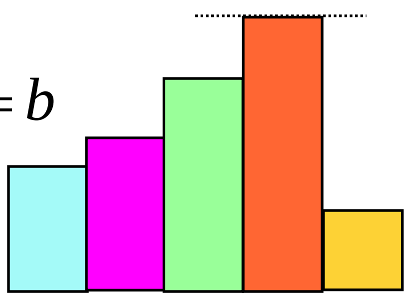
$$c(x_1, \ldots, x_T) = b \sum_T x_t + a \max_t \{x_t\}$$

$$\max_{x_1, \ldots, x_T, z} u(x_1, \ldots, x_T) - b \sum_T x_t - az \text{ s.t. } x_t \le z$$

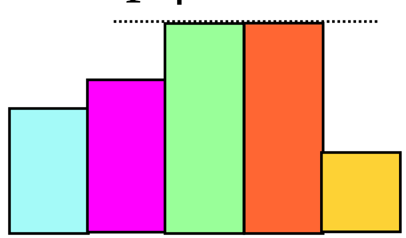
$$L = u(x_1, \ldots, x_T) - b \sum_T x_t - az - \lambda_1 (x_1 - z) - \ldots - \lambda_T (x_T - z)$$

$$p_1 = b + \lambda_1, \ldots, p_T = b + \lambda_T, \; \lambda_1 + \cdots + \lambda_T = a, \text{ and } x_t < z \Rightarrow \lambda_t = 0$$

$p_t^0 = b$

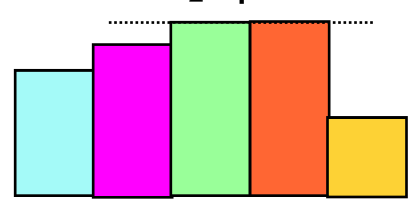


$p_4 = b + d < a$



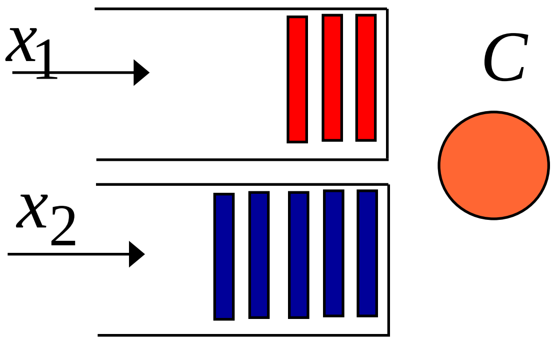
$p_3 = b + d_3,$

$p_4 = b + d_4, d_3 + d_4 = a$



Priority queues

- Real-time and best effort traffic share the same queue
- Real-time traffic is assigned priority, needs $C \geq \rho x, \rho = 3$
- Best-effort uses the left-over capacity
- How to split the capacity marginal cost a?



$$3x_1 \leq C \qquad x_1 + x_2 \leq C$$

$$\max_{x_1, x_2, C} u_1(x_1) + u_2(x_2) - aC$$

$$s.t. \;\; 3x_1 \leq C, x_1 + x_2 \leq C$$

$$p_1 = 3\lambda_1 + \lambda_2, \;\; p_2 = \lambda_2, \;\; \lambda_1 + \lambda_2 = a$$

Priority queues

$$\max_{x_1, x_2, C} u_1(x_1) + u_2(x_2) - aC$$

$$s.t.\quad 3x_1 \le C, x_1 + x_2 \le C$$

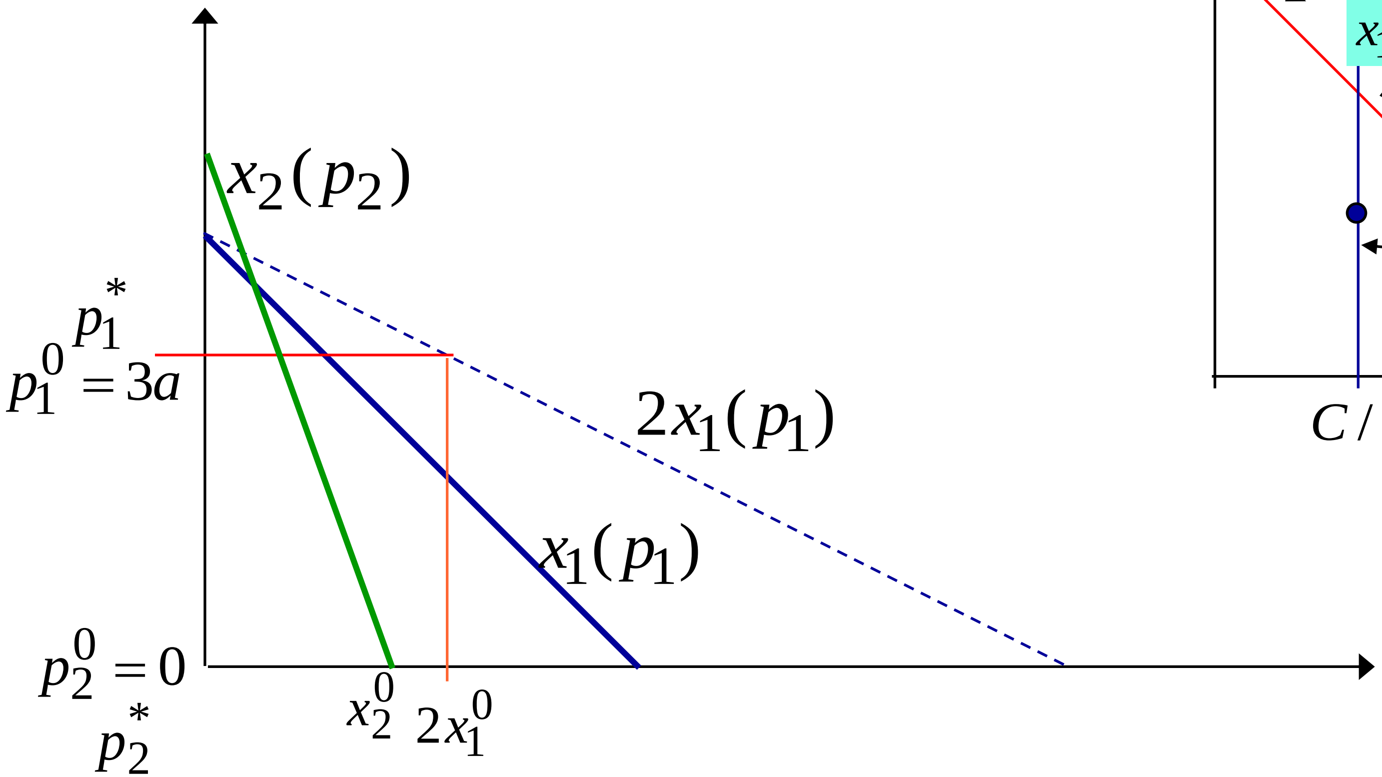
$$\max_{x_1, x_2, C} u_1(x_1) + u_2(x_2) - aC - \lambda_1(3x_1 - C) - \lambda_2(x_1 + x_2 - C)$$

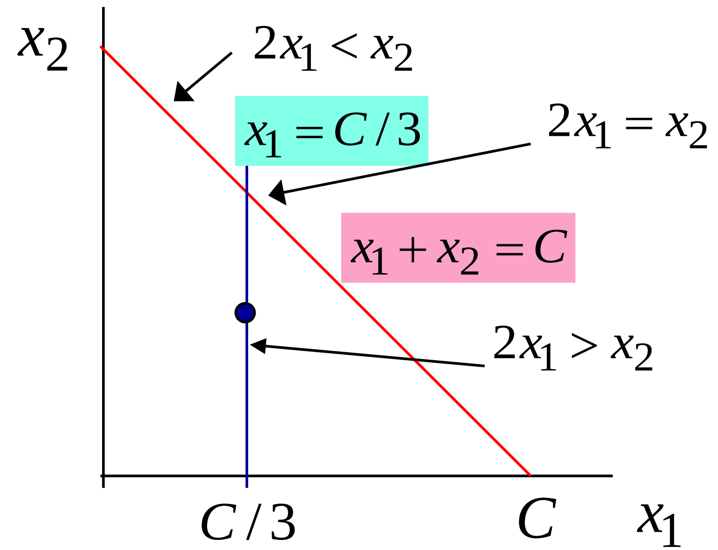
$$\max_{x_1, x_2, C} u_1(x_1) + u_2(x_2) - aC - \lambda_1(3x_1 - C) - \lambda_2(x_1 + x_2 - C)$$

$$\frac{\partial u_1(x_1)}{\partial x_1} = 3\lambda_1 + \lambda_2 = p_1,\ \frac{\partial u_1(x_1)}{\partial x_1} = \lambda_2 = p_2,\ \lambda_1 + \lambda_2 = a$$

Priority queues

Case I: low demand for best-effort



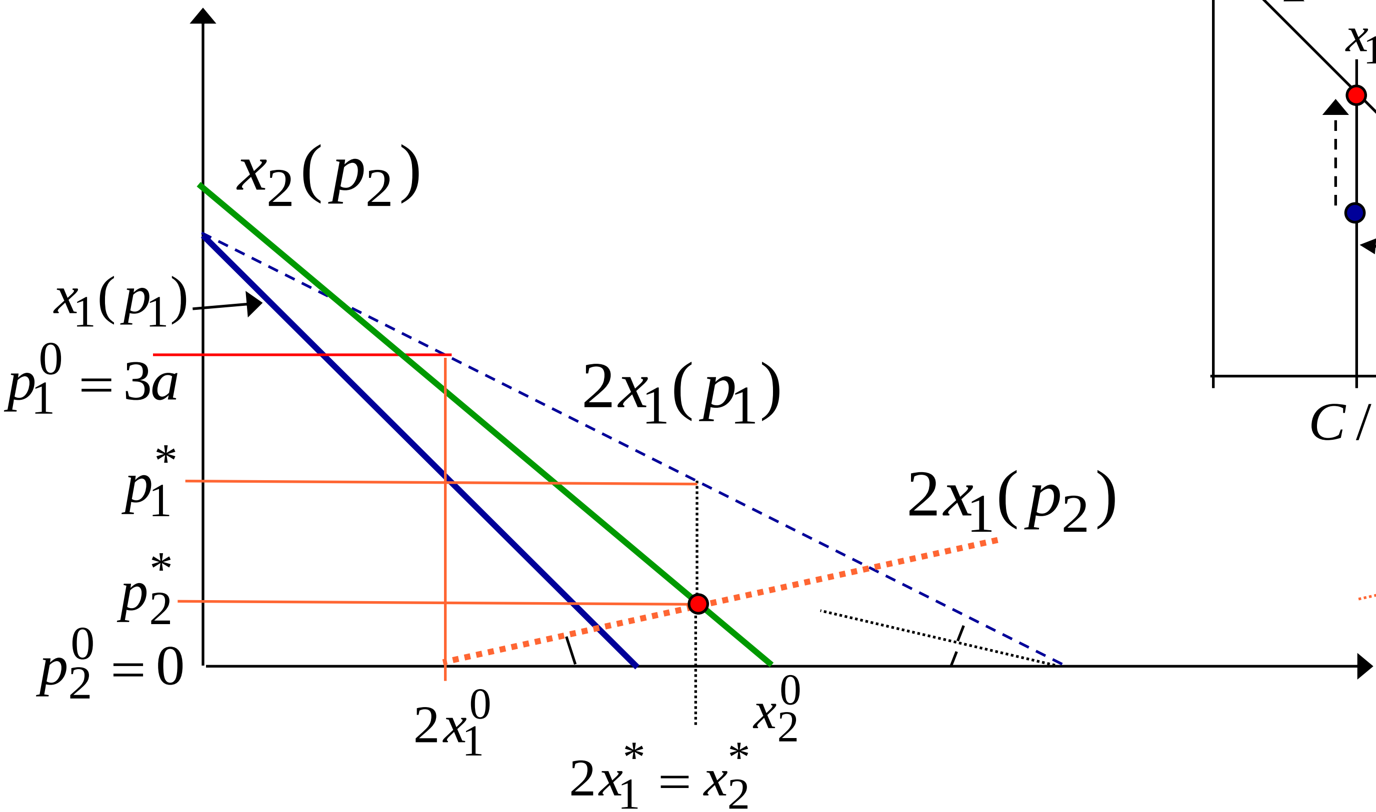


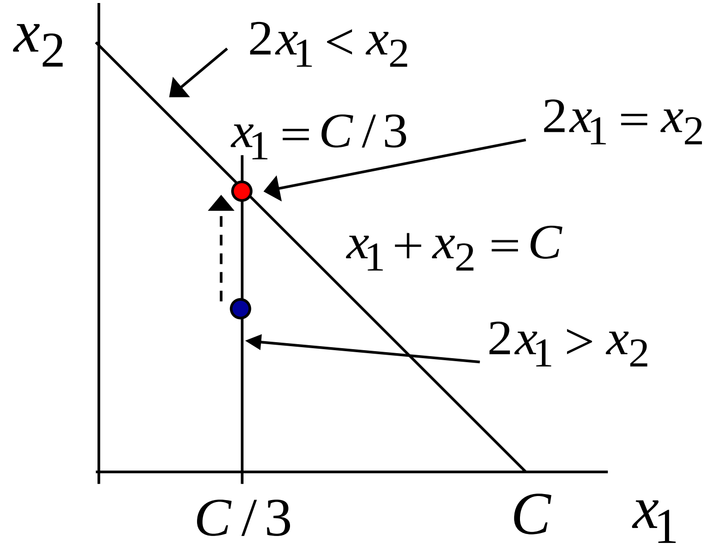
Remember: $p_1 = 3\lambda_1 + \lambda_2,\ p_2 = \lambda_2,\ \lambda_1 + \lambda_2 = a$

Methodology: start with $p_2^0 = 0$ and increase p_2 until the active constraints are consistent with the Lagrange multipliers

Priority queues

Case II: balanced demand



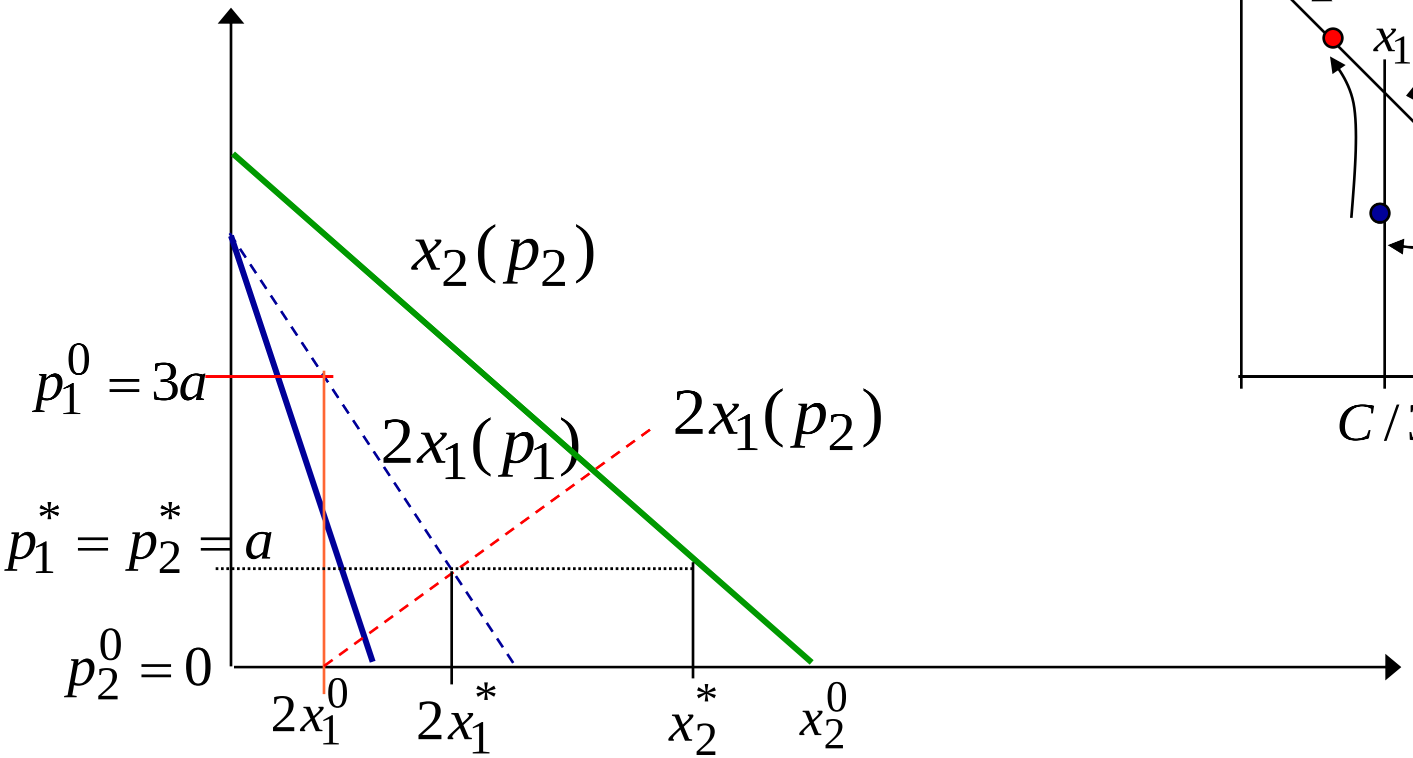


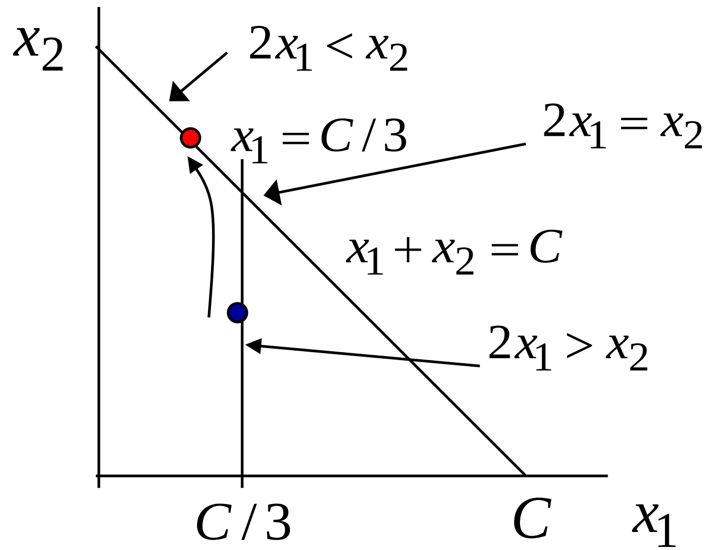
$$p_1 = 3\lambda_1 + \lambda_2,\ \ p_2 = \lambda_2,\ \ \lambda_1 + \lambda_2 = a$$

Methodology: construct $2x_1(p_2)$ where $p_1 = 3a - 2p_2$

Priority queues

Case III: low demand for real-time





Remember: $p_1 = 3\lambda_1 + \lambda_2,\ p_2 = \lambda_2,\ \lambda_1 + \lambda_2 = a$

Conclusions

- Marginal cost pricing is hard to implement in practice
- When joint costs, it is hard to attribute to individual services, depends on demand
- Same problem if joint facility must be configured to accommodate maximum service provisioned
- In communication networks, services may share joint facilities like in priority queues

Pricing

Lock-In

Reference: “Information Rules” by Carl Shapiro and Hal R. Varian

Recognizing lock-In

- Durable investments in complementary assets
  - Hardware
  - Software
- Supplier wants to lock-in customer
- Customer wants to avoid lock-in
- Basic principle: *Look ahead and reason back*
- Examples:
  - Bell Atlantic and AT&T
    - 5ESS digital switch used proprietary operating system
    - Large switching costs to change switches
  - Computer Associates
  - User behavior in the Web

Small switching costs matter

- Small switching cost per customer but large customer bases
  - Phone number portability
  - Email addresses
    - Hotmail (advertising, portability)
    - ACM, CalTech
- Look at lock-in costs on a per customer basis

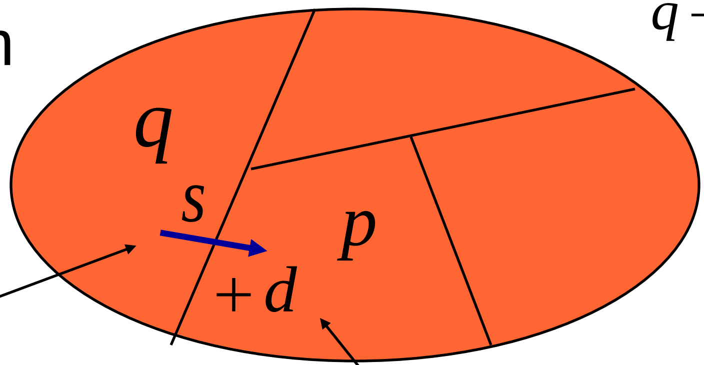
Profits & switching costs in general:

- Profits from a customer = total switching costs + quality/cost advantages
  - Customer C switches from A to "same position" w/ B: Total switching costs = customer costs + B's costs
- In commodity market like telephony, profit per customer = total switching costs per customer
  - Example: ILECs vs CLECs: ILEC profits = customer + CLEC switching costs
- Can answer these questions:
  - How much to invest to get locked-in base
  - Evaluate a target acquisition (e.g., Hotmail)
  - Product and design decisions that affect switching costs

A model of switching cost

$$q + \frac{q}{1+r} + \frac{q}{(1+r)^2} + \ldots = q + \frac{q}{r}$$

q = equilibrium market price



switching cost *s*

new entrant price =*p*, offers discount *d* to switching customer

$$q + \frac{q}{r} = p + \frac{p}{r} + s - d, \quad \leftarrow \text{customer indifferent to switch}$$

$$p - c + \frac{p-c}{r} - d = 0, \quad \longleftarrow \text{new entrant balances costs}$$

$$\Rightarrow q - c + \frac{q-c}{r} = s \Leftrightarrow q = c + \frac{r}{1+r} s$$

Classification of lock-In

- Durable purchases and replacement: declines with time
- Brand-specific training: rises with time
- Information and data: rises with time
- Specialized suppliers: may rise
- Search costs: learn about alternatives
- Loyalty programs: rebuild cumulative usage
- Contractual commitments: damages

Pricing

Price discrimination

Monopoly: linear+uniform prices

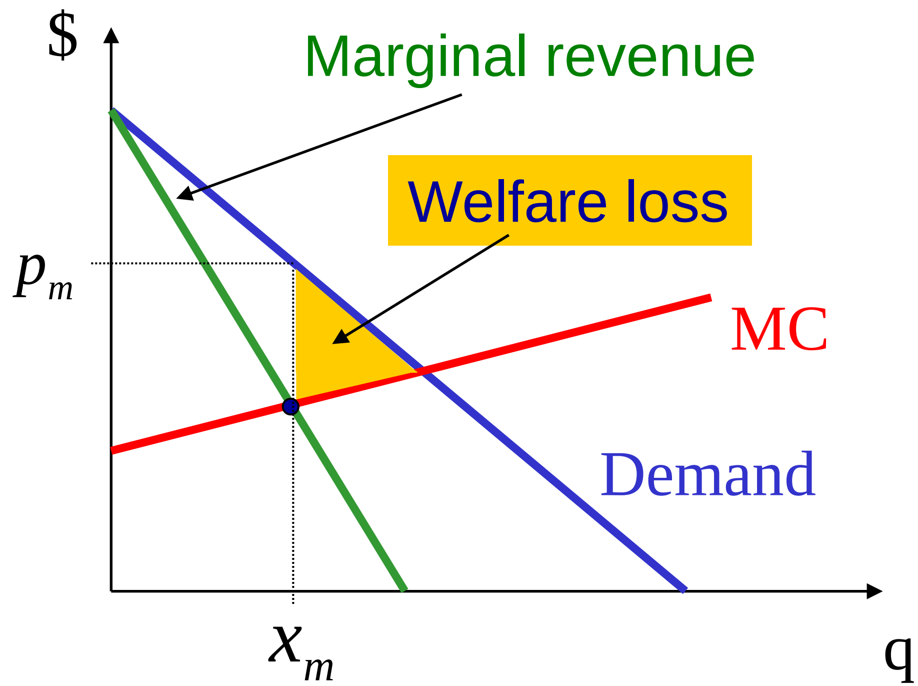
- **Goal**: maximize profits
- **Advantage:** economies of scale (small MC)
- **Disadvantage:** inefficiency, small consumer surplus

$\Rightarrow$ Combine with regulation

$$\max_x \; p(x)^T x - c(x) \Leftrightarrow$$

$$p_i(x) + \frac{\partial p_i}{\partial x_i} x_i = c' \Leftrightarrow$$

$$p_i(x)[1 + \frac{1}{\varepsilon_i}] = c'$$



Oligopoly

- Firms are not price takers
- Individual decisions can influence prices
- Game theory provides appropriate models
- Many models of competition, results sensitive to assumptions
  - Cournot, Bertrand, Stackelberg, etc.

A “rule-of-thumb” result:

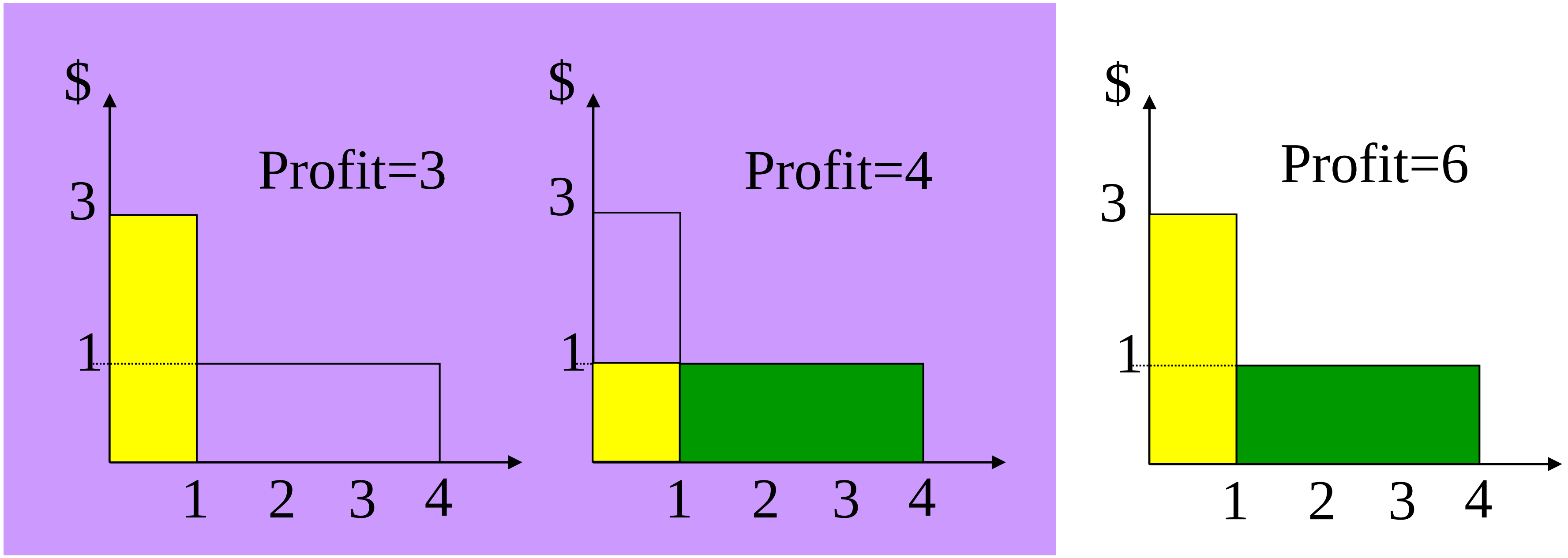
-Assume *n* identical competing firms

-Market demand function = $p(Y)$

Prices: $$p_i(Y)[1+\frac{1}{n\varepsilon_i}]=c'(Y)$$

Price discrimination: an example

Sell a product to different customer types



Price discrimination: personalized pricing, versioning, group pricing

Personalized pricing (1)

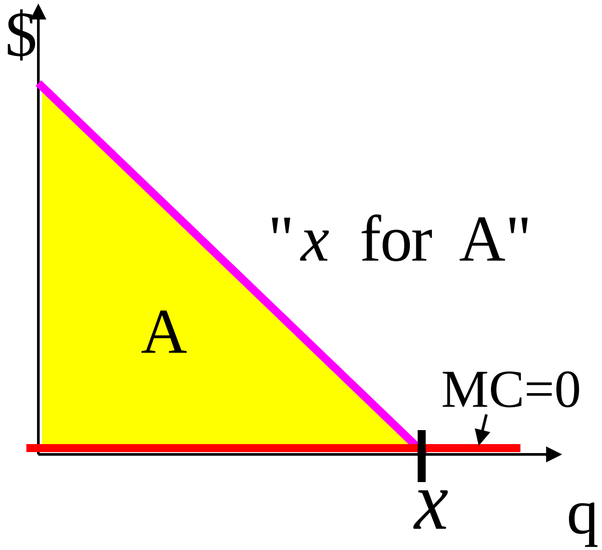
First-degree price discrimination:

- extracts maximum profit from customer
- addresses each customer separately
- "take it or leave it" offer "amount *x* for *m* dollars"
- Pareto efficient operation

$$\max_{x,m} m - c(x) \quad s.t. \quad u(x) - m \geq 0 \Leftrightarrow$$

$$\max_{x} u(x) - c(x) \Leftrightarrow$$

$$u'(x) = c'(x)$$



Two part tariff

Optimal strategy: use a single volume price to maximize social welfare, then take it all customer surplus back using subscription fees

Example: a customer with utility $u(x_i)$, cost $c(x_i)$

If $\lambda = c'(x_i^*)$ is the price at which social welfare is maximized, then use tariff

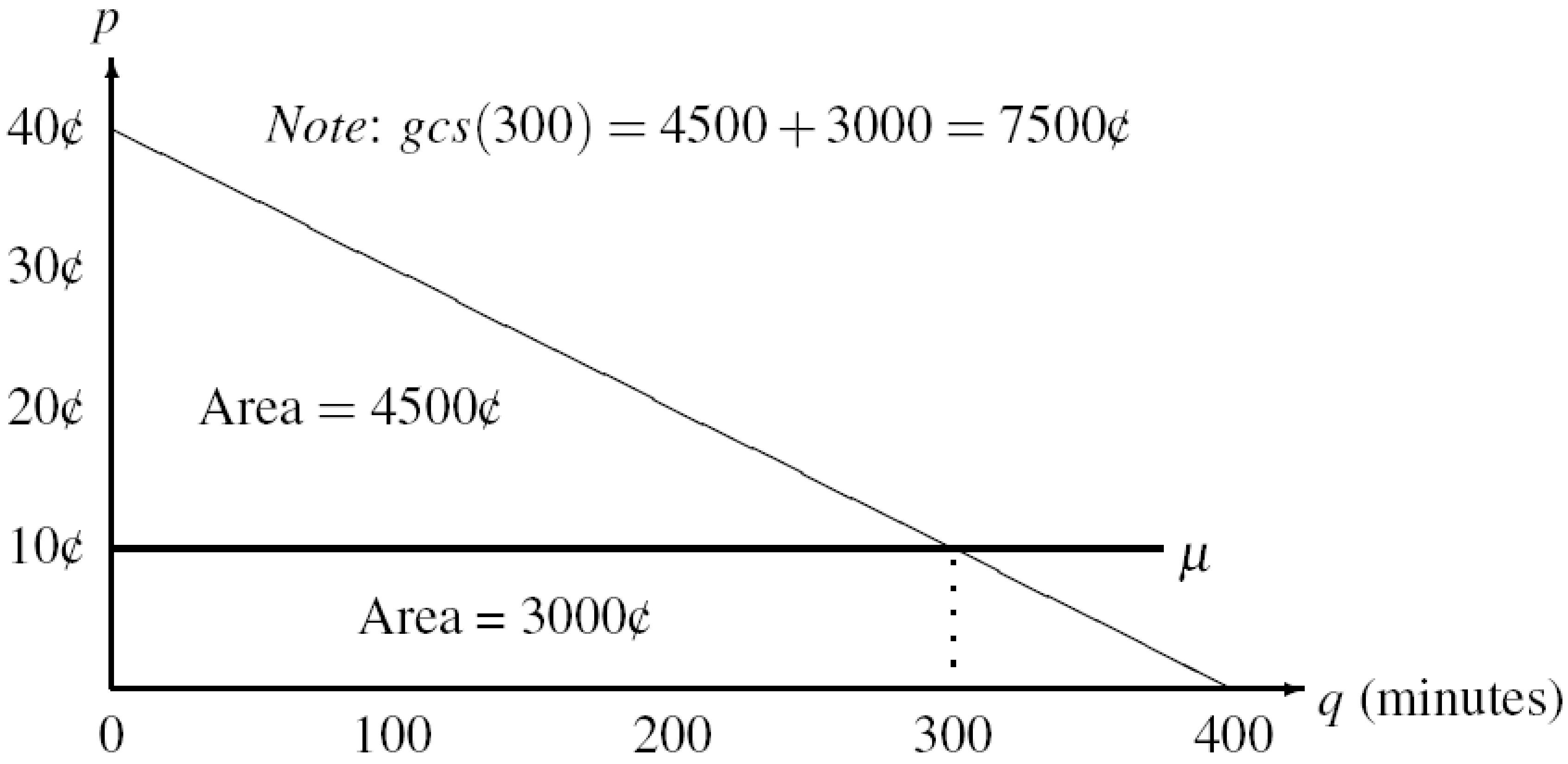
$$\overbrace{[u_i(x_i^*) - \lambda x_i^*]}^{A} + \lambda x_i$$

subscription fee (A) — usage charge (λx_i)

= constant, independent of consumption

Two-part tariff example

- Achieve first-degree price discrimination



Tariff: <f,p>=<$4500,10c>

Personalized pricing (2)

- examples: mail orders, airlines, travel agencies
- information: depends on the kind of enterprise
- price sensitivity of customers is key
  - do market research (promotional pricing)
  - use discount coupons
- Internet: more individualized and interactive
  - price offer depends on what your buying (dynamic)
  - remember customer history
  - inexpensive market research (via promotions)
  - overstock sales

from: Varian and Shapiro: Information Rules

Group pricing (1)

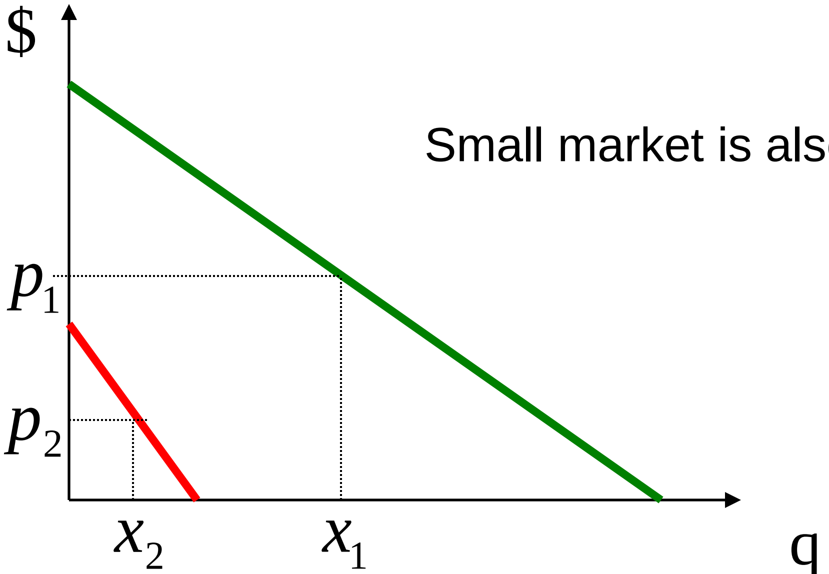
Third-degree price discrimination:

- customer type pricing, no self-selection
- social welfare increases -> increase of output

$$\max_{x_1, x_2} \; p_1(x_1)x_1 + p_2(x_2)x_2 - c(x_1, x_2) \Leftrightarrow$$

$$p_1[1 + \frac{1}{\varepsilon_1}] = c_1'$$

$$p_2[1 + \frac{1}{\varepsilon_2}] = c_2'$$



Small market is also served!

Group pricing (2)

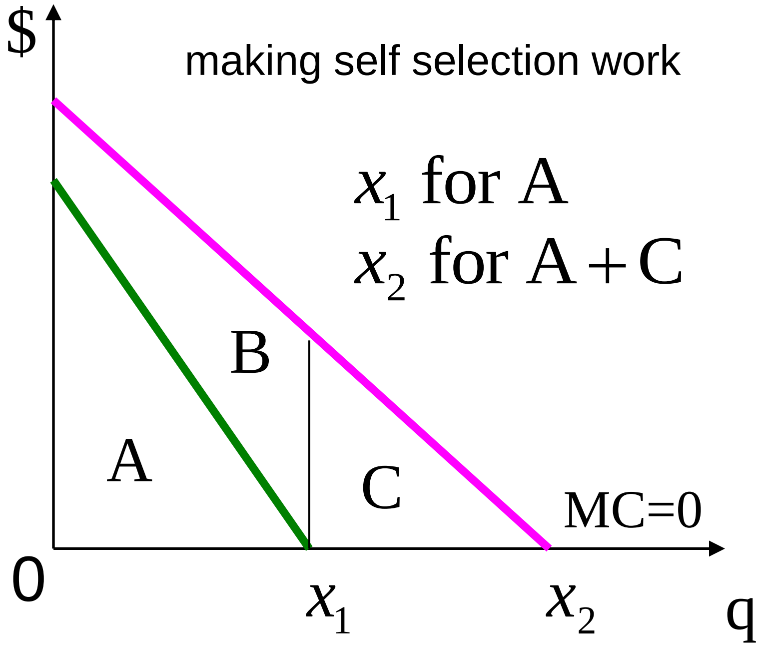
- why sell to groups rather than to end users:
  - price sensitivity: members of different groups differ systematically in price sensitivity
  - network effects: value increases with group ownership
  - lock-in: become ubiquitous in an organization
  - sharing arrangements: pricing for sharing
    - items that are used infrequently by a single user are provided by info intermediaries (libraries, video stores
    - transaction costs determine whether it is better to sell or rent information
    - do even better: offer prices for both sale and rental

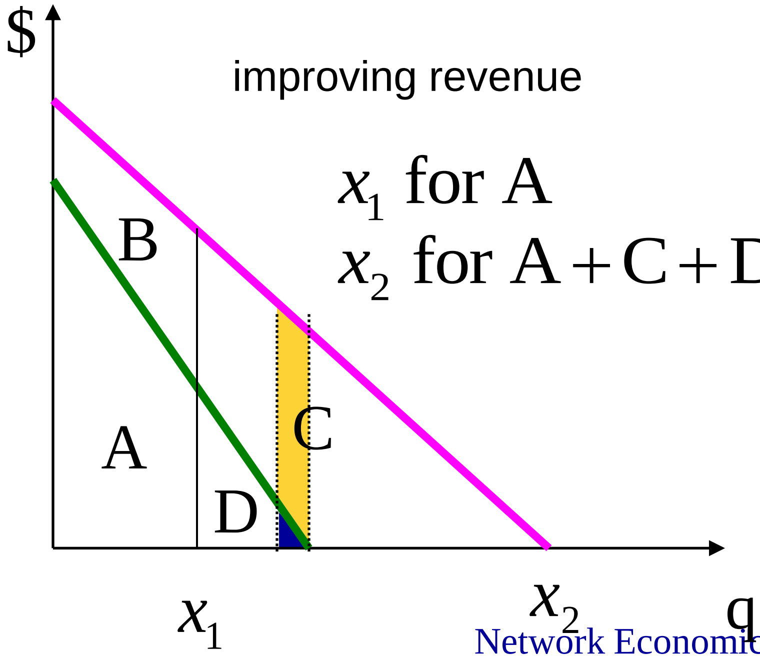
from: Varian and Shapiro: Information Rules

Versioning

Second-degree price discrimination: market segmentation

- set of offers available to all customers
- customers **self select** (incentive compatibility)
- examples: quantity discounts, versioning





Versioning and pricing

- Make prices depend on value to customers
- Don’t need to price by customer identity
- Offer product line, and watch choices
- Design menu of different versions
  - Target different market segments
  - Price accordingly (self selection)
- Traditional information goods:
  - Hardback/paperback
  - Movie/video

from: Varian and Shapiro: Information Rules

Dimensions to use for versions

- Delay
- User Interface
- Image Resolution
- Speed of operation
- Format
- Capability
- Features
- Comprehensiveness

from: Varian and Shapiro: Information Rules

Example

- 40 type As: $100 for speed, $40 for slow
- 60 type Bs: $50 for speed, $30 for slow
- Identity-based pricing: $7000 revenues
- Offer only speedy: $50 is best price, revenues=$5,000
- Offer only slow: not as profitable

from: Varian and Shapiro: Information Rules

Versioning solution

- Try speedy for $100, slow for $30
  - Will this work? Compare benefits and costs
  - 100-100=0, but 40-30=10 > 0
  - Discount the fast version: 100-p=40-30
  - So, p=90
  - Revenues = $5,400 = 90x40 + 30x60

from: Varian and Shapiro: Information Rules

Making self-selection work

- May need to cut price of high end
- May need to cut quality at low end
- Value-subtracted versions
  - May cost more to produce the low-quality version
- In design, make sure you can turn features off!

from: Varian and Shapiro: Information Rules

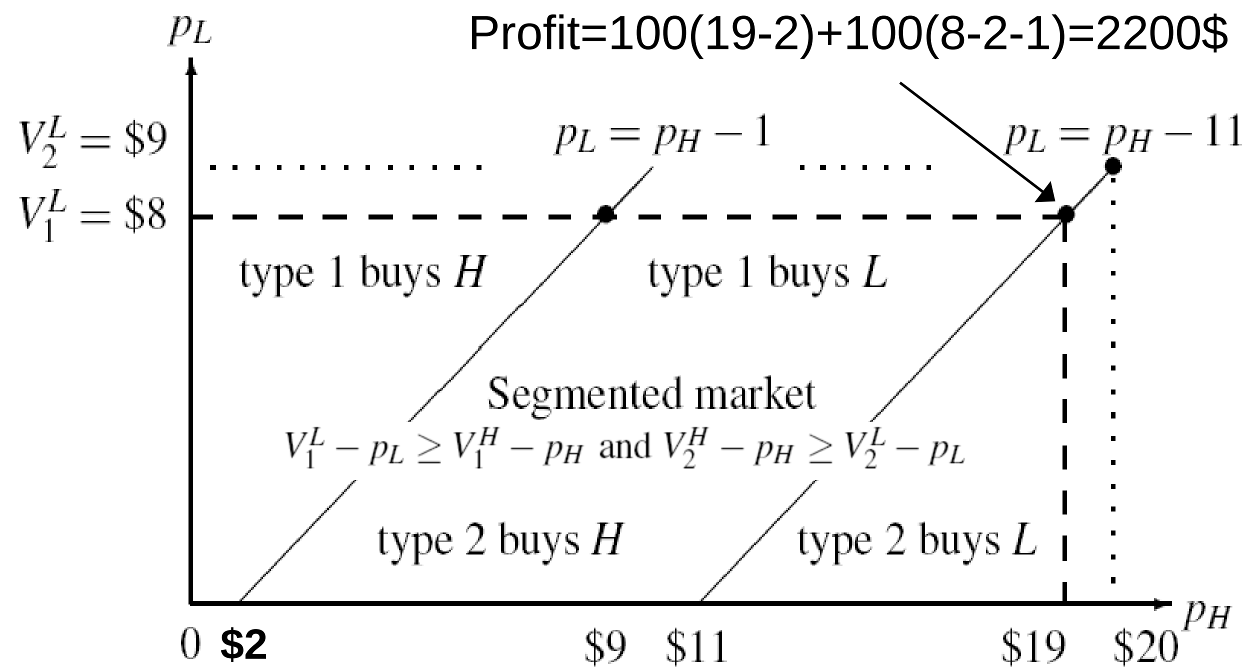
How many versions?

- One is too few
- Ten is (probably) too many
- Two things to do
  - Analyze market
  - Analyze product
- Analyze your market: does it naturally subdivide into different categories? are behaviors sufficiently different?
- Analyze your product: design for high-end, reduce quality for low-end
- Default choice: 3 versions
- Extremeness aversion

from: Varian and Shapiro: Information Rules

Damaged good example

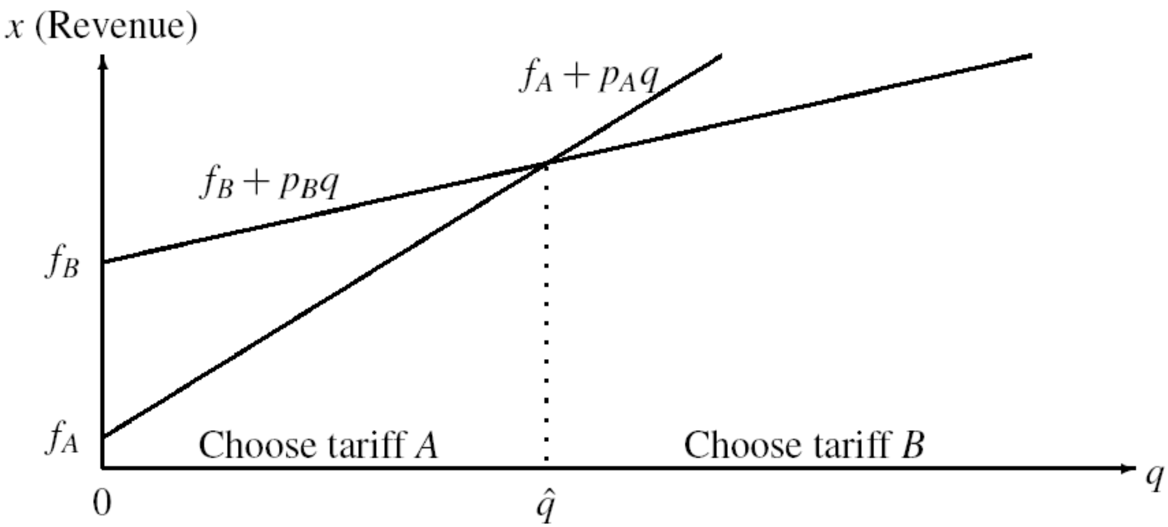
i (Quality)	$\ell = 1$	$\ell = 2$	μ_i (Unit Cost)
H (Original)	$V_1^H = \$10$	$V_2^H = \$20$	$\$2$
L (Damaged)	$V_1^L = \$8$	$V_2^L = \$9$	$\$2 + \$1$
N_ℓ (# consumers)	$N_1 = 100$	$N_2 = 100$	



From "How to Price"
Oz Shy

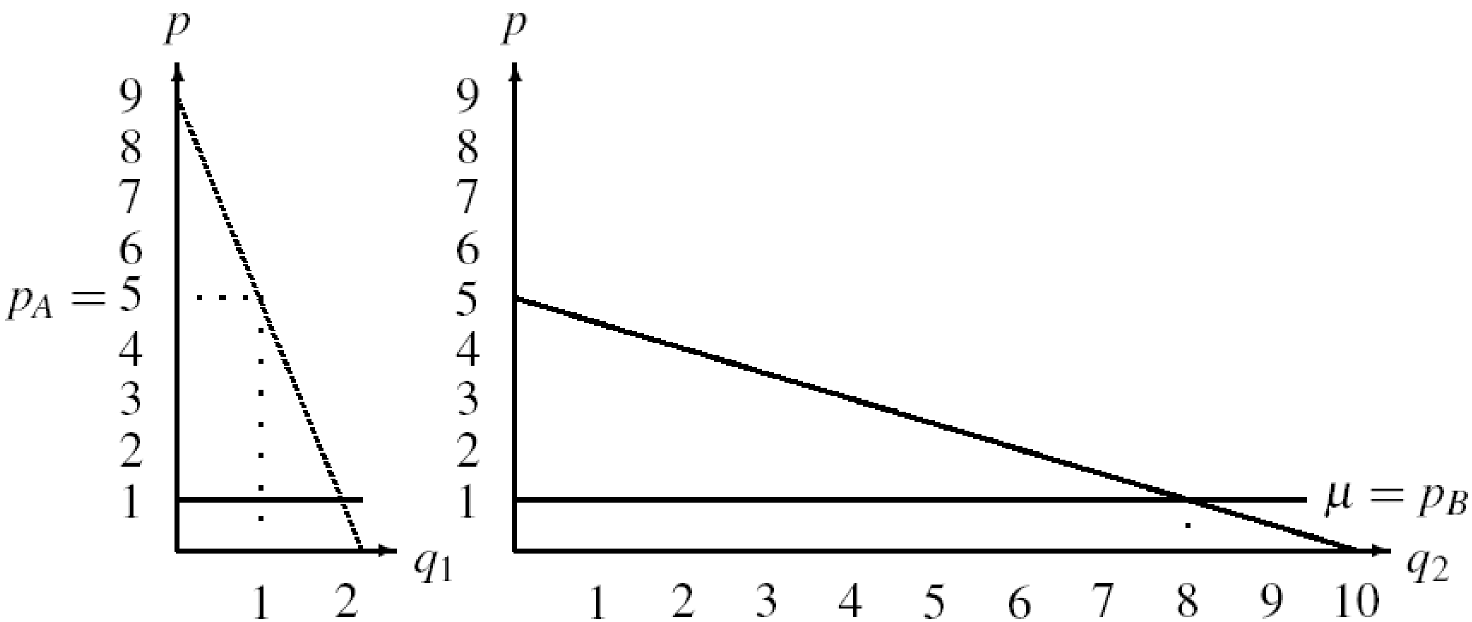
Two-part tariffs

- Same as versioning
  - Price: determines quantity consume
  - Fixed part: lump sum of money requested



Two-part tariffs

- Highest demand customer pays MC
- Lower demand gets zero surplus

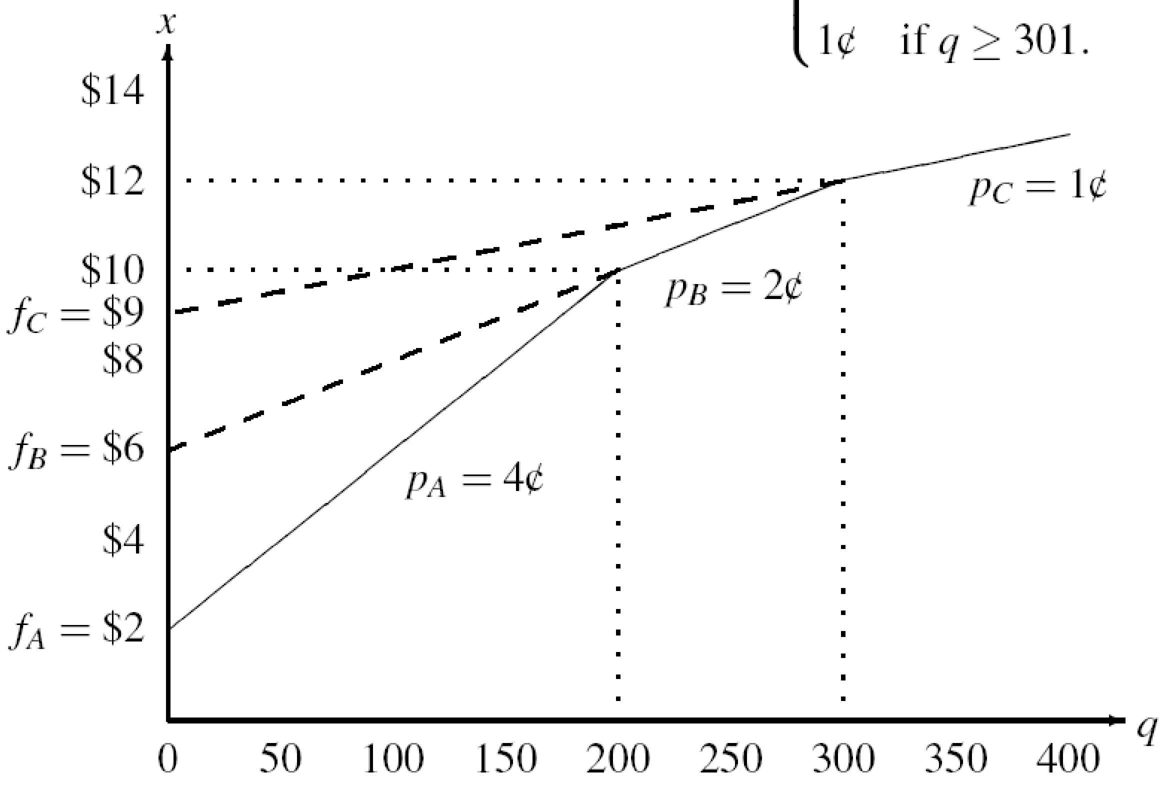


Optimal tariffs: <\$1, \$5>, <\$16,\$1>

Multipart tariffs

- Multipart tariffs are equivalent to multiple 2-part tariffs

$$f^{mp} = \$2 \quad \text{and} \quad p^{mp}(q) = \begin{cases} 4¢ & \text{if } 0 \le q \le 200 \\ 2¢ & \text{if } 201 \le q \le 300 \\ 1¢ & \text{if } q \ge 301. \end{cases}$$



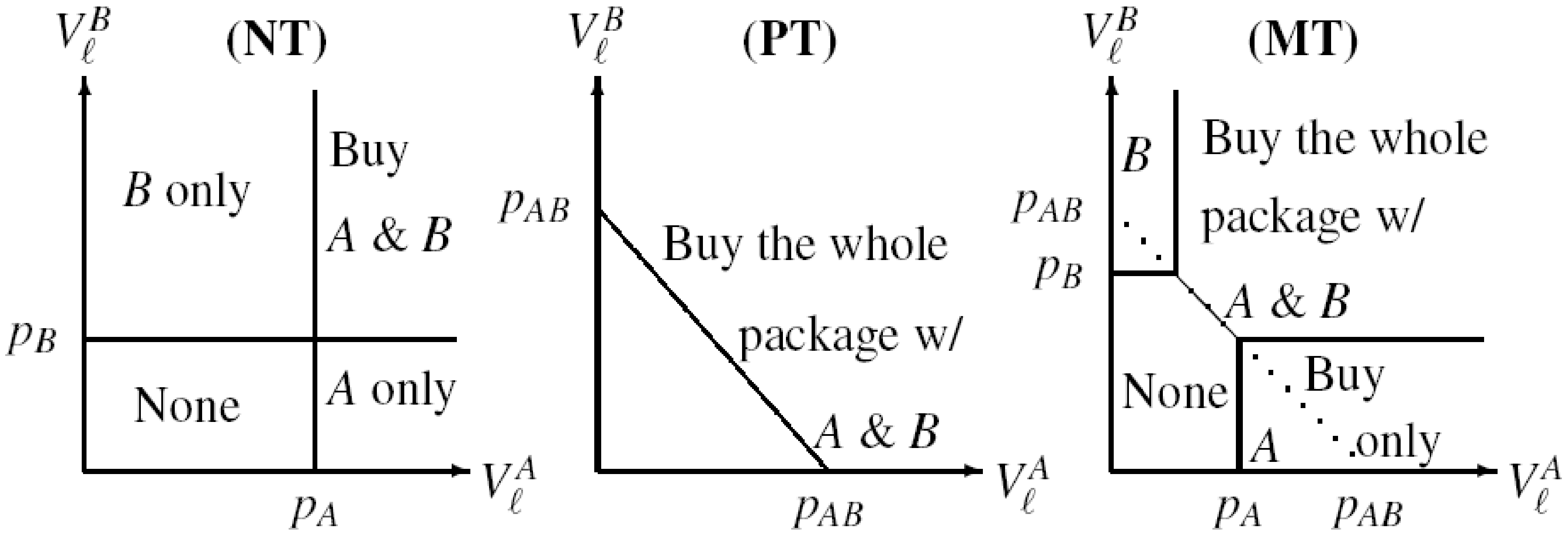
Tying (Bundling)

- Offer many goods as a package
- Example: Microsoft Office
- Added benefit: they work together
- Price of bundle < sum of component prices
  - buy one product, then other is priced less than standalone price
- Reduce dispersion in customer value
  - Example: price separate or together
  - Mark: $120 for WP, $100 for spreadsheet
  - Noah: $100 for WP, $120 for spreadsheet
  - Profits
    - Without tying: $400
    - With tying: $440

from: Varian and Shapiro: Information Rules

Tying

- Consumer choices under no tying, pure tying and mixed tying



Externalities

Externalities

- Externalities: the actions of one agent affect the utility of an other agent: Total utility of agent 1 $= u_1(x_1) \pm g(x_2)$
- Positive (network effects), negative (congestion)
- No externality:

$$SW = u_1(x_1) - c_1(x_1) + u_2(x_2) - c_2(x_2)$$

$$\max SW \Leftrightarrow u_1^{'}(x_1^{*}) - c_1^{'}(x_1^{*}), u_2^{'}(x_2^{*}) - c_2^{'}(x_2^{*})$$

- Externality:

$$SW = u_1(x_1) - c_1(x_1) \pm g_1(x_2) + u_2(x_2) - c_2(x_2)$$

$$\max SW \Leftrightarrow u_1^{'}(x_1^{*}) - c_1^{'}(x_1^{*}), u_2^{'}(x_2^{*}) - c_2^{'}(x_2^{*}) \pm g_1^{'}(x_2^{*})$$

- SW optimal prices can not be determined by the market alone: need special price mechanism that takes account of the externalities

Example

- n identical users, user i consumes x_i, marginal cost =2
- (a) Positive externalities (network effects)

$$u_i(x) = U(x_i) + [x_1, \ldots, x_n]$$

positive effect of other users participating

- (b) Negative externalities (congestion)

$$u_i(x) = U(x_i) - [x_1, \ldots, x_n]$$

negative effect (disutility) because of other users participating

- Price = MC = 2. User i maximizes over x_i

$$(a)\ U(x_i) + [x_1, \ldots, x_n] - 2x_i = U(x_i) - x_i$$

$$(b)\ U(x_i) - [x_1, \ldots, x_n] - 2x_i = U(x_i) - 3x_i$$

- Social planner maximizes:

>>0

$$(a)\ \forall i: U(x_i) + nx_i - 2x_i = U(x_i) + (n-2)x_i$$

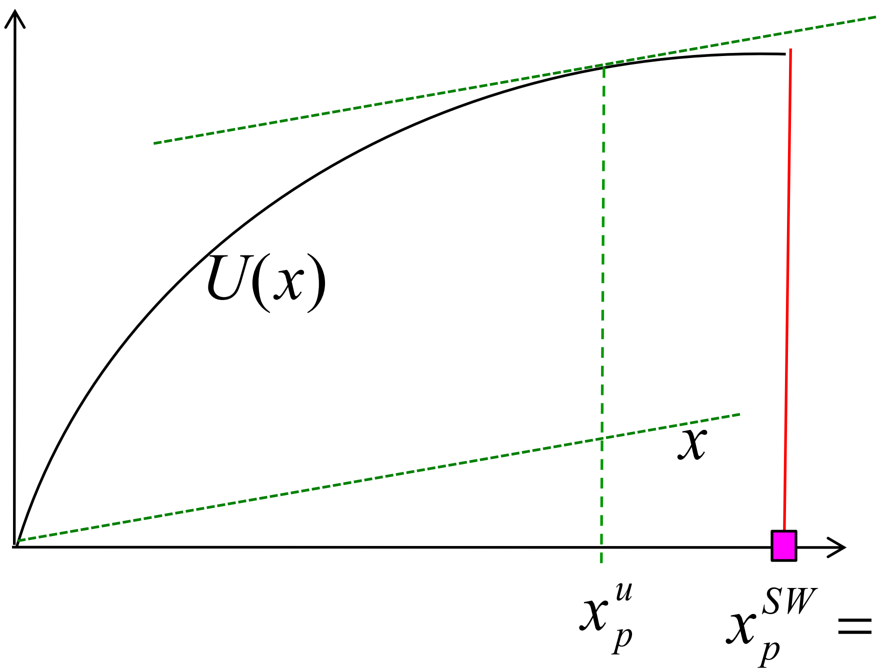
$$(b)\ \forall i: U(x_i) - nx_i - 2x_i = U(x_i) - (n+2)x_i$$

Externalities

Network effects

$$User: U(x_i) + x_i - 2x_i = U(x_i) - x_i$$

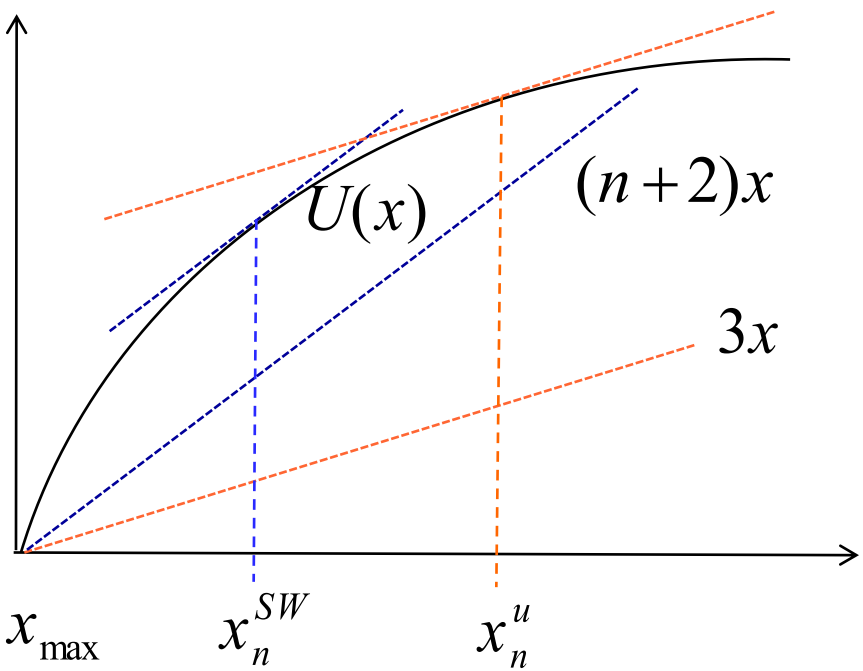
$$SW: U(x_i) + nx_i - 2x_i = U(x_i) + (n-2)x_i$$



Congestion

$$User: U(x_i) - x_i - 2x_i = U(x_i) - 3x_i$$

$$SW: U(x_i) - nx_i - 2x_i = U(x_i) - (n+2)x_i$$



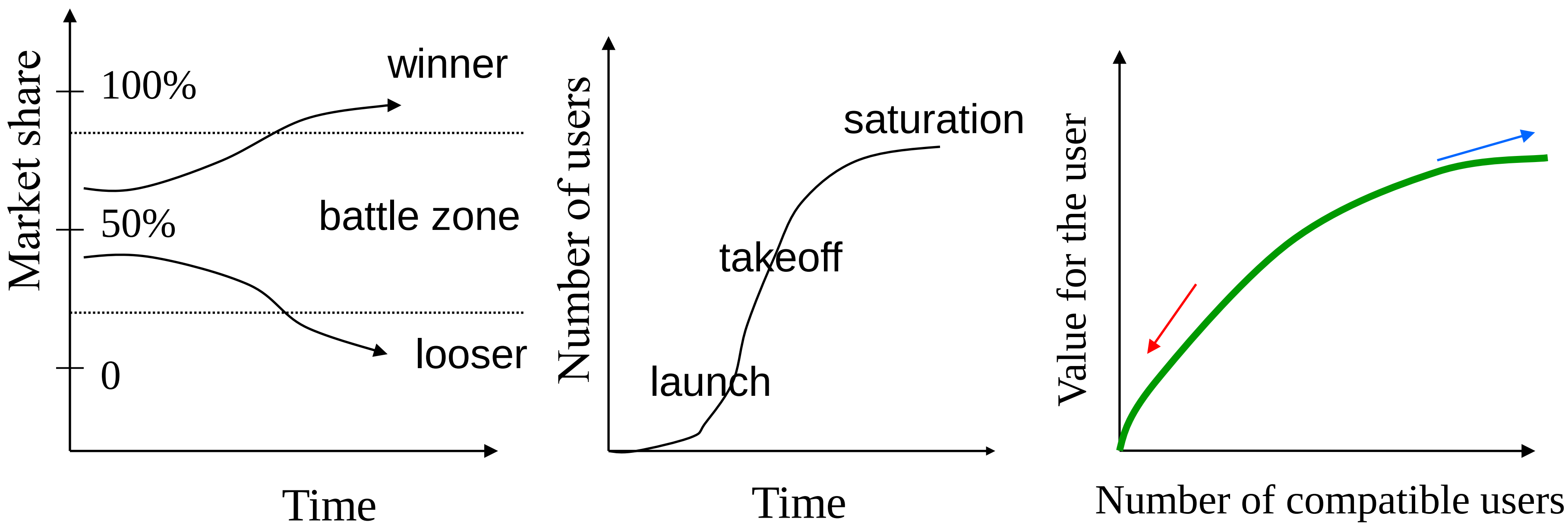
Networks and Positive Externalities

From “Information Rules” by

Carl Shapiro and Hal R. Varian

Positive externalities: positive market feedback

- Positive feedback: strong get stronger, weak get weaker
- Negative feedback: stabilizing effect
- Makes a market “tippy”
- Examples: VHS v. Beta, Wintel v. Apple
- “Winner take all markets”



Sources of positive feedback

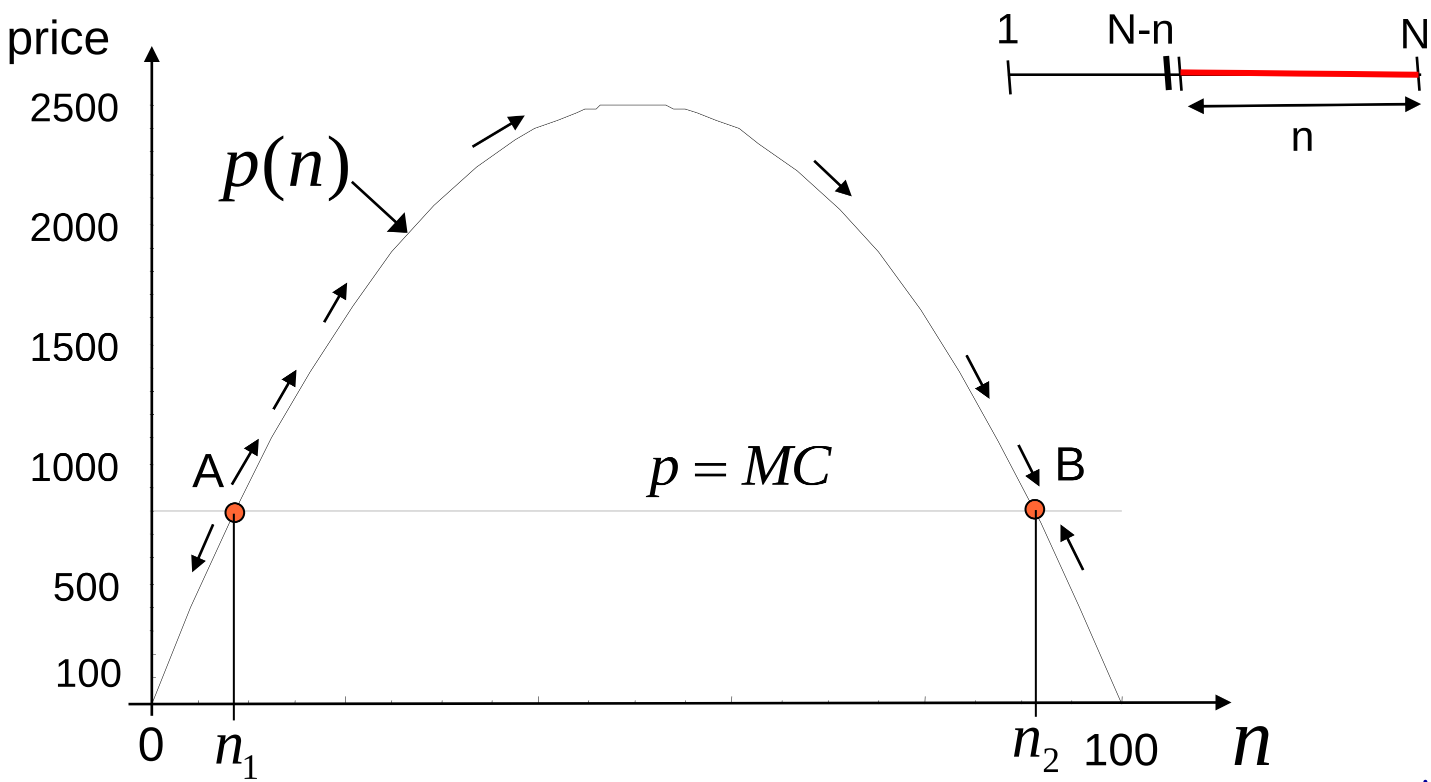
- **Supply side** economies of scale
  - Declining average cost
  - Marginal cost less than average cost
  - Example: information goods
- **Demand side** economies of scale
  - Network effects: virtual networks
    - Network externalities: one market participant affects others without compensation being paid.
  - Examples: fax, email, Web, Sony v. Beta, Wintel v. Apple

Network effects (1)

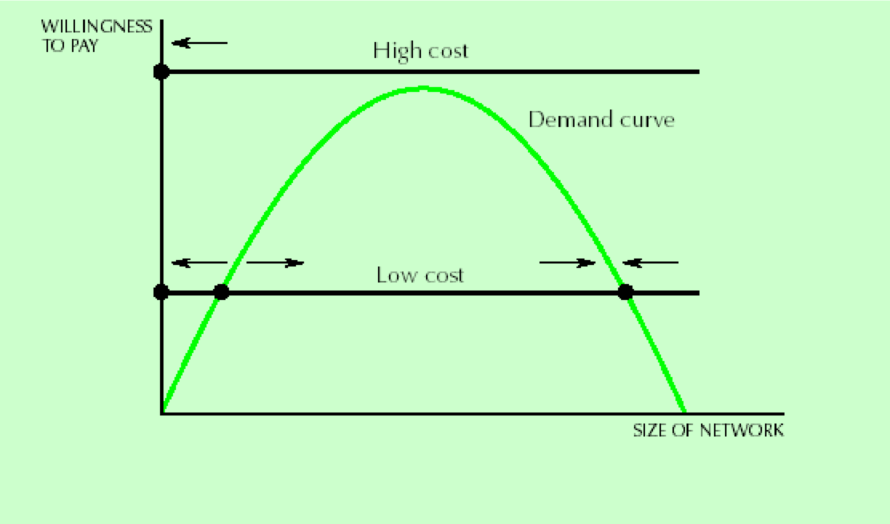
$$i = 1,\ldots,N,\ u_i(n) = ni$$

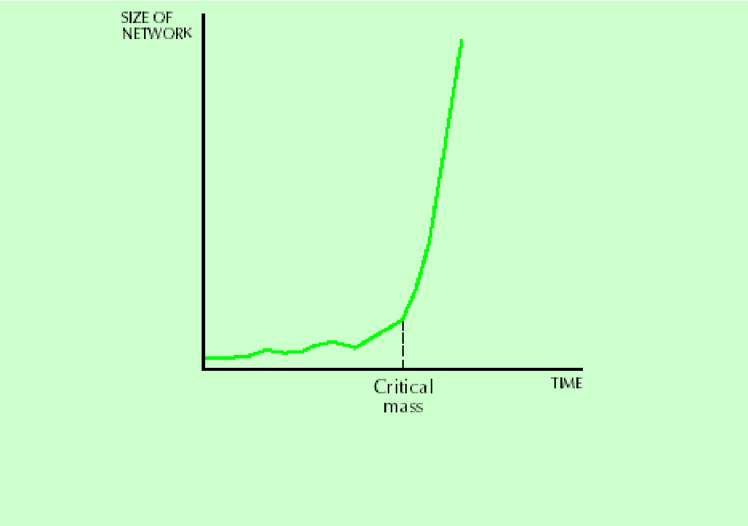
$$\text{assume } p \rightarrow n : N-n+1,\ldots N \text{ consume}$$

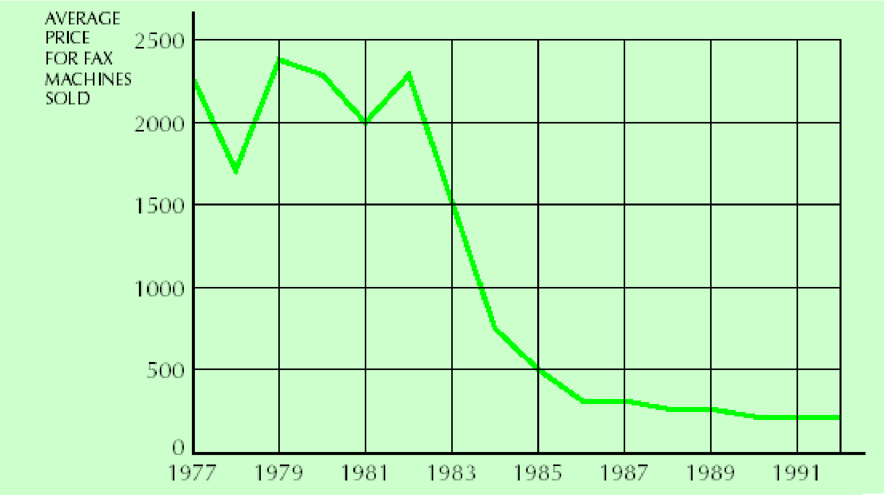
$$\text{marginal customer} = N-n,\ \hat{u} = (N-n)n, \Rightarrow p = n(N-n)$$

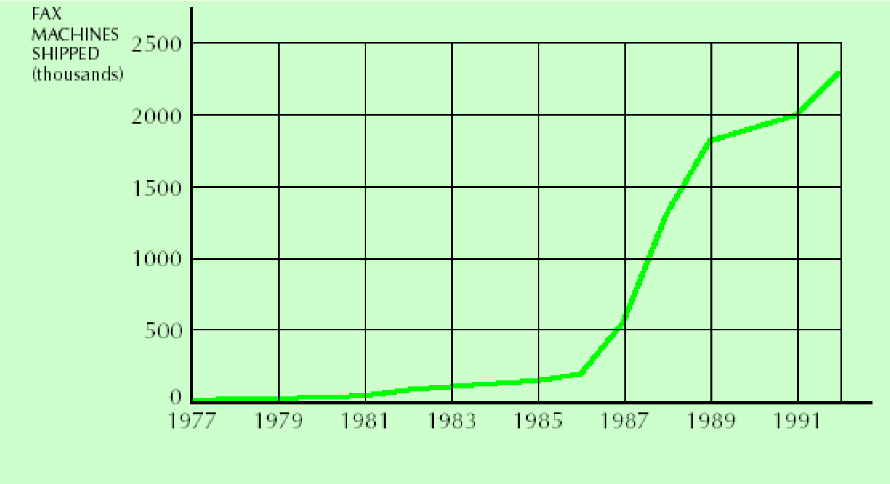


Network effects (2)









Key observations

- Number of users is important
  - Metcalfe's Law:
    Value of network of size n proportional to n^2
  - More likely nlogn
- Importance of expectations
- Network effects lead to substantial collective switching costs: even worse than individual lock-in (due to coordination costs). Example: QWERTY
- Evolution vs revolution, openness vs. control (standards setting)
- Network externalities don't always apply
  - ISPs (but watch out for QoS)
  - PC production

Pricing with (positive) externalities

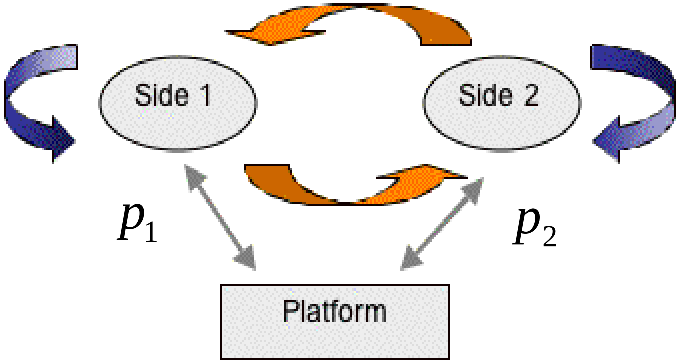
Two sided markets

Definition

- **Two-sided markets** (**two-sided networks**) are economic platforms having two distinct user groups that provide each other with network benefits
- Examples: credit cards (cardholders and merchants); operating systems (end-users and developers), yellow pages (advertisers and consumers); video game consoles (gamers and game developers); communication networks, such as the Internet (end users, content providers)
- Members of each group exhibit a preference regarding the number of users in the other group; these are called cross-side network effects
- Explain many free pricing strategies where one user group gets free use of the platform in order to attract the other user group

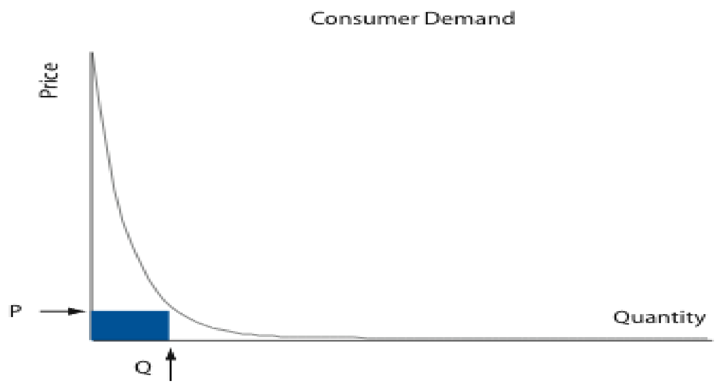
1-sided:
volume of interaction

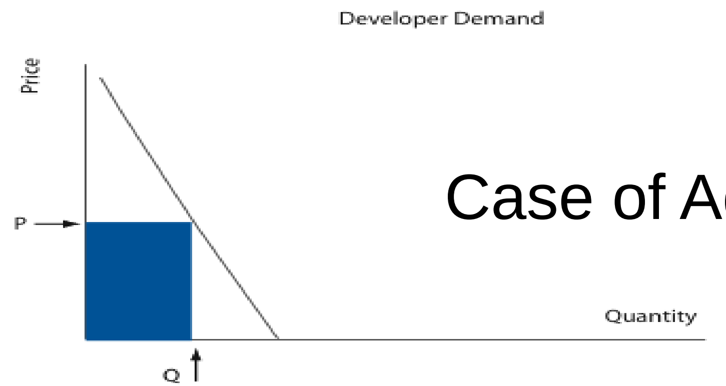
$$V = f(p_1 + p_2)$$



Pricing in two-sided markets

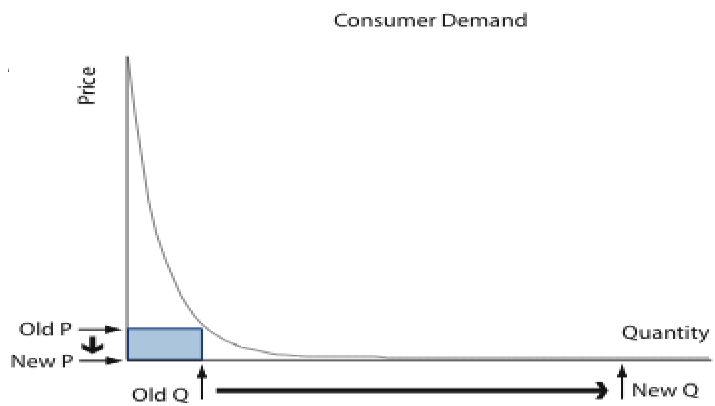
- Pricing each group in a two-sided network must consider network effects

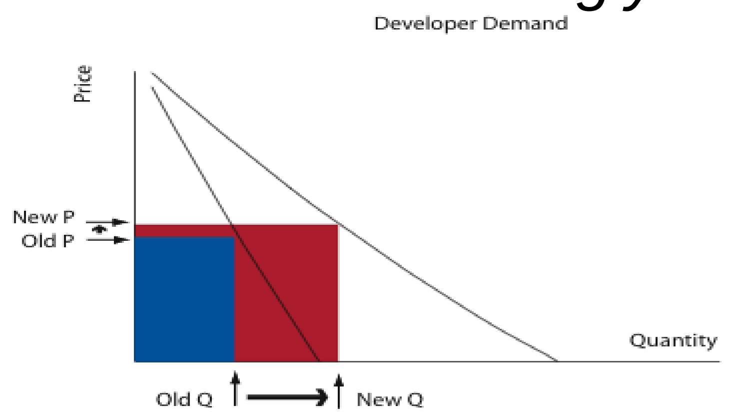




Case of Adobe

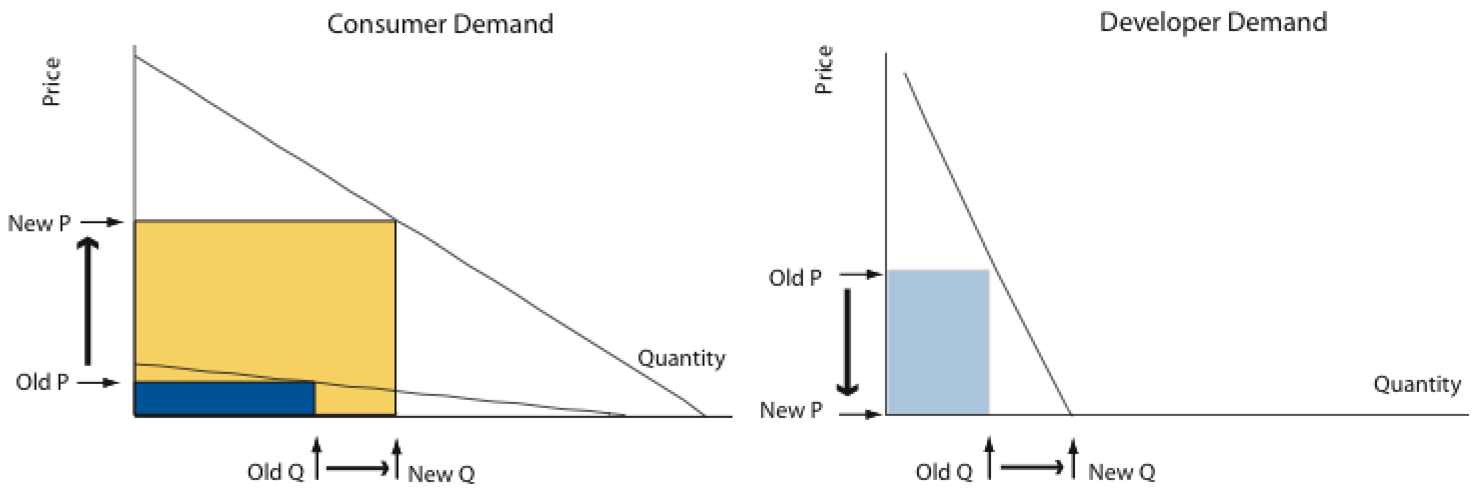
- Rule 1: *subsidize the more price sensitive side, and charge the side whose demand increased more strongly in*





Pricing in two-sided markets

- Rule 2: *subsidize those who add platform value*



Case of Microsoft vs Apple

Negative externalities

Congestion pricing

Defining a congestion price

- Define:

$$B = \text{amount of resource (bandwidth)}$$

$$y = \text{utilization of resource} = \sum_r x_r / B = X / B$$

$$D(y) = \text{packet delay} = \text{for M/M/1: } 1/(B - X)$$

$$g(B) = \text{cost of resource}$$

- The maximization problem including choosing capacity

Congestion depends only on utilization value

$$\max_{\{x_i\},B} \sum_{i=1}^{n} u_i(x_i, D(y)) - c(B)$$

- Here congestion (negative externality) depends only on the utilization parameter (hence it expresses delay, loss probability,…)

Analysis

- The first-order optimality conditions (for fixed B) are

$$\frac{\partial u_i(x_i,D)}{\partial x_i} + \frac{\partial D}{\partial x_i}\sum_j \frac{\partial u_j(x_j,D)}{\partial D} = 0,\ i = 1,\ldots,n \quad \text{(i)}$$

which suggest a congestion price $p_E = -\frac{\partial D}{\partial X}\sum_i \frac{\partial u_i(x_i,D)}{\partial D}$

- Lets check: user i solves $\max_{x_i}\{u_i(x_i,D) - p_E x_i\}$

$$\Leftrightarrow \frac{\partial u_i}{\partial x_i} + \frac{\partial D}{\partial x_i}\frac{\partial u_i}{\partial D} - p_E = \frac{\partial u_i}{\partial x_i} + \frac{\partial D}{\partial x_i}\frac{\partial u_i}{\partial D} + \frac{\partial D}{\partial x_i}\sum_j \frac{\partial u_j(x_j^*,D)}{\partial D} = 0$$

which is the same as (i) when n is large

Remarks

- Note that $p_E = -\frac{\partial D}{\partial x_i} \sum_i \frac{\partial u_i(x_i, D)}{\partial D}$

is the marginal increase of the negative externality for a marginal increase of x_i

- Or the willingness of the users to pay for not increasing the rate
- To compute it we need to know the utility functions of the participants

Capacity expansion

- Do the maximization including the choice of B: maximize

$$W(B) = \sum_i u_i(x_i^*, D^*) - c(B)$$

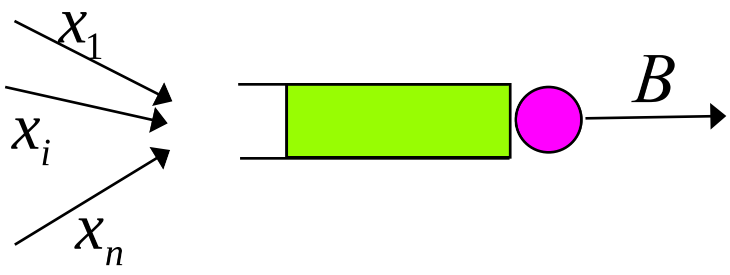
- Algebra: $D(X,B) = const \Leftrightarrow \frac{\partial D}{\partial X}\Delta X + \frac{\partial D}{\partial B}\Delta B = 0 \Leftrightarrow \frac{\partial D}{\partial B} = -\frac{\partial D}{\partial X}\frac{\Delta X}{\Delta B}$ (*)
- First order conditions:

$$W'(B) = \sum_i \frac{\partial W}{\partial x_i^*}\frac{dx_i^*}{dB} + \frac{\partial W}{\partial D}\frac{dD}{dB} - c'(B) = 0 \Leftrightarrow$$

$$\sum_i \frac{\partial u_i}{\partial D}\frac{dD}{dB} - c'(B) = \sum_i \frac{\partial u_i}{\partial D}\frac{\partial D}{\partial X}\frac{\Delta X}{\Delta B} - c'(B) = 0 \Leftrightarrow$$

$$\frac{\Delta X}{\Delta B} p_E = c'(B) \Leftrightarrow \text{expand if } \frac{p_E}{c'} > \frac{\Delta B}{\Delta X} = 1 \text{ for M/M/1}$$

Example: delay cost at a single link



$$U_i(x_i, D) = u_i(x_i) - \gamma_i x_i D$$

$$D(X,B) = \frac{1}{B - \sum_j x_j}$$

$$\text{Max SW}: \max_{x_1, \ldots, x_n} \sum_j [u_j(x_j) - \gamma_j x_j D(\sum_k x_k)]$$

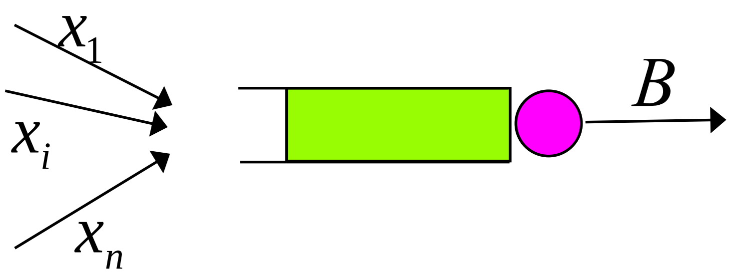
$$\Leftrightarrow u_i' - \gamma_i D - \gamma_i x_i D' - D' \sum_{j \neq i} \gamma_j x_j = 0 \qquad (1)$$

$$\text{Free market equilibrium : User i : } \max_{x_i} [u_i(x_i) - \gamma_i x_i D(\sum_k x_k)]$$

$$\Leftrightarrow u_i' - \gamma_i D - \gamma_i x_i D' = 0 \qquad (2)$$

the system is more congested!

Delay cost at a single link



User i: $u_i(x_i) - \gamma_i x_i D$

$$D(X,B) = \frac{1}{B - \sum_j x_j}$$

$$\text{Max SW} : \max_{x_1,\ldots x_n} \sum_j [u_j(x_j) - \gamma_j x_j D(\sum_k x_k)]$$

$$\Leftrightarrow u_i' - \gamma_i D - \gamma_i x_i D' - D' \sum_{j \neq i} \gamma_j x_j = 0 \qquad (1)$$

same conditions

To maximize SW : charge x_i with price $p_i^c = D' \sum_j \gamma_j x_j$

$$\text{User i} : \max_{x_i} [u_i(x_i) - \gamma_i x_i D(\sum_k x_k) - p_i^c x_i]$$

$$\Leftrightarrow u_i' - \gamma_i D - \gamma_i x_i D' - p_i^c = 0 \qquad (3)$$

For n large use uniform price $p^c = D' \sum_j \gamma_j x_j = \dfrac{1}{(B - \sum_j x_j)^2} \sum_j \gamma_j x_j$

Expanding capacity

- Congestion prices convey information for capacity expansion

$$\frac{\partial}{\partial B}\left[\sum_j [u_j(x_j^*) - \gamma_j x_j^* D(\sum_k x_j^*, B)] - c(B)\right] = 0$$

$$\Leftrightarrow \sum_j \frac{\partial}{\partial x_j} SW(x_1^*, \ldots, x_n^*) \frac{\partial x_j}{\partial B} + \frac{\partial SW}{\partial B} - c'(B) = 0$$

$$\Leftrightarrow \frac{\partial SW}{\partial B} - c'(C) = 0$$

$$\Leftrightarrow \sum_j \gamma_j x_j^* D'(\sum_k x_j^*, B)] - c'(C) = 0$$

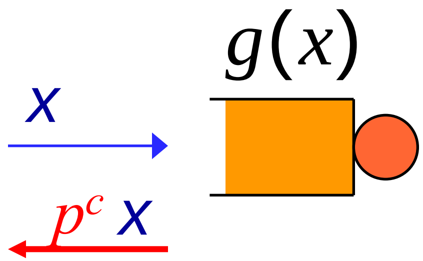
$$\Leftrightarrow p^c = c'(C)$$

Congestion prices on sample paths

- Two practical problems to compute congestion prices
  - to take derivatives we need the form of the utilities
  - need to compute average performance measures in network (slow, inaccurate)
- Instead of constructing deterministic prices that reflect derivatives of some average quantity, construct fluctuating prices that capture temporal congestion effects -> result in same average price
- Charge each packet individually for the cost it imposes to other packets
- How do we learn the delay cost of individual packets if not uniform?

Computing congestion prices

1. Congestion charge rate px is computed on an average basis



$$p^c = \frac{\partial g}{\partial x}$$

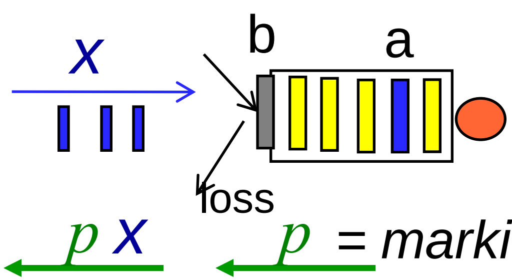
x = average flow

g = average congestion cost

loss, delay

2. Each packet is charged the cost increment that it causes

Assume: cost unit = extra cost caused by a single packet when loss occurs



Packet **a** is charged the **extra cost** it causes

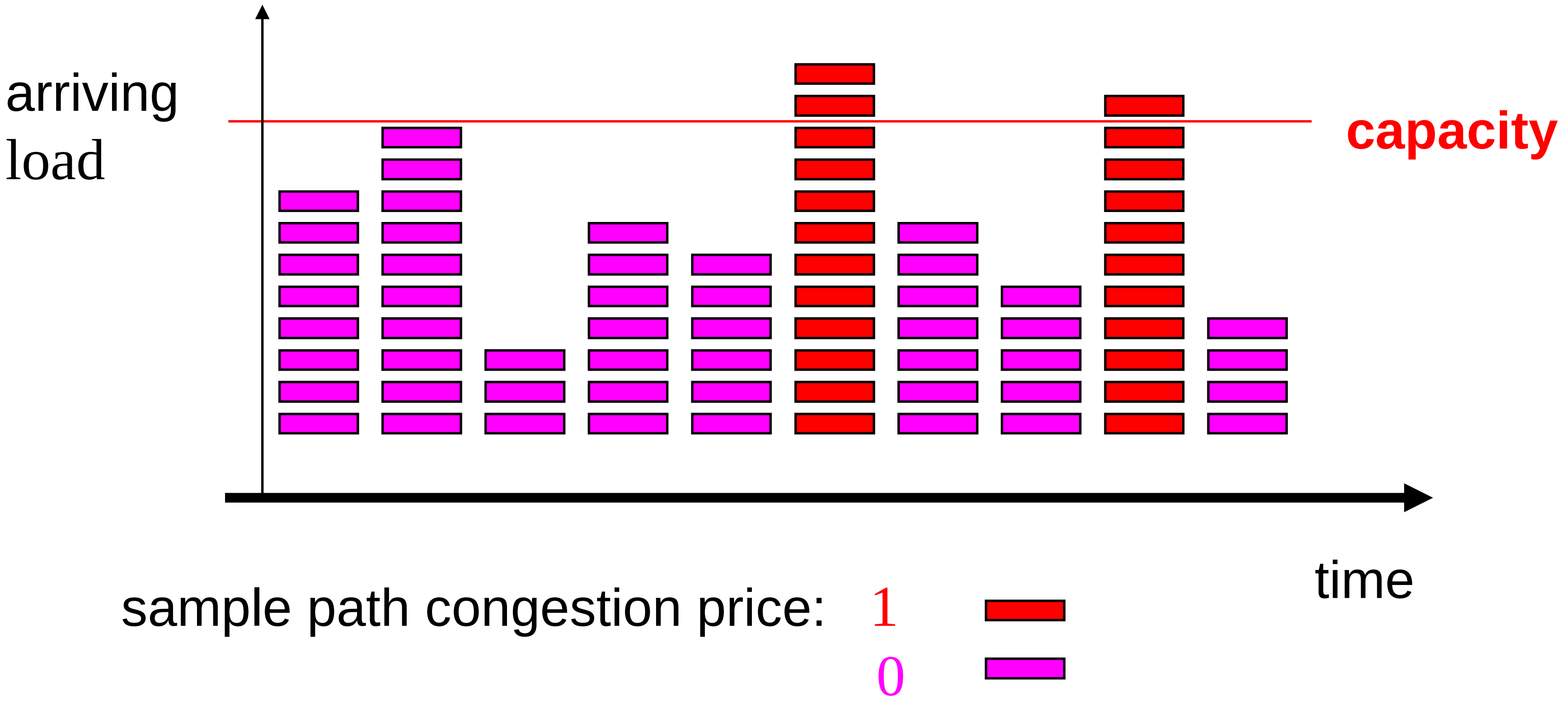
(sender of **a** receives one congestion mark= 1c)

p = *marking probability*

- The rate of charge px is averaged on the particular sample path
- In most systems marking prob $p \approx \frac{\partial g}{\partial x} = p^c$

Sample path congestion prices

Example: Server that serves up to 10 packets in each time slot



How to reveal true congestion cost

- **Need to design a mechanism**
- n packets queue at a router
- Schedule packets to minimize weighted delay cost $\sum_i c_i^* D_i$
- How do we learn the costs c_i^*?
- Mechanism Design paradigm!

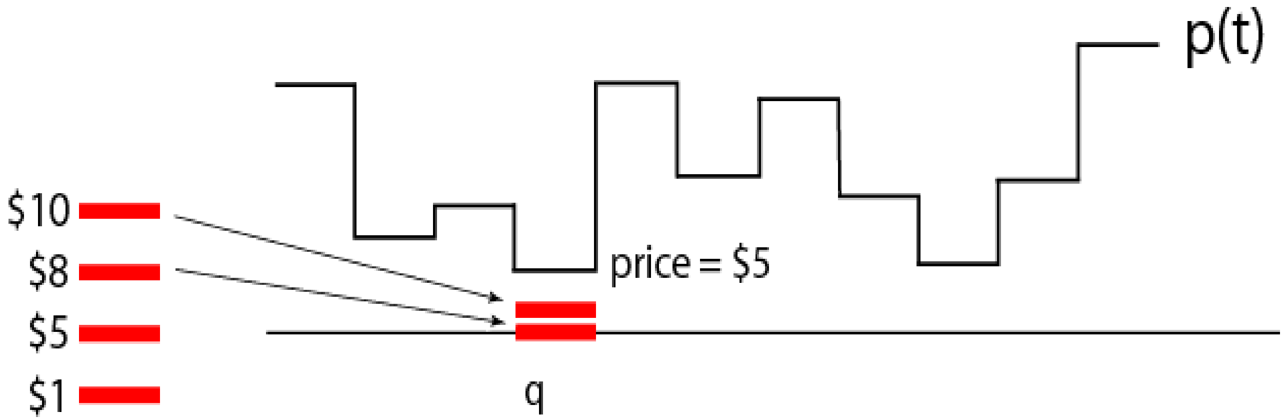
packet service time

1. Use "$c\mu$" rule: serve in decreasing order of c_i^* / T_i
2. Charge each packet the cost it causes to the other packets behind it

- => Incentive compatible and optimal!

Smart markets

- Which k packets to serve in a time slot?
- Use a bandwidth auction in each time slot
- Packets declare maximum price they are willing to pay (bids)
- System accepts the k packets with the highest bids
- Packets pay a uniform price = highest bid of not accepted packet
- Type of congestion charge (why?)
- Incentive compatible



Information issues

A market of lemons

Information

- Economic agents that interact make decisions based on information available regarding the other agents
- Less information available leads to decrease of efficiency
- **Adverse selection** occurs when some type of agent finds it profitable to choose an offer intended for another type. As a result, the seller obtains less profit than anticipated
  - There may be no prices for firm to recover costs
  - $\Rightarrow$ no equilibrium
  - Beneficial for both seller and buyers to **signal** information

Adverse selection and ISPs (1)

- n potential customers, each requiring x units of Internet use, x *uniformly* distributed on [0,1]
- A customer of type x has a utility $u(x) = x \Rightarrow$ he won't buy service if his *surplus* $x - w$ is negative
- The network exhibits economies of scale. The **unit cost** when using total bandwidth b for its customers is $p(b) \leq 1$
  - $p(b)$ includes a **discount factor** that varies linearly from $\alpha < 1$ to 1 with the total amount of bandwidth purchased

$$p(b) = a\frac{b}{n/2} \quad +1 \quad \left(1 - \frac{b}{n/2}\right)$$

Adverse selection and ISPs (2)

- **Complete information**:
  - customer of type x is charged $w(x) = x - \varepsilon$
- All customers subscribe, provider and customers have positive profits

$$p(n/2) = \alpha < 1$$

$\pi(x) = x - \varepsilon - x\alpha = x(1-\alpha) - \varepsilon > 0$ for small enough ε

Adverse selection and ISPs (3)

- **Incomplete information**: price is same for all customers
- **Adverse selection**: price targeted to recover costs for average customer, heavy customers profit and increase average cost => no stable market
- Assume that provider charges w
- $n(1-w)$ heaviest customers subscribe, $b = 1/2n(1-w)(1+w)$
- Typical customer $\bar{x} = 1/2(1+w)$
- Profit from typical customer =

$$\bar{\pi} = w - \frac{1}{2} p(b)(1+w) = w - \frac{1}{2}[1-(1-a)(1-w^2)](1+w)$$

$\bar{\pi} < 0$ if $\alpha > 0.7465$ for all values of w

Game theory

What is game theory?

- Traditional optimization: theory of optimal decision making of a single agent
- Game theory: study of interacting decision makers
- Games: models of interactive decision making
  - strategic form: a player chooses his plan of action once and for all covering all possible contingencies
  - extensive form: explicit description of sequential structure of the decision problems
  - different solution concepts
- one-shot, repeated games

The prisoner's dilemma

Example of strategic game

Description: game matrix (common knowledge)

Nash equilibrium: each player's strategy choice is a best reply to the strategy choices of the other players

Player B

strategies (arrow pointing to the strategy labels)

Player A		cooperate	defect
	cooperate	3,3	0,4
	defect	4,0	1,1

Nash equilibrium = (defect,defect)

= dominant strategy equilibrium

Other concepts

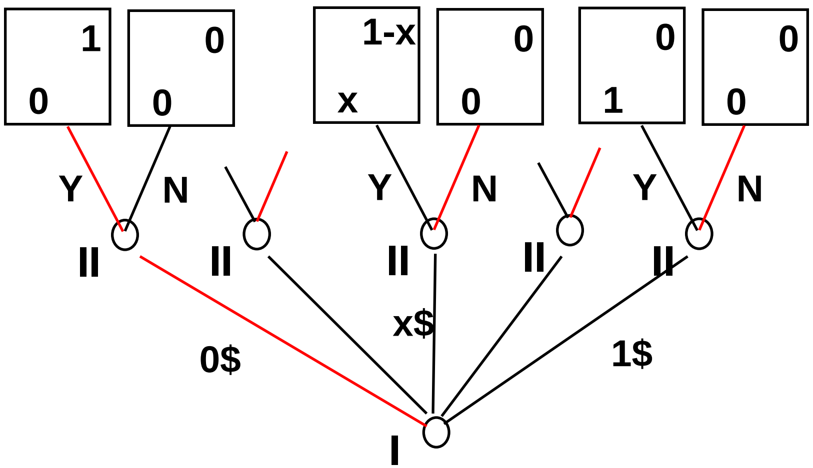
- Nash equilibria may involve mixed (randomized) strategies
- Nash equilibria always exist, but may be many!
- Which one is reasonable to expect?
  - dominant strategy equilibrium: simplify the game by eliminating dominated strategies
  - concept of subgame perfect equilibrium

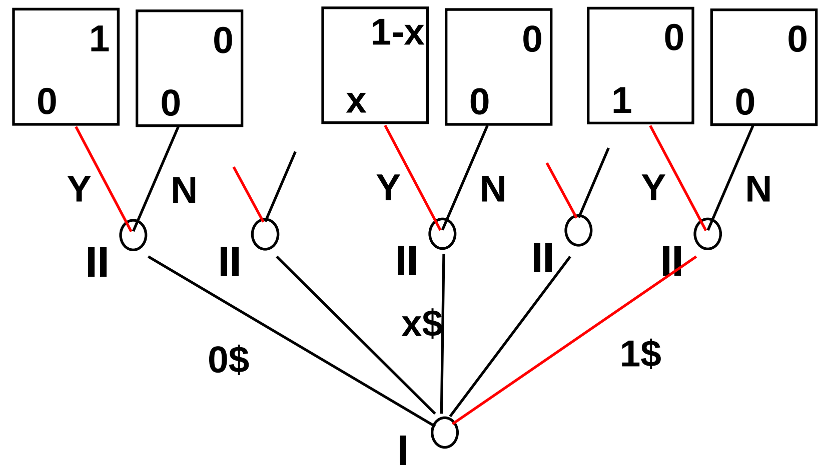
	Bach	Stravinsky
Bach	2,1	0,0
Stravinsky	0,0	1,2

	head	tail
head	1,-1	-1,1
tail	-1,1	1,-1

Subgame-perfect equilibrium

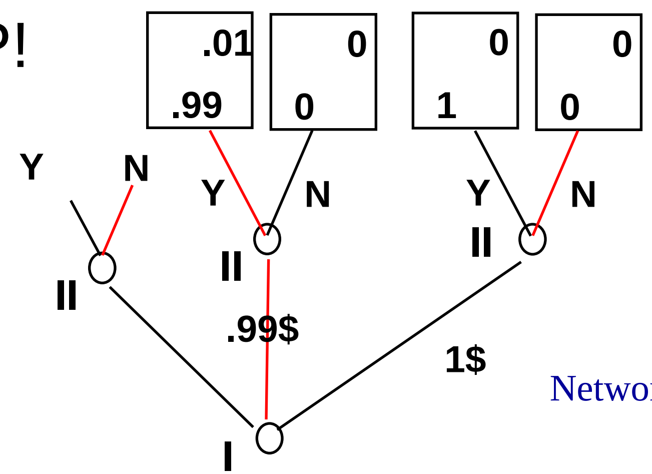
- The ultimatum game
- Some NEs are not rational in the actual game setup





NE1: player 2 gets all: not SGP!

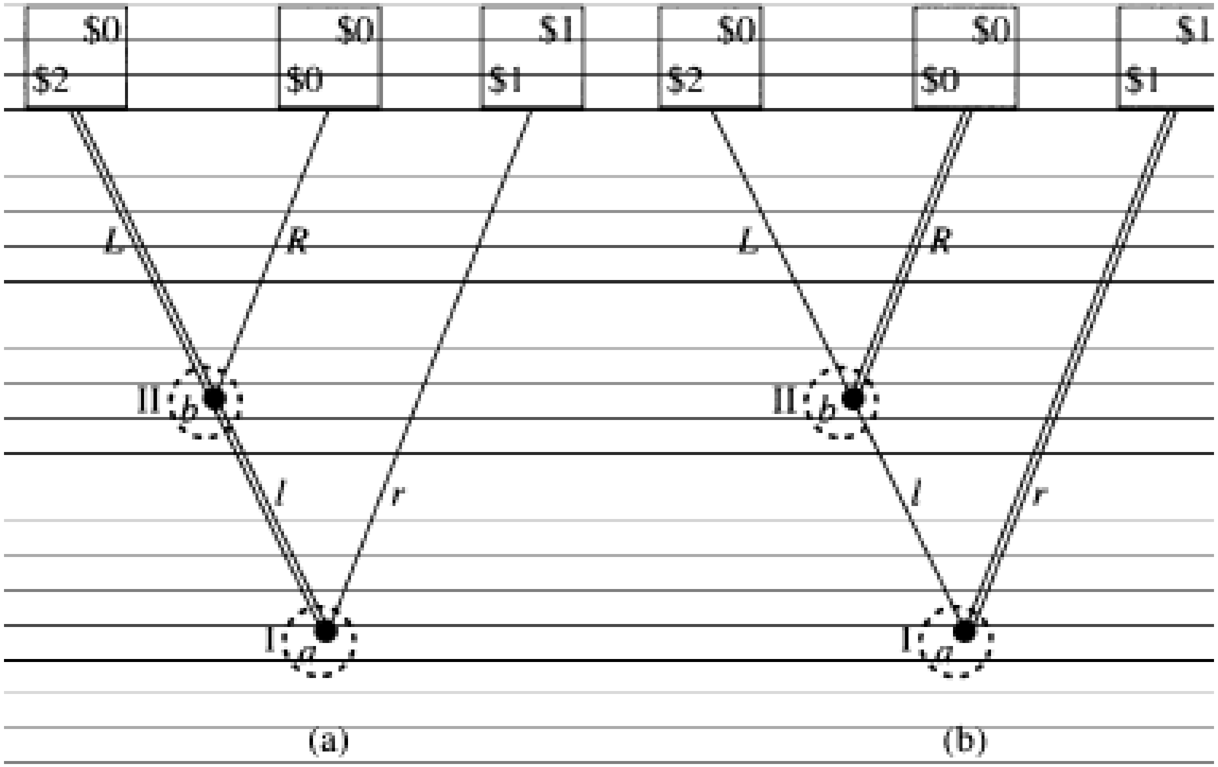
NE2: player 1 gets all



If problem is discrete, then
2 SGP NEs!

Multiple equilibria

- Which one to select?



Repeated games

- Larger strategy space: take account of history
- long-run interest different than short-run interest
- Can enforce cooperation by using punishment strategies
- Cartels

Player B

Player A	cooperate	defect
cooperate	3,3	0,4
defect	4,0	1,1

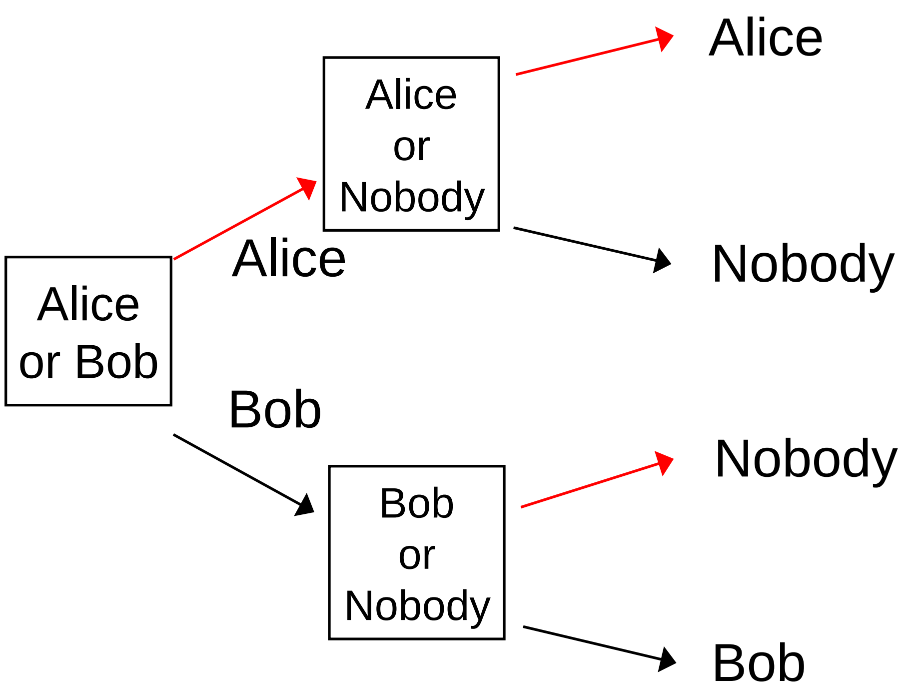
Strategy Grim: cooperate in the current move unless the other player defected in the previous move, in which case defect forever

Payoff with discount *r* = $u_0 + \frac{u_1}{(1+r)} + \frac{u_2}{(1+r)^2} + \cdots$

$$4 + 1/(1+r) + \cdots = 4 + 1/r \qquad 3 + 3/(1+r) + \cdots = 3 + 3/r$$

An example of strategic voting

Boris, Horace and Maurice: membership committee vote: a new member is considered for admission, Alice is in the agenda, but there also a new proposal for Bob to replace Alice



Preferences

Boris	Horace	Maurice	
Alice	Nobody	Bob	1
Nobody	Alice	Alice	2
Bob	Bob	Nobody	3

Strategic voting:
guess other's strategy

from: Ken Binmore, "Fun and Games"

An auction example

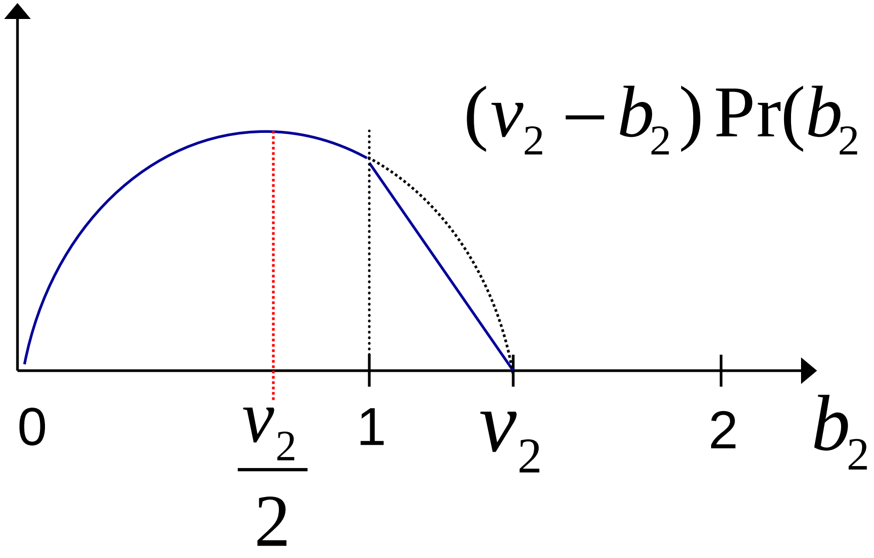
Bidder A: $u_A \in U[0,1]$
Bidder B: $u_B \in U[1,2]$

Auction 1: highest bid wins, pay your bid
Auction 2: highest bid wins, pay loosing bid

Auction2: strategy = tell the truth, always B gets the good
Auction1: strategy = shade bids, sometimes A gets the good!

$$\text{Assume A is TT. Bidder B}: \max_{b_2} (v_2 - b_2)\Pr(b_1 < b_2)$$

$$(v_2 - b_2)\Pr(b_2 < b_1) = \begin{cases} (v_2 - b_2) b_2 \text{ if } b_2 < 1 \\ (v_2 - b_2) \text{ if } b_2 \geq 1 \end{cases}$$



Auction 1 is not achieving max SW!

Public goods

- Non-excludable and non-rival goods
- Incentive problem in provisioning: the free-rider problem

Example: provision a common facility of size = 1,2

$$u_i(1) = 2, u_i(2) = 4, c_i(1) = 3$$

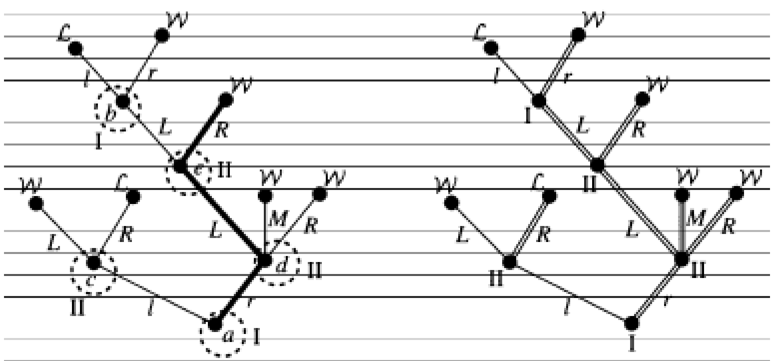
Player B

		provision 1	provision 0
Player A	provision 1	1,1	-1,2
	provision 0	2,-1	0,0

Free-riding: player i prefers the other player to contribute
Free-market fails to provision optimum amount of public goods

Strictly competitive games

- Strategic and extensive forms



	LLL	LLR	LML	LMR	LRL	LRR	RLL	RLR	RML	RMR	RRL	RRR
ll	W	W	W	W	W	W	L	L	L	L	L	L
lr	W	W	W	W	W	W	L	L	L	L	L	L
rl	L	W	W	W	W	W	L	W	W	W	W	W
rr	W	W	W	W	W	W	W	W	W	W	W	W

Zermelo’s algorithm: work backwards on subgames first

Theorem: Any finite strictly competitive game with perfect Information has a value

Player II can force an outcome in here.

u_1	u_2	...	$v = u_j$	u_{j+1}	...	u_k

Player I can force an outcome in here.

Offering different qualities

Different qualities

- Two virtual links with different quality, max loading bandwidth = C^{high}, C^{low}
- Maximization problem:

$$\max_{\{x_i^h, x_i^l\}} \sum_i u_i(x_i^h, x_i^l) \quad s.t. \quad \sum_i x_i^h \le C^h, \sum_i x_i^l \le C^l \qquad (1)$$

- Mathematical solution: we maximize the Lagrangian

$$\max_{\{x_i\}} \sum_i u_i(x_i^h, x_i^l) - \lambda^h (\sum_i x_i^h - C^h) - \lambda^l (\sum_i x_i^l - C^l)$$

The optimal point of (1) is characterized by

$$\sum_i x_i^h = C^h, \sum_i x_i^l = C^l, \ \frac{\partial u_i}{\partial x_i^h} = \lambda^h, \frac{\partial u_i}{\partial x_i^l} = \lambda^l$$

- Solution with market mechanism: use prices $p^h = \lambda^h, p^l = \lambda^l$
- The user solves: $\dfrac{\partial u_i}{\partial x_i^h} = p^h, \dfrac{\partial u_i}{\partial x_i^l} = p^l$
- Under general market conditions prices p^h, p^l converge to λ^h, λ^l
- How do we ensure that $p^h > p^l$?